

Experiment 3 - Transmission Lines, Part 1

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Introduction

0.1 Prelab Exercise

- 1. Define the terms: Relative permittivity, dielectric loss tangent ($\tan \delta$), skin depth and distributed elements.
- 2. Refer to Figure 1, find the frequency that will cause a 2π radians phase difference between the two RG-58 coaxial cables, assume $\mu_R = 1$, $\epsilon_r = 2.3$.
 - a. Find the relative amplitude of the two signals.
 - b. Verify your answers in the laboratory.

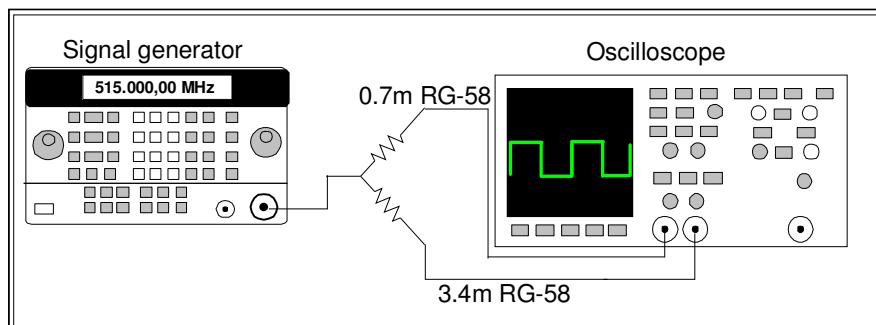


Figure 1 -Phase difference between two coaxial cables.

0.2 Background Theory

Transmission lines provide one media of transmitting electrical energy between the power source to the load. Figure 2 shows three different geometry types of lines used at microwave frequencies.

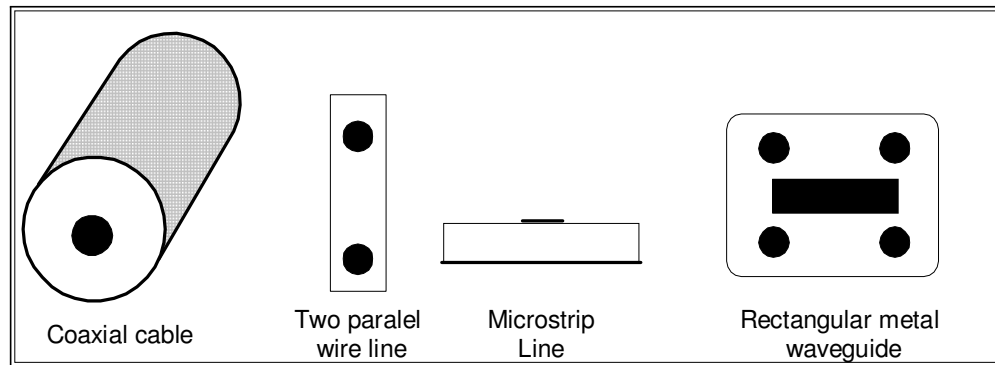


Figure 2 - Popular transmission lines.

The open two-wire line is the most popular at lower frequencies, especially for TV application. Modern RF and microwave devices practice involves considerable usage of coaxial cables at frequencies from about 10 MHz up to 30 GHz and hollow waveguides from 1 to 300 GHz.

A uniform transmission line can be defined as a line with distributed elements, as shown in Figure 3.

R' = Series resistance per unit length of line (Ω/m).

Resistance is related to the dimensions and conductivity of the metallic conductors, resistance is depended on frequency due to skin effect.

G' = Shunt conductance per unit length of line (\mathcal{U}/m).

G' is related to the loss tangent of the dielectric material between the two conductors. It is important to remember that G' is not a reciprocal of R' . They are independent quantities, R' being related to the various properties of the two conductors while G' is related to the properties of the insulating material between them.

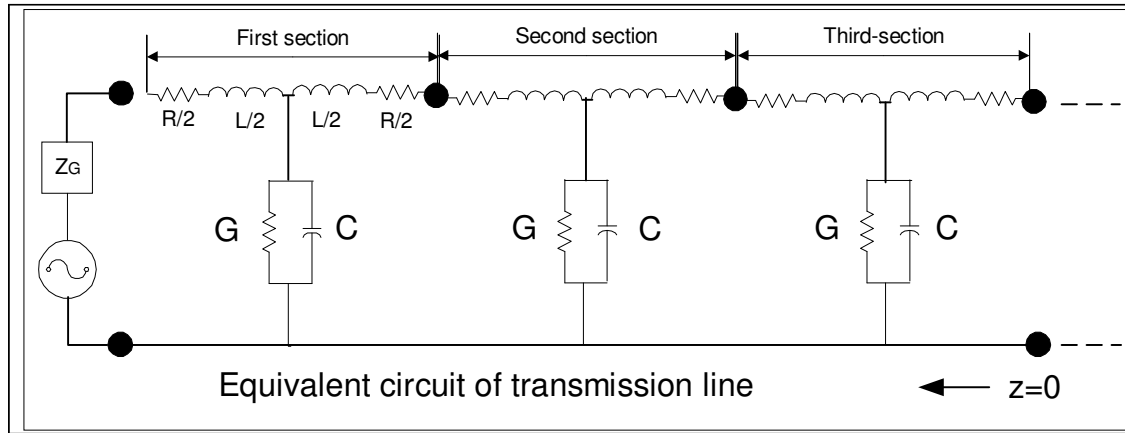


Figure 3 -Transmission lines model

L' = Series inductance per unit length of line (H/m).

L' - Is associated with the magnetic flux between the conductors.

C' = Shunt capacitance per unit length of line (F/m).

C' - Is associated with the charge on the conductors.

Naturally, a relatively long piece of line would contain identical sections as shown. Since these sections can always be chosen to be small as compared to the operating wavelength. Hence the idea is valid at all frequencies. The series impedance and the shunt admittance per unit length of the transmission line are given by:

$$Z = R' + J\omega L'$$

$$Y = G' + J\omega C'$$

The expressions for voltage and current per unit length are given respectively by equations (1) and (2):

$$\frac{dV(z)}{dz} = -I(z)(R' + J\omega L') \quad (1)$$

$$\frac{dI(z)}{dz} = -V(z)(G' + J\omega C') \quad (2)$$

Where the negative sign indicates on a decrease in voltage and current as z increases. The current and voltage are measured from the receiving end; at $z = 0$ and line extends in negative z -direction. The differentiating equations, (3) and (4), associate the voltage and current:

$$\begin{aligned}\frac{d^2V(z)}{dz} &= -(R' + J\omega L')\frac{dI(z)}{dz} = (G' + J\omega C')(R' + J\omega L')V(z) = \gamma^2V(z) \\ \frac{d^2I(z)}{dz} &= -(G' + J\omega C)\frac{dV(z)}{dz} = (G' + J\omega C')(R' + J\omega L')I(z) = \gamma^2I(z)\end{aligned}\quad (4)$$

Where

$$\gamma = \sqrt{ZY} = \sqrt{(G' + j\omega C')(R' + j\omega L')} \quad (5)$$

The above equations are known as wave equations for voltage and current, respectively, propagating on a line. The solutions of voltage and current waves are:

$$\begin{aligned}V(z) &= V_1e^{-\gamma z} + V_2e^{+\gamma z} \\ I(z) &= I_1e^{-\gamma z} + I_2e^{+\gamma z}\end{aligned}\quad (6)$$

These solutions are shown as the sum of two waves; the first term, V_1 , indicates the wave traveling in positive z-direction, and is called the incident wave, while the second term, V_2 , indicates the wave traveling in the negative z-direction, and is called the reflected wave. γ is a complex number that is called the propagation constant and can be defined as:

$$\gamma = \alpha + j\beta \quad (7)$$

α is called the attenuation constant of the propagating wave, α is the real part of Eq. (2.7) while β is the imaginary part and is called the phase constant. Thus, propagation constant γ is the phase shift and attenuation per unit length along the line. Separating equation (2.5) into real and imaginary parts, we can get:

$$\alpha = \left[\frac{\sqrt{(G'^2 + \omega^2 C'^2)(R'^2 + \omega^2 L'^2)} + (R'G' - \omega^2 L'C')}{2} \right]^{\frac{1}{2}} \quad (8)$$

$$\beta = \left[\frac{\sqrt{(G'^2 + \omega^2 C'^2)(R'^2 + \omega^2 L'^2)} - (R'G' - \omega^2 L'C')}{2} \right]^{\frac{1}{2}} \quad (9)$$

α is measured in nepers per unit length of the transmission line (1 neper = 8.686dB). β is measured in radians per unit length of the transmission

line. That means that β can be calculated as:

$$\beta = \frac{2\pi}{\lambda} \quad (10)$$

Where λ is the wavelength or distance along the line corresponding to a phase change of 2π radians. If the wavelength in free space is denoted by λ_0 , then:

$$\lambda_0 = \frac{c}{f_0} = \frac{v_p \sqrt{\epsilon_R \mu_R}}{f} = \lambda \sqrt{\epsilon_R \mu_R} \quad (11)$$

Where c is the velocity of light in free space and v_p is the velocity of electric wave in a dielectric material.

0.2.1 Characteristic Impedance

If RF voltage V is applied across the conductors of an infinite line, it causes a current I to flow. By this observation, the line is equivalent to an impedance, which is known as the characteristic impedance, Z_0 :

$$Z_0 = \frac{V(z)}{I(z)}$$

The expression for current I , using Eqs. (2.1) and (2.6), is given by:

$$I = -\frac{1}{(R' + J\omega L')} \frac{\partial V}{\partial z} = -\frac{1}{(R' + J\omega L')} (-\gamma)(V_1 e^{-\gamma z} + V_2 e^{+\gamma z})$$

$$I(z) = I_1 e^{-\gamma z} + I_2 e^{+\gamma z}$$

Where

$$I_1 = \frac{V_1}{(R' + J\omega L')} (\gamma) \quad \text{and} \quad I_2 = \frac{V_2}{(R' + J\omega L')} (\gamma)$$

Infinite line has no reflection, therefore $I_2 = V_2 = 0$ and:

$$\frac{V(z)}{I(z)} = \frac{V_1}{I_1} = \frac{(R' + J\omega L')}{\sqrt{(R' + J\omega L')(G' + J\omega C')}}}$$

Thus the characteristic impedance of infinite line can be calculated by (12):

$$Z_0 = \frac{V(z)}{I(z)} = \sqrt{\frac{(R' + J\omega L')}{(G' + J\omega C')}} \quad (12)$$

Line Impedance

We can rewrite the current and voltage along the line as:

$$\begin{aligned} V(z) &= V_1 e^{-\gamma z} + V_2 e^{+\gamma z} \\ I(z) &= \frac{V_1}{Z_0} e^{-\gamma z} + \frac{V_2}{Z_0} e^{+\gamma z} \end{aligned} \quad (13)$$

and the line impedance (the impedance at point z) as:

$$Z(z) = \frac{V(z)}{I(z)} = \frac{Z_0 (V_1 e^{-\gamma z} + V_2 e^{+\gamma z})}{V_1 e^{-\gamma z} - V_2 e^{+\gamma z}} \quad (14)$$

0.2.2 Terminated Lossless Line

Consider a lossless line, length l , terminated with a load Z_L , as shown in Figure 4.

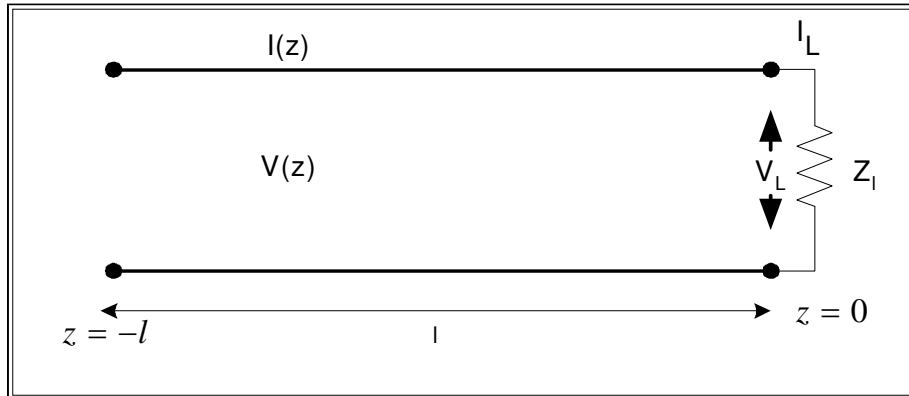


Figure 4 - A transmission line, terminated with a load.

Using equation (2.13) and recalling that $\alpha = 0$, one can define the current and voltage on the load terminal (at $z = 0$) as:

$$\frac{V_L}{I_L} = Z_L \quad (15)$$

$$\begin{aligned} V(z = 0) &= V_1 e^{-j\beta(0)} + V_2 e^{j\beta(0)} = V_1 + V_2 \\ I(z = 0) &= \frac{V_1}{Z_0} e^{-j\beta(0)} - \frac{V_2}{Z_0} e^{j\beta(0)} = \frac{V_1}{Z_0} - \frac{V_2}{Z_0} \end{aligned} \quad (16)$$

We can also add the boundary conditions:

$$\begin{aligned} V(z = 0) &= V_L \\ I(z = 0) &= I_L \end{aligned} \quad (17)$$

Using equation (2.16) we can rewrite (2.15) as equation (2.18):

$$Z_L = \frac{V_L}{I_L} = \frac{V(z = 0)}{I(z = 0)} = \frac{V_1 + V_2}{\frac{V_1}{Z_0} + \frac{V_2}{Z_0}} \quad (18)$$

Rearranging equation (2.18) one can rewrite:

$$\frac{V_2}{V_1} = \frac{Z_L - Z_0}{Z_L + Z_0} \triangleq \Gamma$$

Where Γ is the complex reflection coefficient. It relates to the magnitude and phase of the reflected wave (V_2) emerging from the load and to the magnitude and phase of the incident wave (V_1), or

$$V_2 = \Gamma V_1 \quad (19)$$

Equation (2.13) can be rewrite as:

$$\begin{aligned} V(z) &= V_1 e^{-j\beta z} + \Gamma V_1 e^{j\beta z} \\ I(z) &= \frac{V_1}{Z_0} e^{-j\beta z} - \Gamma \frac{V_1}{Z_0} e^{j\beta z} \end{aligned} \quad (20)$$

The reflection coefficient at an arbitrary point along the transmission line is:

$$\Gamma(z) = \frac{V_2 e^{-j\beta z}}{V_1 e^{-j\beta z}} = \Gamma(0) e^{-2j\beta z}$$

While the time average power flow along the line at point z is:

$$\begin{aligned} P_{av} &= \frac{1}{2} \operatorname{Re}[V(z)I(z)^*] \\ &= \frac{1}{2} \frac{V_1^2}{Z_0} \operatorname{Re}(1 - \Gamma^* e^{-j2\beta z} + \Gamma e^{j2\beta z} - |\Gamma|^2) \end{aligned} \quad (21)$$

When recalling that $Z - Z^* = 2j \operatorname{Im} Z$, equation (2.21) can be simplified to:

$$P_{av} = \frac{1}{2} \frac{V_1^2}{Z_0} (1 - |\Gamma|^2) \quad (22)$$

Which shows that the average power flow is constant at any point on the lossless transmission line. If $\Gamma = 0$ (perfect match), the maximum power is delivered to the load, while all the power is reflected for $\Gamma = 1$.

0.2.3 The Impedance Transformation

According to the transmission line theory, in a short circuit line, the impedance become infinite at a distance of one-quarter wavelength from the short. The ability to change the impedance by adding a length of transmission line is a very important attribute to every RF or microwave designer. If we look at the transmission line (lossless line), as illustrated in Figure 5, and use equation (2.20), the line impedance at $z = -l$ (input impedance) is:

$$Z_{in} = \frac{V(z = -l)}{I(z = -l)} = Z_0 \left(\frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} \right) \quad (23)$$

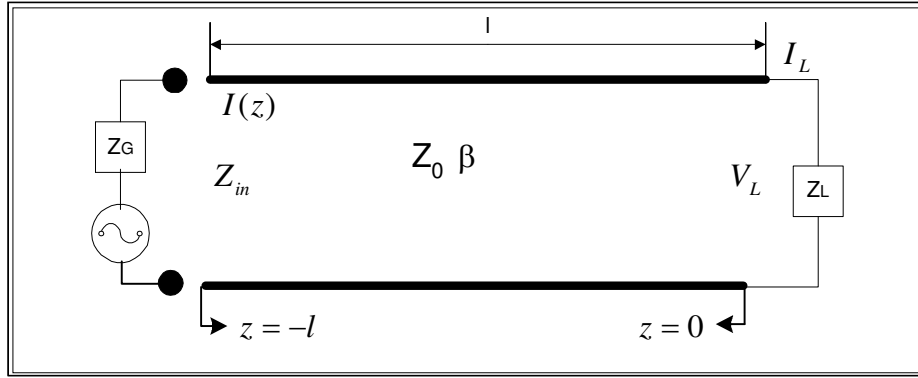


Figure 5 -Transmission line terminated in arbitrary impedance Z_L .

If we use the relationship $\Gamma = (Z_L - Z_0) / (Z_L + Z_0)$ with equation (2.23) we get:

$$Z_{in} = Z_0 \left(\frac{Z_L (e^{j\beta l} + e^{-j\beta l}) + Z_0 (e^{j\beta l} - e^{-j\beta l})}{Z_L (e^{j\beta l} + \Gamma e^{-j\beta l}) - Z_0 (e^{j\beta l} - e^{-j\beta l})} \right) \quad (24)$$

By recalling Euler's equations:

$$\begin{aligned} e^{j\beta l} &= \cos \beta l + j \sin \beta l \\ e^{-j\beta l} &= \cos \beta l - j \sin \beta l \end{aligned}$$

We can get:

$$Z_{in} = Z_0 \left(\frac{Z_L \cos \beta l + j Z_0 \sin \beta l}{Z_0 \cos \beta l + j Z_L \sin \beta l} \right)$$

Or:

$$Z_{in} = Z_0 \left(\frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right) \quad (25)$$

Lets examine special cases:

0.3 Shorted Transmission Line

The voltage and current along a transmission line as a function of time is known as:

$$V(z, t) = A \cos \left[\omega \left(t - \frac{z}{v_p} \right) + \theta \right] + B \cos \left[\omega \left(t + \frac{z}{v_p} \right) + \phi \right] \quad (26)$$

$$I(z, t) = \frac{1}{Z_0} \left\{ A \cos \left[\omega \left(t - \frac{z}{v_p} \right) + \theta \right] - B \cos \left[\omega \left(t + \frac{z}{v_p} \right) + \phi \right] \right\}$$

Where $z = 0$, at the generator side, and $d = 0$ at the load side, as shown in Figure 6.

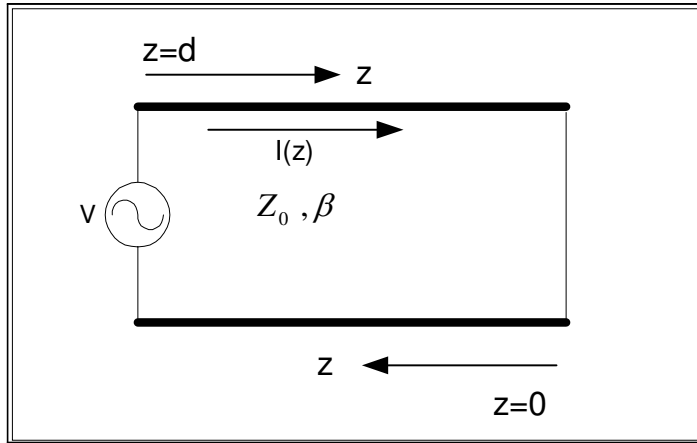


Figure 6 -Shorted transmission line.

This expression can be rewrite in phasor form as:

$$V(z) = V_1 e^{-j\beta z} + V_2 e^{j\beta z} \quad (27)$$

$$I(z) = \frac{1}{Z_0} (V_1 e^{-j\beta z} - V_2 e^{j\beta z})$$

Where $V_1 = A e^{j\theta}$ is the forward wave, $V_2 = B e^{j\phi}$ is the reflected wave and $\beta = \omega/v_p$. The boundary condition for the short circuit transmission line at $z = 0$ is that the voltage across the short circuit is zero:

$$V(0) = 0 \quad (28)$$

Using equations (27) and (28), we obtain:

$$\begin{aligned} V(0) &= V_1 e^{j\beta 0} + V_2 e^{-j\beta 0} = 0 \\ V_1 &= -V_2 \end{aligned} \quad (29)$$

Inserting Eq (29) into Eq (28), we get:

$$\begin{aligned} V(z) &= V_1 e^{j\beta z} - V_1 e^{-j\beta z} = 2jV_1 \sin \beta z \\ I(z) &= \frac{1}{Z_0} (V_1 e^{j\beta z} + V_1 e^{-j\beta z}) = 2\frac{V_1}{Z_0} \cos \beta z \end{aligned} \quad (30)$$

Which shows that the $V = 0$ at the load end, while the current is a maximum there. The ratio between $V(z)$ and $I(z)$ is the input impedance and is equal to:

$$Z_{in} = jZ_0 \tan \beta z \quad (31)$$

Which is purely imaginary for any length of z . The value of $Z_{in(sc)}$ vary between $+j\infty$ to $-j\infty$, every $\pi/2$ ($\lambda/4$), by changing z , the length of the line, or by changing the frequency. The voltage and current as a function of time and distance are:

$$\begin{aligned} V(z, t) &= \text{Re} [V(z)e^{j\omega t}] \\ &= \text{Re} (2e^{j\pi/2} |V_1| e^{j\theta} e^{j\omega t} \sin \beta z) \\ &= -2 |V_1| \sin \beta z \sin (\omega t + \theta) \end{aligned} \quad (32)$$

$$\begin{aligned} I(z, t) &= \text{Re} [I(z)e^{j\omega t}] \\ &= \text{Re} \left(2\frac{V_1}{Z_0} \cos \beta z e^{j\omega t} \right) \\ &= |V_1| \frac{2}{Z_0} \cos \beta z \cos (\theta + \omega t) \end{aligned} \quad (33)$$

Where $V_1 = |V_1| e^{j\theta}$ and $j = e^{j\pi/2}$.

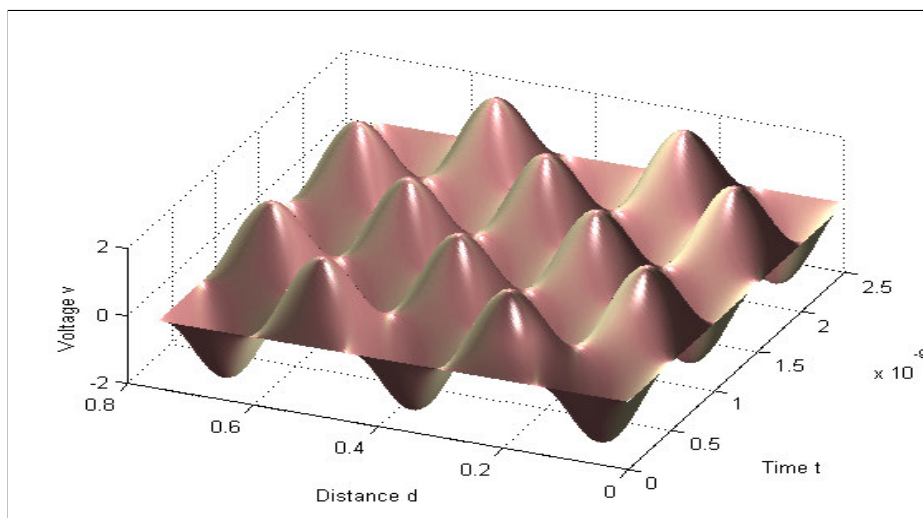


Figure 7 -Voltage along a shorted transmission line, as a function of time and distance from the load. Frequency 1GHz.

The result of the voltage on the shorted transmission line is shown in figure 7, assume that $V_1 = \sin 2\pi 10^9 t$ ($\lambda = 30cm$), which shows that :

- The line voltage is zero for $\beta z = 0 + n\pi$ $n = 0, 1, 2, \dots$ for all value of time.
- The voltage at every point of z is sinusoidal as a function of time. The maximum absolute value of the voltage is known as a standing wave pattern.

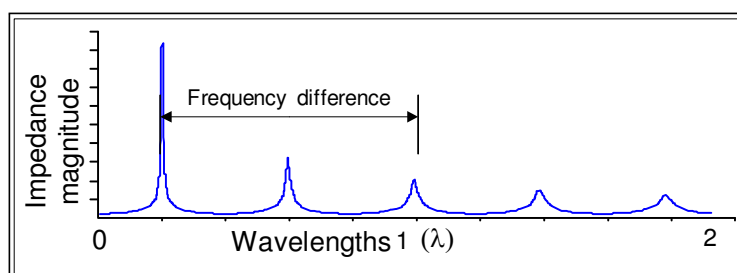


Figure 8 - Input impedance of shorted low loss coaxial cable.

0.3.1 Open Circuit Line-

For lossless line $\alpha \neq 0$ and $Y_L = 0$, equation (2.25) is reduce to:

$$Y_{in} = Y_0 \tan \beta z$$

or in impedance form:

$$Z_{in(os)} = \frac{Z_0}{\tan \beta z} \quad (34)$$

Using Eqs. (2.24), (2.34) and (2.35), Z_0 can be computed from the short and open circuit, by the relation:

$$Z_0 = \sqrt{Z_{in(sc)} Z_{in(os)}}$$

0.3.2 Low Loss Line

In many cases, the loss of microwave transmission lines is small. In these cases, some approximation can be made that simplify the expression of propagation constant γ and characteristic impedance Z_0 . Equation (5) can be rearranged as:

$$\begin{aligned} \gamma &= \sqrt{(j\omega C' j\omega L') \left(1 + \frac{R'}{j\omega L'}\right) \left(1 + \frac{G'}{j\omega C'}\right)} \\ &= j\omega \sqrt{L'C'} \sqrt{1 - j \left(\frac{R'}{\omega L'} + \frac{G'}{\omega C'}\right) - \frac{R'G'}{\omega^2 L'C'}} \end{aligned} \quad (35)$$

If we assume that $R' \ll \omega L'$ and $G' \ll \omega C'$ for low loss line, than $R'G' \ll \omega^2 L'C'$, therefore Eq. (36) is reduced to:

$$\gamma = j\omega \sqrt{L'C'} \sqrt{1 - j \left(\frac{R'}{\omega L'} + \frac{G'}{\omega C'}\right)} \quad (36)$$

By using Taylor approximation $\sqrt{1+x} \approx 1 + \frac{x}{2}$, the propagation constant can be approximated by:

$$\gamma \approx j\omega \sqrt{L'C'} \left[1 - \frac{j}{2} \left(\frac{R'}{\omega L'} + \frac{G'}{\omega C'}\right)\right]$$

0.4 CALCULATING TRANSMISSION LINE PARAMETERS BY MEASURING $Z_0, \alpha,$

so $\text{Re } \gamma = \alpha$ is the attenuation constant and $\text{Im } \gamma = \beta$ is the phase constant, therefore:

$$\begin{aligned}\alpha &\approx \frac{1}{2} \left(R' \sqrt{\frac{C'}{L'}} + G' \sqrt{\frac{L'}{C'}} \right) \\ &= \frac{1}{2} \left(\frac{R'}{Z_0} + G' Z_0 \right) = \alpha_c + \alpha_d\end{aligned}\quad (37)$$

Where α_c is the conductor loss and α_d is the dielectric loss. The phase constant is equal to:

$$\beta \approx \omega \sqrt{L' C'} \quad (38)$$

Where $Z_0 = \sqrt{\frac{L'}{C'}}$.

Note that the characteristic impedance of a low loss transmission line can be approximated by:

$$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}} \approx \sqrt{\frac{L'}{C'}}$$

0.4 Calculating Transmission Line parameters by Measuring $Z_0, \alpha,$ and β

By knowing the primary parameters $Z_0, \alpha,$ and β , the equivalent parameters R, L, C and G can be extracted by multiplying the general expressions of Z_0 and γ :

$$\begin{aligned}R' + j\omega L' &= Z_0 \gamma \\ &= (R_0 + jx_0)(\alpha + j\beta) \\ &= (R_0\alpha - x_0\beta) + j(R_0\beta + x_0\alpha)\end{aligned}\quad (39)$$

Where R_0 is the real part of Z_0 and x_0 is its imaginary part. By Equating the real and imaginary part we get:

$$R' = R_0\alpha - x_0\beta \frac{\Omega}{m} \quad (40)$$

and:

$$L' = \frac{R_0\beta + x_0\alpha}{\omega} \frac{H}{m} \quad (41)$$

By dividing the general expressions of Z_0 and γ , we get

$$\begin{aligned} & \frac{G' + J\omega C'}{\gamma} \\ &= \frac{\gamma}{Z_0} \end{aligned} \quad (42)$$

$$\begin{aligned} &= \frac{\alpha + j\beta}{R_0 + jx_0} * \frac{R_0 - jx_0}{R_0 - jx_0} \\ &= \frac{R_0\alpha + x_0\beta}{R_0^2 + x_0^2} + j \frac{R_0\beta - x_0\alpha}{R_0^2 + x_0^2} \end{aligned} \quad (43)$$

By equating real and imaginary parts, we get:

$$G' = \frac{R_0\alpha + x_0\beta}{R_0^2 + x_0^2} \quad (44)$$

and:

$$C' = \frac{R_0\beta - x_0\alpha}{(R_0^2 + x_0^2)\omega} \quad (45)$$

0.4.1 Calculating Coaxial Line Parameters by Measuring Physical Dimensions

Referring to Figure 9, if the center conductor is charged to $+q$ and the outer conductor (shielding) is charged to $-q$, then the electric field lines will direct radially outward, while the magnetic lines will surround the inner conductor.

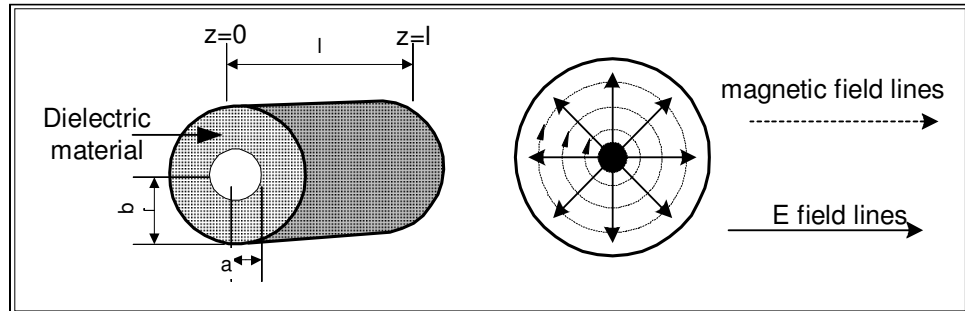


Figure 9 - Geometry and electromagnetic field lines of coaxial cable.

By applying Gauss law to a cylindrical structure like a coaxial cable of length l and radius r , where $a < r < b$:

$$\int_0^l \int_0^{2\pi} Drd\phi dz = q$$

0.4 CALCULATING TRANSMISSION LINE PARAMETERS BY MEASURING $Z_0, \alpha,$

D is independent of z and ϕ therefore:

$$D = \frac{q}{2\pi r l} \text{ and } E = \frac{q}{2\pi\epsilon_0\epsilon_r r l}$$

The voltage difference between the conductors:

$$V = - \int_a^b E \cdot dr = \frac{q}{2\pi\epsilon_0\epsilon_r l} \ln \frac{b}{a}$$

Hence:

$$C = \frac{q}{V} = \frac{2\pi\epsilon_0\epsilon_r l}{\ln \frac{b}{a}} \quad (46)$$

All dielectric materials have lossy at microwave frequencies. Hence one can look at a coaxial cable as a capacitor with parallel conductance or

$$Y = G + j\omega C = j\omega C \left(1 - j \frac{G}{\omega C}\right)$$

The quantity $j \frac{G}{\omega C}$ called material loss tangent and assign as $\tan \delta$. This quantity indicates the relative magnitude of the loss component. It is often used to specify the loss properties of dielectrics:

$$G = j\omega C \tan \delta = \frac{2\pi\epsilon_0\epsilon_r l}{\ln \frac{b}{a}} \omega \tan \delta \quad (47)$$

Inductance of Transmission line

The inductance of solenoid known as

$$L = \frac{N\Phi}{I} \quad (48)$$

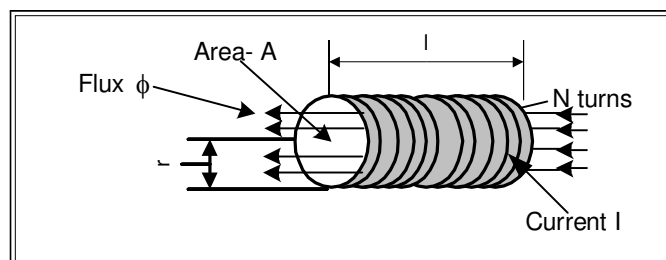


Figure 10 -Inductance of solenoid.

where N is the number of turns, l is the length and I is the current in the solenoid. Applying ampere law to a circle radius r ($b < r > a$) around any turn of the solenoid will result in:

$$H = \frac{I}{2\pi r}$$

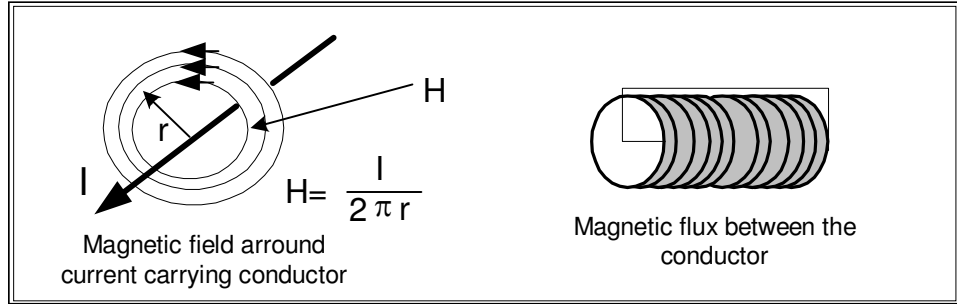


Figure 11 -Magnetic field around conductor and solenoid.

If one wants to measure the total flux between the conductors, it's the flux enter the turn shown in Figure10 and Figure11. Therefore

$$\Phi = \int_0^l \int_a^b B dr dz = \mu_0 \mu_r l \int_a^b \frac{I}{2\pi r} dr = \frac{\mu_0 \mu_R l I}{2\pi} \ln \frac{b}{a}$$

Coaxial cable could be considered as a solenoid with one turn. By using equation 1.50, the expression for Φ and $N=1$ becomes:

$$L = \frac{\mu_0 \mu_R l}{2\pi} \ln \frac{b}{a} \quad (49)$$

Resistance of Transmission Line

The high frequency resistance of a coaxial cable is equal to the D.C. resistance of a equivalent coaxial cable composed of two hollow conductors with the radii, a and b , respectively, and with thickness equal to the skin depth penetration δ . Skin depth penetration is defined as

$$\delta_{skin} = \frac{1}{\sqrt{\pi f \mu_0 \mu_R \sigma}} \text{ meters}$$

Where σ is the conductivity of the conductor. Therefore the resistance per unit length is:

$$R' = \frac{1}{2\pi a \delta_{skin} \sigma} + \frac{1}{2\pi b \delta_{skin} \sigma} = \frac{b+a}{2\pi a \delta_{skin} \sigma b} \quad (50)$$

Experiment Procedure

0.5 Required Equipment

1. Network Analyzer *HP – 8714B*.
2. Arbitrary Waveform Generators (*AWG*) *HP – 33120A*.
3. Keysight ADS software.
4. Termination- 50Ω .
5. Standard 50Ω coaxial cable .
6. Open Short termination.
7. SMA short termination.
8. 50 cm 50Ω FR4 microstrip line

0.6 Phase Difference of Coaxial Cables, Simulation and Measurement

In this part of the experiment you will verify your answers to the prelab exercise.

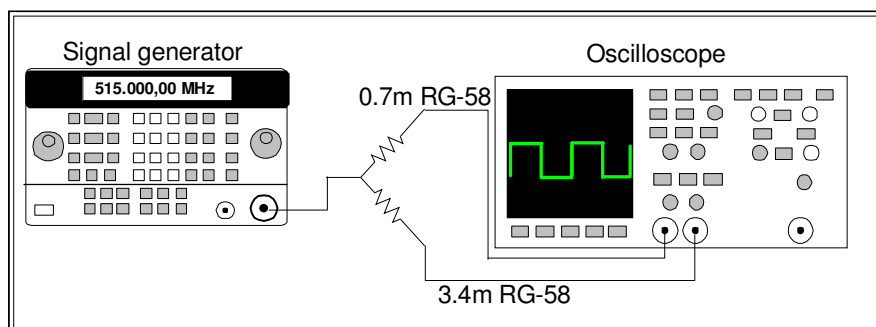


Figure 1 - Phase difference between two coaxial cables

1. Simulate the system as indicated in Figure 2 (see also Figure 1).

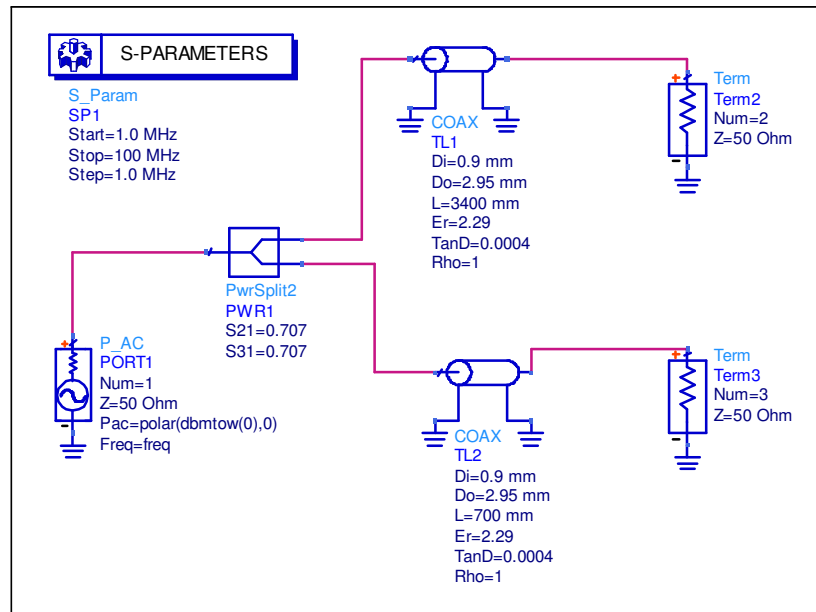


Figure 2 - Phase Difference Simulation.

2. Plot S_{21} (phase) and S_{31} (phase) on the same graph, find the frequency where the phases of the two cables are equal. **Save the data.**

Compare this frequency to the frequency you calculated in the prelab exercise.

3. Connect coaxial cables to an oscilloscope using a power splitter, as indicated in Figure 1.

4. Set the signal generator to the frequency according to your simulation, and verify that the phase difference is near 0 degree. **Save the data on magnetic media.**

0.7 Wavelength of Electromagnetic Wave in Dielectric Material

1. Select a 50 cm length FR4 microstrip line, $Width = 3mm$, $Height = 1.6mm$, $\epsilon_r = 4.6$, $Z_0 = 50\Omega$, $v_p = 0.539c$, $\epsilon_{eff} = 3.446$.

0.7 WAVELENGTH OF ELECTROMAGNETIC WAVE IN DIELECTRIC MATERIAL

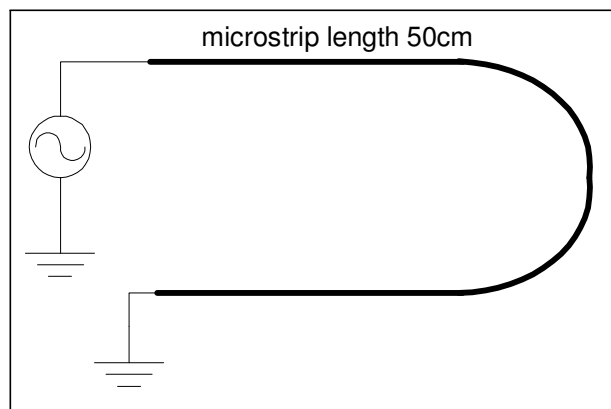


Figure 3 - 50 cm short circuit microstrip line.

- Calculate the frequency that half the wavelength in the dielectric material is equal to 50 cm ($\lambda/2 = 50\text{cm}$). Record this frequency. Verify your answer using ADS: Menu -> Tools -> LineCalc.

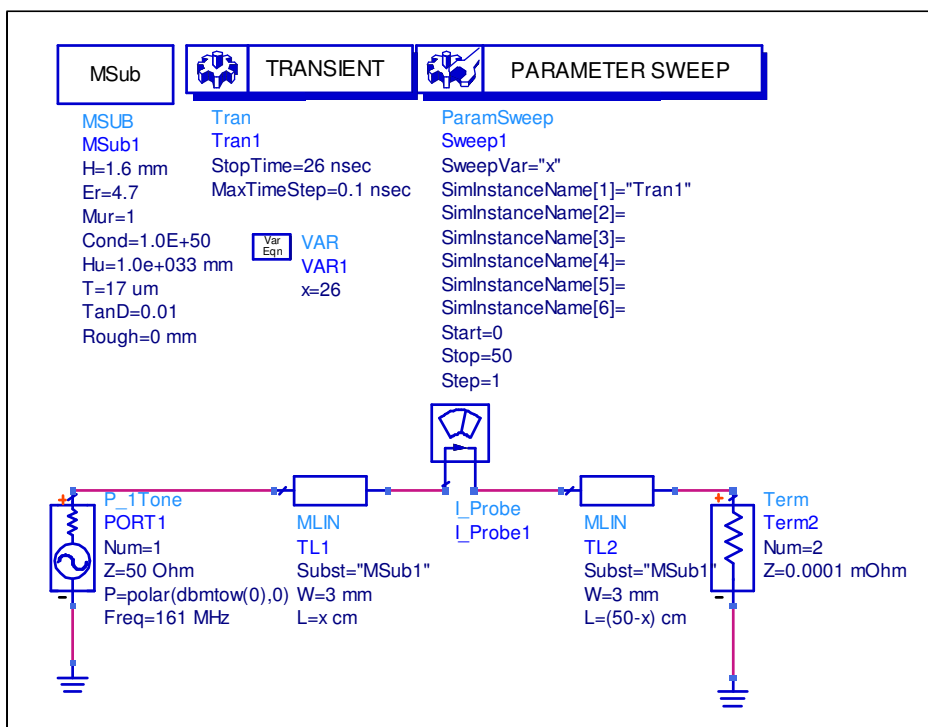


Figure 4 - Simulation of short circuit microstrip transmission line.

3. Simulate a short circuit $\lambda/2$ microstrip line according to Figure 4.
Draw a graph of the current as a function of time.
Explain the graph.
Find the distance where the current amplitude is minimum all the time.
Find the time where the current amplitude is minimum at every point on the microstrip line. **Save the data.**
4. Find the position where the current amplitude is maximum and change the simulation accordingly.
Draw a graph of the current as a function of time to prove your answer. **Save the data.**
5. Find the position where the current amplitude is minimum. Draw a graph of the current as a function of time to prove your answer. **Save the data.**
6. Connect the short circuit microstrip line to a signal generator, adjust the frequency to the calculated frequency and the amplitude to 0 dBm.
7. Measure the amplitude of the signal, using wideband oscilloscope at load end and signal generator end, why the amplitude of the signal at both ends is close to 0 volt.
8. Measure the voltage along the line using the probe of the oscilloscope, verify that the voltage is maximum at the middle of the line.
9. Calculate the frequency for $\lambda = 50$ cm in the dielectric material. Measure the voltage along the line, how many points of maximum voltage exist along the line?

0.8 Input Impedance of Short - Circuit Transmission Line

1. Simulate a transmission line terminated by a short circuit, as shown in Figure 5.

0.8 INPUT IMPEDANCE OF SHORT - CIRCUIT TRANSMISSION LINE²¹

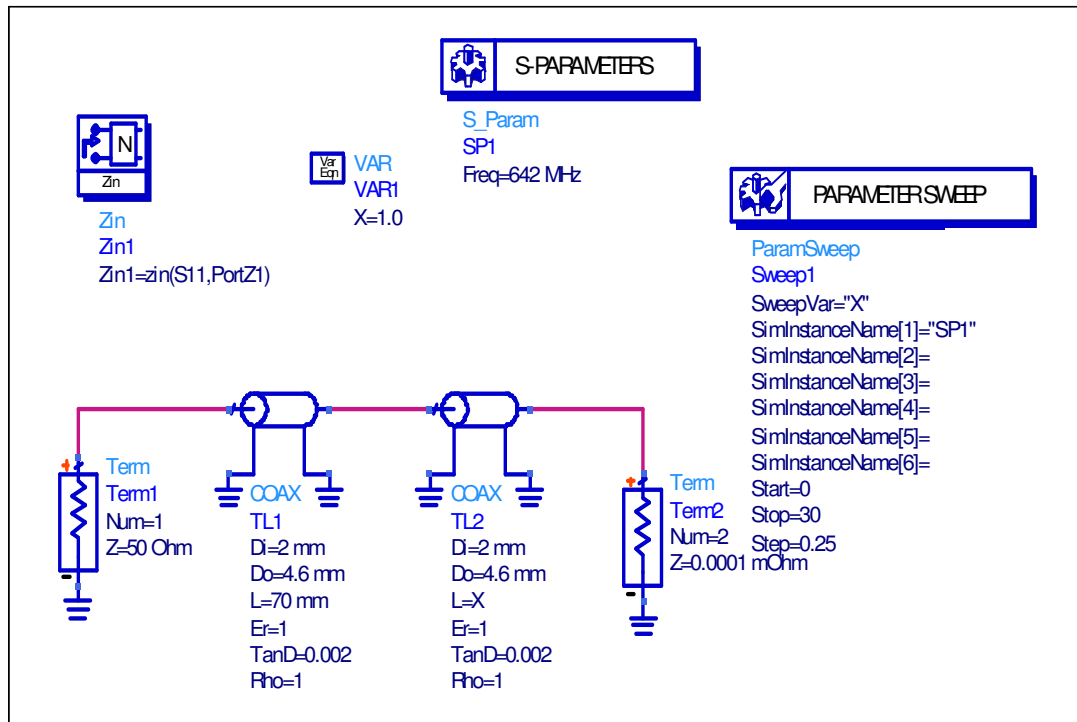


Figure 5 - Input impedance of a short circuit coaxial transmission line.

2. Draw a graph of Z_{in} as a function of length x , **save the data**.

3. If you use fix length of transmission line, which parameter of the simulation you have to change in order to get the same variation of Z_{in} , prove your answer by simulation.

4. Connect line stretcher terminated in a short circuit to a network analyzer (see Figure 6).

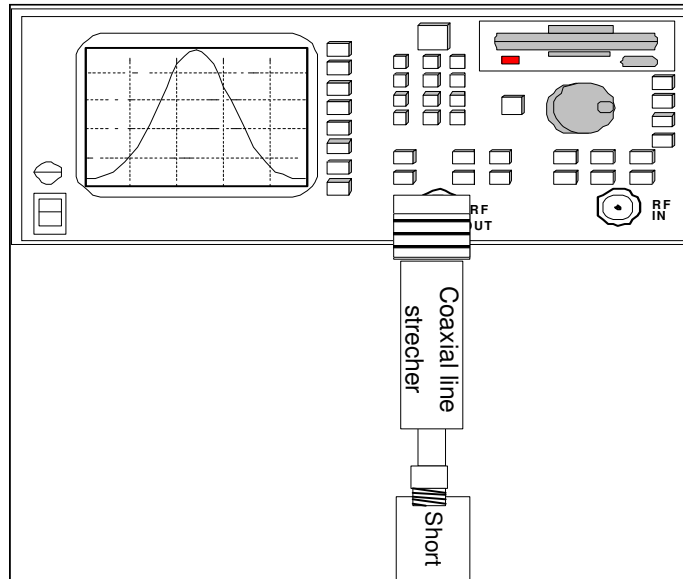


Figure 6 - Input impedance measurement of short circuit coaxial transmission line.

5. Calculate the frequency for the wavelength $1.5\lambda = 70\text{cm}$ (line stretcher at minimum overall length).
6. Set the network analyzer to smith chart format and find the exact frequency (around the calculated frequency) for the input impedance $Z_{in} = 0$. (line stretcher at minimum overall length), set the network analyzer to CW frequency equal to this measured frequency.
7. Stretch the line and analyze the impedance in a smith chart.

0.9 Impedance Along a Short - Circuit Microstrip Transmission Line

1. Simulate a short circuit transmission line (see Figure 7) with length 50cm, width 3mm, height 1.6 mm and dielectric material FR4 $\epsilon_{eff} = 3.446$.

0.9 IMPEDANCE ALONG A SHORT - CIRCUIT MICROSTRIP TRANSMISSION LI

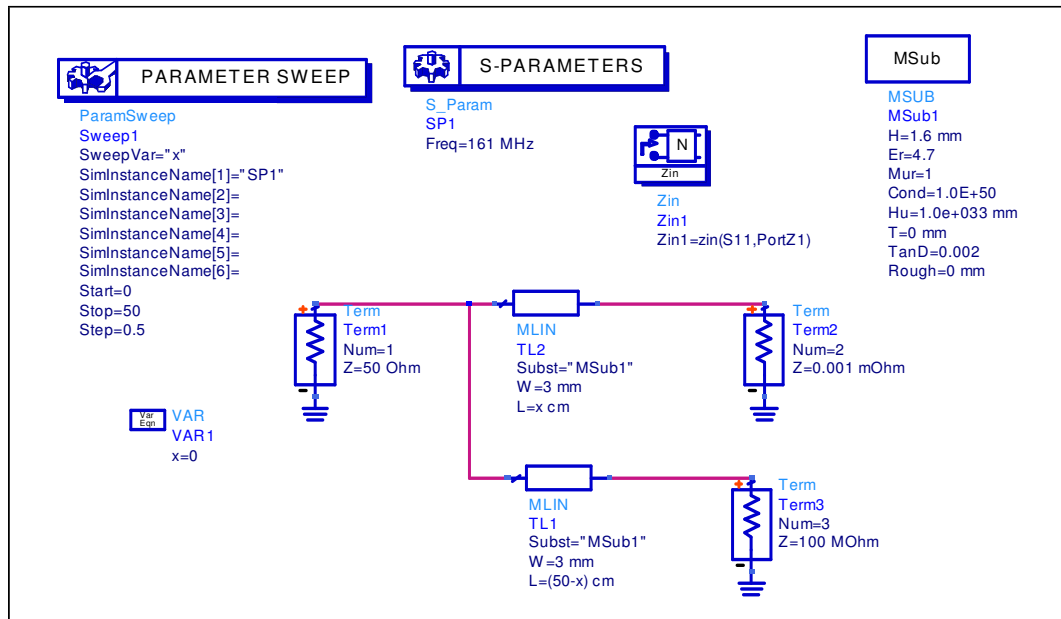


Figure 7 - Simulation of a short and open circuit microstrip transmission lines.

Pay attention that the simulation is parametric (x is the distance parameter).

Verify that for $x \neq \lambda/2$ the input impedance is a superposition of a short and an open circuit transmission line connected in parallel and explain why is this so.

2. Draw a graph of the input impedance as a function of the distance x .
Save the data.

3. Connect the microstrip to the network analyzer (as shown in Figure 8), set the network analyzer to impedance magnitude measurement.

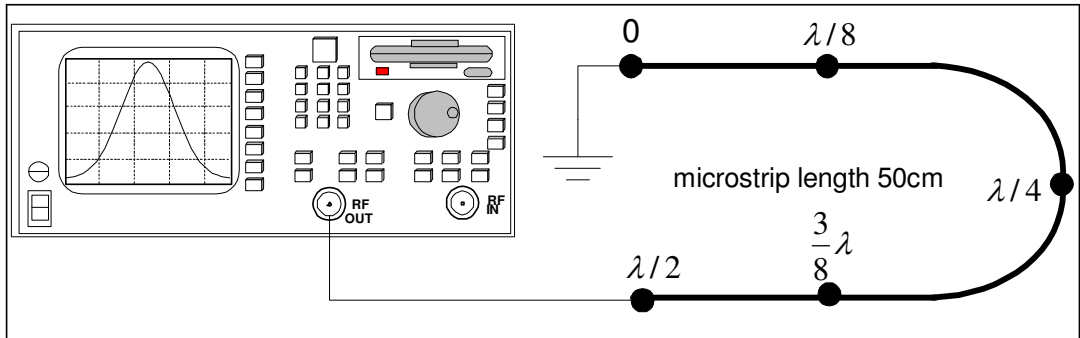


Figure 8 - Measurement of input impedance of short circuit microstrip transmission line

4. Set the network analyzer to Continuous Wave (CW), at the calculated frequency for $\lambda/2 = 50\text{cm}$. Calibrate the network analyzer to 1 port (reflection) at the end of the cable.

4. What is the expected value of the impedance at each point?

Measure the impedance magnitude at $\lambda/2, 3\lambda/8, \lambda/4, \lambda/8, 0$. **Save the data on magnetic media.**

0.10 Final Report

1. Attach and explain all the simulation graphs.

2. Using MATLAB, draw a 3D graph of the current as a function of time and distance of a short circuit microstrip line for a frequency of 1GHz (similar to Figure 7 from chapter 1). Choose the ranges of time and distance as you wish and attach the matlab code.

3. Using MATLAB, draw a graph of the input impedance, Z_0 , as a function of the length, x , for a lossless short circuit coaxial line for a frequency of 161 MHz. Compare this graph to your measurement and ADS simulation. Attach the matlab code.

4. Using the physical dimension of a coaxial cable RG-58 that you measured, find the following parameters versus frequency (if apply) and draw a graph of:

- Shunt capacitance C per meter.
- Series inductance L per meter.
- Series resistance R per meter.
- Shunt conductance G per meter.

0.11 APPENDIX-1 - ENGINEERING INFORMATION FOR RF COAXIAL CABLE R

0.11 Appendix-1 - Engineering Information for RF Coaxial Cable RG-58

Component	Construction details
Inner conductor	Nineteen strands of tinned copper wire. each strand 0.18 mm diameter Overall diameter: 0.9 mm.
Dielectric core	Type A-1: Solid polyethylene. Diameter: 2.95 mm
Outer conductor	Single screened of tinned cooper wire. Diameter: 3.6 mm .
Jacket	Type Ila: PVC. Diameter: 4.95mm.

Engineering Information:

Impedance: 50Ω

Continuous working voltage; 1,400 V RMS, maximum.

Operating frequency: 1 GHz, maximum.

Velocity of propagation: 66% of speed light.

Dielectric constant of polyethylene: 2.29

Dielectric loss tangents of polyethylene: $\tan \delta = 0.0004$

Operating temperature range: -40 C to +85 C.

Capacitance: $101 \frac{pF}{m}$.

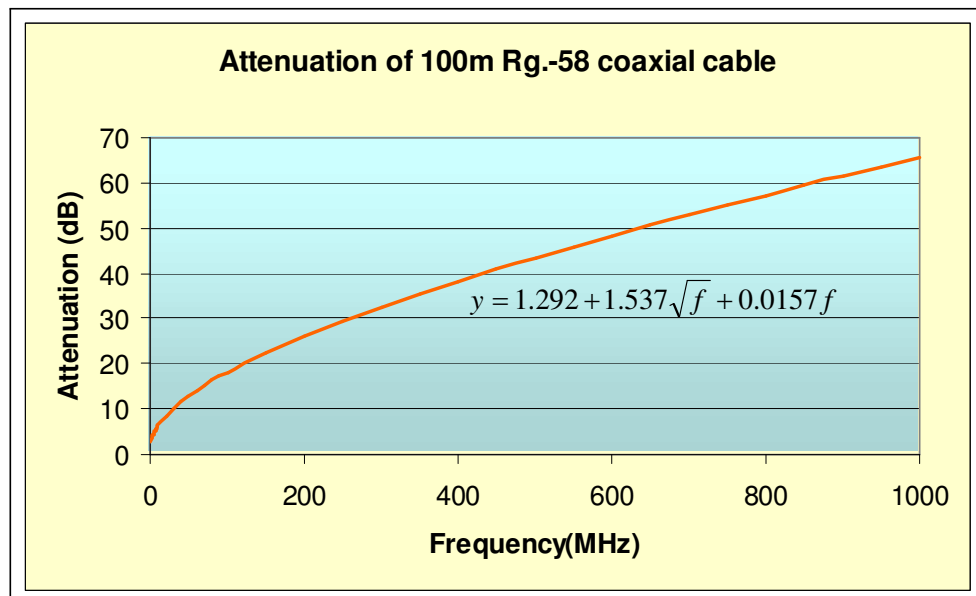


Figure 9 - Attenuation versus frequency of a coaxial cable RG-58.