Nonmonotonic plasma density profile due to neutral-gas depletion

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The dependencies in a gas discharge of the plasma density and of the neutral-gas depletion on the magnetic-field intensity and on the plasma particle flux are studied. It is shown that if plasma particle flux density outward of the discharge is fixed, varying the magnetic field intensity does not affect neutral-gas depletion. When there are plasma end losses along magnetic-field lines while neutral-gas is depleted, an increase in the magnetic field intensity results in a nonmonotonic plasma density profile across field lines. The plasma density then has a local minimum at the center of the discharge. © *2010 American Institute of Physics*. [doi:10.1063/1.3313352]

I. INTRODUCTION

Space and laboratory plasmas can be significantly affected when ionization is so intense that the neutral-gas density is modified. The effect of neutral-gas density modification in gas discharges, usually neutral-gas depletion, has already been addressed in early studies.[1–4] However, only in recent years, with the growing use of lower pressure and higher power radio-frequency discharges, is the importance of neutral-gas density modification becoming fully recognized.[5–34]

The effect of neutral-gas depletion was first studied theoretically in unmagnetized plasmas.[23–31] Recently, we extended our studies of neutral-gas depletion to magnetized plasmas.[32] Plasma diffusion across a magnetic field has been studied extensively for decades,[35–47] but the coupled dynamics of plasma and neutral-gas during such a diffusion has been less investigated. We examined in Ref. 32 how, in magnetized plasmas, the linear dependence of the plasma density on the cross-field radial plasma particle flux is modified due to strong ionization. We showed that the nonlinear dependence can result in a larger-than-linear increase in the plasma density with an increase in the radial plasma flux density, or in a smaller-than-linear increase, depending on the mechanism of the cross-field transport. For example, if electron-neutral collisions induce the cross field transport, neutral-gas depletion results in an improved plasma confinement, and, consequently, in a larger-than-linear such an increase in the plasma density with an increase in the radial plasma particle flux. If, however, these are electron-ion collisions that cause the cross-field transport, the increase in plasma density results in a reduced plasma confinement, and, as a result, the increase in plasma density with the increase in the radial plasma flux density is smaller than linear. These examples correspond to ambipolar flow, but we also studied in Ref. 32 the dependence of the plasma density on the plasma particle flux density for a nonambipolar flow. In addition, we examined the modification of the plasma radial density profile due to strong ionization, and we identified cases in which the plasma density profile becomes more peaked radially, or flatter.[32] Liard *et al.*[33] then considered neutral depletion in a magnetized plasma in which ambipolar cross-field diffusion is induced by electron-neutral collisions; one of the cases we analyzed in Ref. 32. However, while we studied in Ref. 32 how neutral-gas depletion and the magnetized plasma are affected by varying the deposited power (and the plasma particle flux), Liard *et al.*[33] examined how they are affected by varying the intensity of the magnetic field. Liard *et al.*[33] found that, for a specified plasma density at the center of the discharge, the increase in the magnetic field intensity results in a smaller neutral-gas depletion. In the present paper, we also address the relation between the intensity of the magnetic field and the rate of neutral-gas depletion. We show that the rate of neutral-gas depletion is determined by the total plasma particle outward flux at the boundaries, and that the neutral-gas depletion increases when that flux increases. Thus, varying the magnetic field intensity while the plasma particle flux is kept constant does not affect the neutral-gas depletion. Note that the plasma maximal density (the density at the center of the discharge) does increase if the magnetic field increases while the plasma particle flux is kept constant. Thus, if the plasma maximal density is constant, the plasma particle flux decreases when the magnetic field increases. This decrease in plasma particle flux should result in a decrease in neutral-gas depletion, according to our claim here, which explains the effect described in Ref. 33.

The analysis we performed is for a two-dimensional (2D) configuration, in which plasma transport is both along and across magnetic-field lines. We show that, when neutral-gas depletion is significant, the two-dimensionality of the plasma flow results in a nonmonotonic plasma density profile across field lines. The plasma density then has a local minimum at the center of the discharge. In the neighborhood of the discharge center, the plasma diffuses inward across magnetic-field lines, rather than outward, and then flows outward along magnetic-field lines.

In Sec. II we present the model for the plasma and for the neutral gas. The plasma is assumed to move freely along magnetic field lines and to diffuse due to electron-ion collisions across the field lines. Neutrals move ballistically and are coupled to the plasma through volume ionization and wall recombination only, as in Refs. 28 and 30. In Sec. III we derive a set of equations for dimensionless variables, specify the boundary conditions, and derive relations between those variables. In Sec. IV we solve the case of low neutral-gas

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depletion where neutral-gas density is approximately uniform. In Sec. V we write the equations for the case of no end losses along field lines, so that transport is across magnetic field lines only, and in Sec. VI we solve the asymptotic limit of a large neutral-gas depletion in this case of cross-field transport only. In Sec. VII we present numerical solutions for the 2D case which includes end losses along magnetic field lines. Nonmonotonic plasma density profiles across field lines are found, exhibiting a local density minimum at the center of the discharge.

II. THE MODEL

We consider a plasma immersed in a uniform magnetic field of an intensity B, which lies parallel to the z axis. The plasma is contained in a rectangular vessel of dimensions $2L$ in the z direction, $2a$ in the x direction, and $2b$ in the y direction. In such a configuration of open magnetic field lines, the plasma flow is sometimes claimed to be nonambipolar.[36,39–44] In our previous study,[32] we indeed examined cases of a nonambipolar flow. In the present paper, we assume that the flow is ambipolar. We assume that $a \ll b$, so that the y direction is ignorable, and also that $a \ll L$, so that

$$a \ll L,b. \tag{1}$$

Although a cylindrical geometry is more common, we prefer, for simplicity, to analyze such a 2D planar geometry. The plasma is symmetric with respect to the middle plane located at $x=0$, and also with respect to the plane located at $z=0$. We are mostly interested in the cross-field transport in the x direction, although we would also like to approximately model losses along field lines. We therefore write the plasma continuity equation as

$$\frac{d\Gamma}{dx} + \frac{nc}{L} = \beta N n. \tag{2}$$

Here Γ is the component of the plasma particle flux density in the x direction, N is the neutral-gas density, n is the plasma density, and β is the ionization rate constant due to electron-atom ionizing collisions, a constant which strongly depends on T, the assumed-uniform electron temperature. The second term on the left hand side (LHS) of the equation approximately describes losses along field lines, when ions are collisionless. This is because when ions are collisionless, the average plasma velocity along field lines is a fraction of the ion acoustic velocity,

$$c \equiv \sqrt{\frac{T}{m}}, \tag{3}$$

where m is the ion (and neutral) mass.[40,45] In Eq. (2), we approximate that fraction as unity. The variables in Eq. (2) are axially (over z) averaged. The plasma cross-field diffusion is described as

$$D\frac{dn}{dx} = -\Gamma, \tag{4}$$

where D, the cross-field diffusion coefficient, is

$$D = \frac{T\nu_e}{m_e\omega_e^2} = \frac{Tk_{ei}n}{m_e\omega_e^2}. \tag{5}$$

Here, $\omega_e \equiv eB/m_e$ is the electron cyclotron frequency, e is the elementary charge, and m_e is the electron mass. The cross field diffusion is induced by electron collisions, where ν_e is the electron collision frequency. Here we assume that the dominant collisions of the electrons are with ions, $\nu_e = k_{ei}n$, k_{ei} being the electron-ion collision rate constant.

Similarly to what we did in Refs. 25, 28, and 30, we assume that the neutrals are collisionless, that they move ballistically, and that they are coupled to the plasma only through volume ionization and recombination at the boundaries. This neutral-gas dynamics is different from that of thermalized neutrals,[23,26,34] and, as shown in Refs. 28 and 30, the pressure of the neutrals as described here is higher where their density is lower. We assume here that most of the neutral-gas flux into the discharge is along the x direction, because of the inequality in Eq. (1). A further simplifying assumption in our model is that all neutrals have a velocity of the same magnitude v_a in the x direction only, either to the right (positive x direction) or to the left (negative x direction). The neutral-gas particle flux density is written as

$$\Gamma_N(x) = \Gamma_1(x) - \Gamma_2(x), \tag{6}$$

being composed of a flux density to the right $\Gamma_1(x)$ and a flux density to the left $\Gamma_2(x)$. Because of the symmetry with respect to $x=0$,

$$\Gamma_1(x) = \Gamma_2(-x), \quad -a \le x \le a. \tag{7}$$

The neutral-gas density is expressed as

$$N(x) = \frac{\Gamma_1(x) + \Gamma_2(x)}{v_a}. \tag{8}$$

From the separate continuity equations for the two counterstreaming neutral beams,

$$\frac{d\Gamma_1}{dx} = -\beta n\frac{\Gamma_1}{v_a}, \quad \frac{d\Gamma_2}{dx} = \beta n\frac{\Gamma_2}{v_a}, \tag{9}$$

it follows that

$$\Gamma_1(x)\Gamma_2(x) = \Gamma_0^2, \tag{10}$$

where $\Gamma_0 \equiv \Gamma_1(0) = \Gamma_2(0)$. Adding Eqs. (2) and (9) and employing Eq. (8), we obtain the following relation between the plasma particle flux density and the neutral-gas particle flux density:

$$\Gamma_1(x) - \Gamma_2(x) + \Gamma(x) + \frac{c}{L}\int_0^x dx' n(x') = 0. \tag{11}$$

The total net mass flow from the boundaries is zero; the neutral-gas particle flow inward along the x direction is balanced by a plasma particle flow outward along both x and z directions. Since there are plasma losses in the z direction along magnetic field lines, the neutral-gas flow from the boundaries in the x direction is larger than the plasma particle flux impinging on the these boundaries in the x direction. Equivalently to Eq. (11), we write

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$$\Gamma_2(x) - \Gamma_1(x) = \Gamma_t(x) \equiv \Gamma(x) + \frac{c}{L}\int_0^x dx' n(x'), \tag{12}$$

where $\Gamma_t(x)$ is the total plasma particle flux per unit area, which we call the plasma particle flux density.

Employing Eqs. (8), (10), and (12), we write an expression for the density of the neutrals,

$$N(x) = \frac{\sqrt{\Gamma_t^2(x) + 4\Gamma_0^2}}{v_a}. \tag{13}$$

The neutral-gas density at the plasma boundary is

$$N_W \equiv N(a) = \frac{\sqrt{\Gamma_m^2 + 4\Gamma_0^2}}{v_a}, \tag{14}$$

where $\Gamma_m \equiv \Gamma_t(a)$ is the maximal plasma particle flux density, the plasma particle flux density at the plasma boundary. Employing Eq. (14), we express $4\Gamma_0^2$ in Eq. (13) as $-\Gamma_m^2 + N_W^2 v_a^2$, so that the neutral-gas density is now expressed as

$$N(x) = \frac{\sqrt{\Gamma_t^2(x) - \Gamma_m^2 + N_W^2 v_a^2}}{v_a}. \tag{15}$$

With the expressions for $N(x)$ [Eq. (15)] and for $\Gamma_t(x)$ [Eq. (12)], the governing equations are Eqs. (2) and (4) for Γ and for n. We can solve these equations once we specify boundary conditions for Γ and for n, say at $x=0$. Because the solution is symmetric with respect to $x=0$, the plasma particle flux density is zero at the midplane, $\Gamma(x=0)=0$, while we are free to specify the value of $n_0 \equiv n(x=0)$. We also require the plasma density to vanish at the boundary, $n(x=a)=0$. This additional boundary condition does not allow us to freely specify the values of all the parameters in the equations. As is usually done,[40,45,50] we view the electron temperature T as an eigenvalue and its value is found by solving the equations with the above-mentioned requirement that the plasma density vanish at the boundary, once the values of all other parameters are specified. When n_0 is specified, the plasma particle flux density at the boundary, $\Gamma_t(a)$, found through a solution of the equations has to be the same as Γ_m which is substituted in the expression for the neutral-gas density in Eq. (15). Thus, if we specify the value of n_0, we should view both T and Γ_m that appear in the equations as eigenvalues. Their values in Eqs. (2), (4), and (15) should be determined by the requirements that $n=0$ and that $\Gamma_t=\Gamma_m$ at the boundary $x=a$. When the magnetic field (and ω_e^2 in the expression for D) is varied, while n_0 is kept fixed, T and Γ_m should also vary.[27]

Experimentally, we do not directly control the plasma density n_0. Instead of B and n_0, it is more reasonable to consider B and the total deposited power P as control parameters. However, the power P does not appear in our equations. We need to relate P to the parameters that do appear in our model. The total plasma particle flux, the number of plasma particles that reach the plasma boundaries per unit time, is $8bL\Gamma_m$. According to the power balance in Ref. 40, the power is related to the maximal plasma particle flux density through

$$P = 8bL\Gamma_m \varepsilon_T(T), \tag{16}$$

where $\varepsilon_T(T)$, the energy invested per ion-electron pair, is a decreasing function of T for the electron temperatures of interest. However, since the relation between the power P and the plasma particle flux, Eq. (16), could be debatable, we prefer in this paper to view B and Γ_m as control parameters. If we specify Γ_m, we view both T and n_0 as eigenvalues. Values for T and for n_0 should be found, so that the solution of the equations should yield $n=0$ and $\Gamma_t=\Gamma_m$ at the boundary $x=a$. Thus, we specify Γ_m, and examine how the discharge parameters, such as T and n_0, vary when we vary B.

The expression for the neutral-gas density profile, Eq. (15), exhibits the relation between the neutral-gas density $N(x)$ and the plasma particle flux density $\Gamma_t(x)$. In particular, the maximal neutral-gas depletion, the minimum of the neutral-gas density, occurs at the center of the discharge. The neutral-gas density there is determined by Γ_m, the maximal value of $\Gamma_t(x)$. That minimal neutral-gas density, at the center of the discharge, is

$$N(0) = N_W\sqrt{1 - \left(\frac{\Gamma_m}{N_W v_a}\right)^2}. \tag{17}$$

For specified neutral-gas parameters at the wall, N_W and v_a, the neutral-gas depletion, which we denote here by $N(0)/N_W$, is determined by the value of the plasma particle flux density Γ_m. Therefore, it is clear already at this stage of the analysis that varying the magnetic field while Γ_m is fixed does not change the neutral-gas depletion. The fact that the rate of neutral-gas depletion is constant once Γ_m is specified, even if B (and consequently n_0 and T) varies, is a major conclusion of this analysis.

We would like to explore how once the geometry (L, b, and a), the neutral-gas density at the boundary N_W, the flow velocity of the neutrals v_a, and the maximal plasma particle flux density Γ_m are all specified, the plasma density $n(x)$, the electron temperature T, and the neutrals density $N(x)$ vary as functions of the magnetic field B. To that end, we start in the next section by writing the governing equations in dimensionless form.

III. THE DIMENSIONLESS GOVERNING EQUATIONS

The governing equations, Eqs. (2) and (4), and the neutral-gas density, Eq. (15), are rewritten in a dimensionless form as

$$\frac{d\bar{\Gamma}_t}{d\xi} = \alpha\bar{n}\sqrt{\bar{\Gamma}_t^2 + \delta^2}, \tag{18}$$

$$\frac{d\theta}{d\xi} = \alpha\bar{n}, \tag{19}$$

$$\bar{n}\frac{d\bar{n}}{d\xi} = \frac{\alpha}{3}\left(2d_l\theta - \frac{d_p}{\delta}\bar{\Gamma}_t\right), \tag{20}$$

and

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$$\bar{N} = \sqrt{\bar{\Gamma}_t^2 + \delta^2}, \tag{21}$$

for the dimensionless variables,

$$\bar{\Gamma}_t \equiv \frac{\Gamma_t}{N_W v_a}, \quad \bar{n} \equiv \frac{n}{n_0}, \quad \theta \equiv \alpha \int_0^{\xi} d\xi' \bar{n}(\xi'), \quad \bar{N} \equiv \frac{N}{N_W}, \tag{22}$$

while the dimensionless parameters and coordinate are

$$\alpha \equiv \frac{\beta n_0 a}{v_a}, \quad d_l \equiv \frac{3c}{2L\beta^2} \frac{v_a^2 m_e \omega_e^2}{n_0^3 T k_{ei}},$$
$$d_p \equiv \frac{3N_W}{\beta} \frac{v_a^2 m_e \omega_e^2}{n_0^3 T k_{ei}} \delta, \quad \delta^2 \equiv 1 - \left(\frac{\Gamma_m}{N_W v_a}\right)^2, \quad \xi \equiv \frac{x}{a}. \tag{23}$$

Here $n_0 \equiv n(x=0)$, as defined above. It is easy to derive algebraic relations between the four dependent variables, θ, $\bar{n}$, $\bar{N}$, and $\bar{\Gamma}_t$. From Eqs. (18) and (19) we express $\bar{\Gamma}_t$ as a function of θ:

$$\bar{\Gamma}_t = \delta \sinh \theta. \tag{24}$$

From Eqs. (19) and (20) and this last expression, Eq. (24), we obtain the following relation between θ and $\bar{n}$:

$$\bar{n}^3 - 1 = d_l \theta^2 + d_p(1 - \cosh \theta). \tag{25}$$

Substituting $\bar{n}(\theta)$ from Eq. (25) into Eq. (19), we obtain the profile $\xi(\theta)$ as

$$\alpha \xi = \int_0^{\theta} \frac{d\theta'}{[1 + d_l \theta'^2 + d_p(1 - \cosh \theta')]^{1/3}}. \tag{26}$$

We now apply the boundary conditions at $\xi=1$. Using the relation (24) and the boundary condition, $\bar{\Gamma}_t(\bar{n}=0) = \sqrt{1-\delta^2}$, we obtain that

$$\theta(\bar{n} = 0) = \cosh^{-1}(1/\delta). \tag{27}$$

Substituting this expression and $\xi(\bar{n}=0)=1$ into Eqs. (25) and (26), we obtain two solvability conditions. They are

$$1 = d_p\left(\frac{1}{\delta} - 1\right) - d_l\left[\cosh^{-1}\left(\frac{1}{\delta}\right)\right]^2 \tag{28}$$

and

$$\alpha = \int_0^{\cosh^{-1}(1/\delta)} \frac{d\theta}{[1 + d_l \theta^2 + d_p(1 - \cosh \theta)]^{1/3}}. \tag{29}$$

These solvability conditions should be used to determine the values of two parameters. Liard *et al.*[27] considered n_0 as specified and solved for Γ_m and T, as B is varied. We, instead, view Γ_m as specified and solve for n_0 and T, as B is varied. Once $\xi(\theta)$ is found by solving Eq. (26) with the solvability conditions (28) and (29), $\bar{\Gamma}_t(\xi)$ and $\bar{n}(\xi)$ can be found by use of Eqs. (24) and (25), while for the neutral-gas density $\bar{N}(\xi)$ we use

$$\bar{N} = \delta \cosh \theta. \tag{30}$$

As explained in the discussion following Eq. (17), or, equivalently, as Eq. (30) in which we substitute $\theta(\bar{n}=1)=0$ implies, the neutral-gas depletion, $\bar{N}(0)$, depends only on the parameter δ. If the plasma particle flux density Γ_m is kept fixed as B is varied, the neutral-gas depletion, $\bar{N}(0)$, does not vary.

For the calculation, we need to know the ionization and the collision rate constants. We assume that the electron distribution is Maxwellian and choose the approximated expression for the ionization rate constant,[40]

$$\beta = \sigma_0 v_{te} \exp\left(-\frac{\epsilon_i}{T}\right), \tag{31}$$

where $v_{te} \equiv (8T/\pi m_e)^{1/2}$ is the electron thermal velocity and $\sigma_0 \equiv \pi(e^2/4\pi\epsilon_0\epsilon_i)^2$, ϵ_0 being the permittivity of the vacuum and ϵ_i (=15.6 eV for argon) the ionization energy. The electron-ion collision rate constant is[32,48]

$$k_{ei} = 2.91 \times 10^{-6} \ln \Lambda T^{-3/2} \ \text{cm}^3/\text{s}, \tag{32}$$

where $\ln \Lambda = 10$.

In studying the effect of the magnetic field intensity on the plasma and neutral-gas variables, we will examine the two opposite limits of low and high neutral-gas depletion. In Sec. IV we study the limit of low neutral-gas depletion, obtained, according to Eq. (17), when $\delta \cong 1$.

IV. LOW NEUTRAL-GAS DEPLETION

When $\Gamma_m \ll N_W v_a$, neutral-gas depletion is low. The parameter δ is then

$$\delta \cong 1. \tag{33}$$

In this case, we approximate $\cosh^{-1}(1/\delta) = \sqrt{2(1-\delta)}$. Equation (28) then becomes

$$2(1 - \delta)(d_p/2 - d_l) = 1. \tag{34}$$

Using this relation in the integral in Eq. (29) and the approximate relation $1 - \cosh \theta = -\theta^2/2$ for a small θ, we obtain that

$$\frac{\alpha}{\sqrt{2(1-\delta)}} = \int_0^1 \frac{ds}{(1 - s^2)^{1/3}} = 1.2936. \tag{35}$$

Approximating $\sqrt{2(1-\delta)} = \Gamma_m/(N_W v_a)$ and employing the explicit expressions for the dimensionless quantities, we obtain that

$$\frac{2n_0^3 T k_{ei} \beta N_W}{3m_e \omega_e^2 \Gamma_m^2} + \frac{c}{L N_W \beta} = 1, \tag{36}$$

and

$$\frac{\beta N_W n_0 a}{\Gamma_m} = 1.2936. \tag{37}$$

When neutral-gas depletion is small, the electron temperature T is usually found through particle balance, and its value is independent of the value of n_0, which depends linearly on the plasma particle flux density Γ_m (or on the deposited power).[40,49–52] Here, however, T and n_0 are coupled and T depends on Γ_m, since the equations are nonlinear, even when the neutral-gas depletion is low. The nonlinearity is expressed in the appearance of n_0^3/Γ_m^2 in the first term on the LHS of Eq. (36). This nonlinearity stems from the depen-

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dence of the diffusion coefficient D on the plasma density n, since the cross field diffusion results from electron-ion collisions. The relations (36) and (37) have been derived in Ref. 32 for the case that axial end losses are negligible.

If we substitute the expression for n_0 from Eq. (37) into Eq. (36), we obtain the dependence of T on B. It is apparent that as B is increased for a fixed plasma particle flux density Γ_m, the electron temperature T decreases. This decrease in electron temperature with the increase in B, when Γ_m is fixed, usually occurs even in the nonlinear regime in which neutral-gas depletion is significant, as we show in the next sections.

In Sec. V we examine the case that axial end losses along magnetic field lines are negligible.

V. MAGNETIC FIELD AND NEUTRAL-GAS DEPLETION: NO AXIAL END LOSSES

If the discharge is long enough, so that end losses can be neglected, we approximate

$$d_l = 0. \tag{38}$$

The solvability condition (28) becomes

$$d_p = \frac{\delta}{1-\delta}. \tag{39}$$

Using Eqs. (38) and (39), we write the solvability condition (29) as

$$\alpha = (1-\delta)^{1/3}\int_0^{\cosh^{-1}(1/\delta)} \frac{d\theta}{(1-\delta\cosh\,\theta)^{1/3}}. \tag{40}$$

The dimensionless profiles are now expressed as functions of θ and of the single parameter δ. These are $\bar{n}(\delta,\theta)$,

$$\bar{n} = \left(\frac{1-\delta\cosh\,\theta}{1-\delta}\right)^{1/3}, \tag{41}$$

and $\xi(\delta,\theta)$,

$$\xi = \frac{\int_0^{\theta} d\theta'(1-\delta\cosh\,\theta')^{-1/3}}{\int_0^{\cosh^{-1}(1/\delta)} d\theta(1-\delta\cosh\,\theta)^{-1/3}}. \tag{42}$$

The dimensionless plasma particle flux $\overline{\Gamma}_t(\delta,\theta)$ and neutral-gas density $\bar{N}(\delta,\theta)$ are described by Eqs. (24) and (30).

It is clear that once δ is specified, the parameters d_p and α, and the dimensionless profiles, $\overline{\Gamma}_t(\xi)$, $\bar{n}(\xi)$, and $\bar{N}(\xi)$, are determined. In this case of no losses along magnetic field lines, the profiles are not affected at all by the magnetic field intensity, as long as δ is constant.

Let us examine now how the plasma density on axis n_0 and the electron temperature T vary when the magnetic field varies. Substituting the explicit expressions for the dimensionless quantities into the solvability conditions, we derive the following relation:

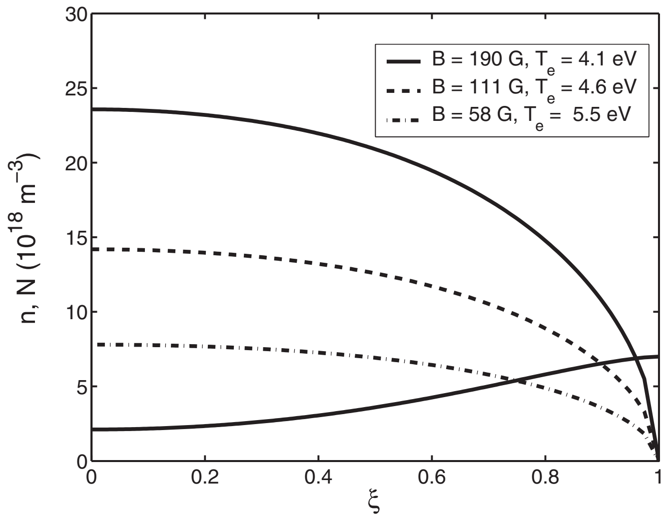

FIG. 1. Plasma and neutral-gas cross-field density profiles for three values of the magnetic field intensity, no axial end losses ($L=\infty$). The normalized neutral-gas density profile (the solid line with a minimum at $\xi=0$) is identical for all values of the magnetic field intensity. In the legend are written the values of B and also of the corresponding electron temperature, denoted by T_e. The gas is argon and the discharge parameters are $N_W=7\times10^{18}$ m^{-3}, $v_a=405$ m s^{-1}, $\Gamma_m=2.8208\times10^{21}$ m^{-2} s^{-1} ($\delta=0.1$), and $a=0.05$ m.

$$\omega_e^2\frac{\beta^2}{Tk_{ei}} = \frac{v_a}{3N_W m_e a^3}\left[\int_0^{\cosh^{-1}(1/\delta)}\frac{d\theta}{(1-\delta\cosh\,\theta)^{1/3}}\right]^3. \tag{43}$$

When δ is specified and the RHS of this equation is constant, the increase in the magnetic field (B and ω_e^2) is followed by a decrease in the quantity $\beta^2(T)/Tk_{ei}(T)$, which happens if T decreases. The plasma density increases as the electron temperature (and β) decreases, since $\alpha=\beta n_0 a/v_a$ is constant for a fixed δ. The decrease in T and increase in n_0 for a fixed Γ_m when B is increased, which we found in Sec. IV for the case of a low neutral-gas depletion, are thus shown here to occur in the general case of an arbitrary neutral-gas depletion (but without axial end losses).

If instead of a fixed Γ_m, we consider n_0 to be fixed, we find that the increase in B results in a smaller Γ_m and, according to Eq. (17), in a lower neutral-gas depletion. This effect in a plasma in which the cross-field diffusion is determined by electron-neutral collisions, rather than electron-ion collisions as here, was identified in Ref. 33 as a competition between the magnetic field and neutral gas depletion.

In this paper we focus on the case that the plasma particle flux is fixed and the numerical calculations demonstrate results for this case. Figure 1 shows the profiles of the plasma density and of the neutral-gas density for cases of negligible axial end losses. For the calculation, we assumed that $N_W=7\times10^{18}$ m^{-3} and $v_a=405$ m s^{-1} (corresponding to a thermal velocity of argon atoms when the neutral-gas temperature is 300 K). The plasma flux density at the boundary is taken as $\Gamma_m=2.8208\times10^{21}$ m^{-2} s^{-1}, corresponding to $\delta=0.1$. The plasma half width is $a=0.05$ m. For three values of the magnetic field intensity B, we calculate T and n_0 from the algebraic relations above. As we showed here, the nor-

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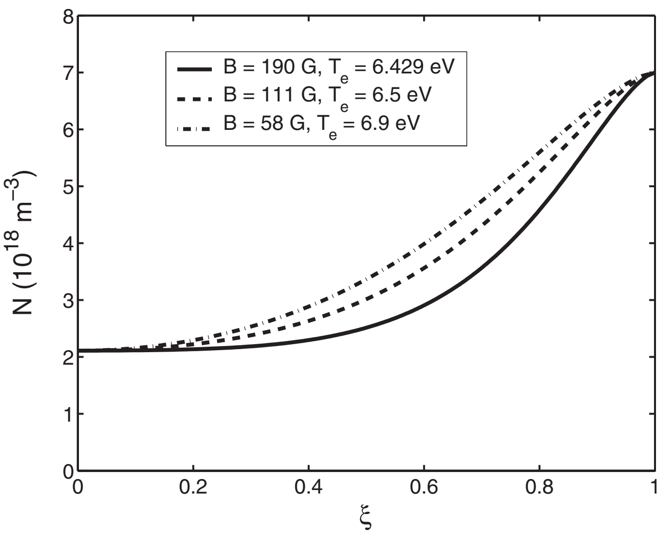

FIG. 2. Neutral-gas cross-field density profiles for the same three values of the magnetic field intensity as in Fig. 1, but with axial end losses ($L=0.35$ m). The gas is argon and the rest of the parameters are as in Fig. 1: $N_W=7\times10^{18}$ m^{-3}, $v_a=405$ m s^{-1}, $\Gamma_m=2.8208\times10^{21}$ m^{-2} s^{-1} ($\delta=0.1$), and $a=0.05$ m. The density profiles are different for different values of the magnetic field but the density at $\xi=0$ is the same. In the legend are written the values of B, the same as in Fig. 1, and also of the corresponding electron temperature, denoted as T_e (which is different than in Fig. 1).

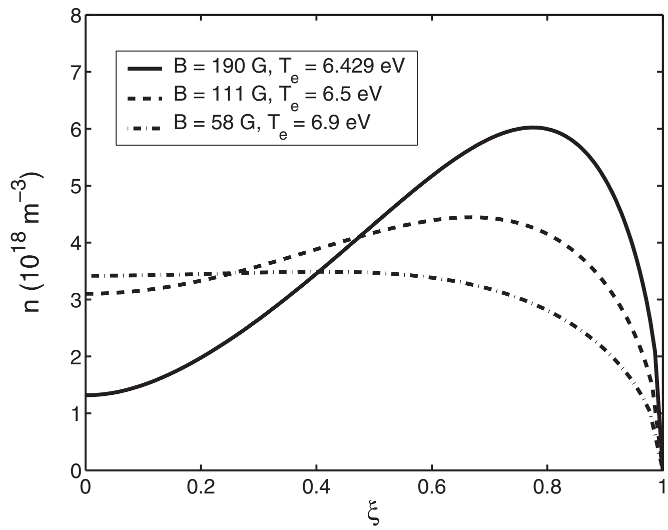

FIG. 3. Plasma cross-field density profiles for the case shown in Fig. 2. There are axial end losses ($L=0.35$ m), the gas is argon, and the rest of the parameters are as in Figs. 1 and 2: $N_W=7\times10^{18}$ m^{-3}, $v_a=405$ m s^{-1}, $\Gamma_m=2.8208\times10^{21}$ m^{-2} s^{-1} ($\delta=0.1$), and $a=0.05$ m. As B increases the plasma density profile becomes nonmonotonic; the plasma density at the center, at $\xi=0$, decreases and its maximal value, away from the center, increases.

malized profiles $\bar{N}(\xi)$, $\overline{\Gamma_t}(\xi)$, and $\bar{n}(\xi)$ are identical for the three cases. We plot the three density profiles $n(\xi)=n_0\bar{n}(\xi)$ for the three values of B, which differ by the values of n_0 only.

Before we turn to the case with axial end losses along magnetic field lines, we examine the limit of high neutral-gas depletion without axial end losses.

VI. HIGH NEUTRAL-GAS DEPLETION: NO AXIAL END LOSSES

We discuss here the limit of a high plasma flux density, $\Gamma_m\cong N_W v_a$, for which

$$\delta\ll 1. \tag{44}$$

We approximate the solvability condition, Eq. (28), as

$$d_p=\delta. \tag{45}$$

In the solvability condition, Eq. (40), we approximate $\cosh\theta\cong\exp\theta$. We then define a new variable $t\equiv[1-(\delta/2)\exp\theta]^{1/3}$, so that $\alpha\cong\int_0^{(1-\delta/2)^{1/3}}[1/(t-1)+(t-1)/(t+t^2+1)]dt$. The integral of the first term is $-\ln[1-(1-\delta/2)^{1/3}]\cong\ln(6/\delta)$, while the integral of the second term is approximated, for $\delta=0$, as $-0.357\,59$. Thus, for $\delta\ll1$, Eq. (40) is approximated as

$$\alpha=\ln(4.1962/\delta). \tag{46}$$

Substituting the explicit expressions for the dimensionless parameters into the approximate expressions, Eqs. (45) and (46), we obtain that

$$\frac{3N_W}{\beta}\frac{v_a^2 m_e\omega_e^2}{n_0^3 Tk_{ei}}=1, \tag{47}$$

and

$$\frac{\beta n_0 a}{v_a}=\ln[4.1962/\sqrt{1-(\Gamma_m/N_W v_a)^2}]. \tag{48}$$

We view both T and n_0 as unknown. From these two last relations we write an expression for T only:

$$\omega_e^2\frac{\beta^2}{Tk_{ei}}=\frac{v_a}{3m_eN_Wa^3}\{\ln[4.1962/\sqrt{1-(\Gamma_m/N_Wv_a)^2}]\}^3. \tag{49}$$

This equation is the approximate form of Eq. (43) at the limit of $\delta\ll1$. The decrease in T and increase in n_0, as B (and ω_e^2) is increased for a fixed Γ_m, found in Sec. V for the more general case, occur also here, when neutral depletion is high.

In the next section we continue to explore the effect of magnetic field intensity on the plasma density profile for a fixed plasma particle flux density, with finite axial end losses along magnetic-field lines.

VII. MAGNETIC FIELD AND NEUTRAL-GAS DEPLETION: WITH AXIAL END LOSSES

When end losses are finite, so that $d_l\neq0$, the parameter δ does not determine the values of the other dimensionless parameters. For the same value of δ, for the same plasma particle flux density, varying B results in a modification of the profiles $\overline{\Gamma_t}(\xi)$, $\bar{n}(\xi)$, and $\bar{N}(\xi)$. The neutral-gas density $\bar{N}(\xi)$ may be modified, although, as we mentioned above, the minimal value, $\bar{N}(0)$, is determined by the plasma particle flux density and does not vary when B varies [Eq. (17)].

Figures 2 and 3 show neutral-gas and plasma density profiles when there are axial end losses. In Fig. 2 we show the neutral-gas density profiles and in Fig. 3 the plasma den-

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sity profiles for the same parameters and the same values of the magnetic field B as in the example shown in Fig. 1, as described in the previous section, except for the length of the plasma, which, in Figs. 2 and 3, is $L=0.35$ m. It is seen in Fig. 2, that, as explained above, the minimal neutral-gas density does not change when the magnetic field varies. Except for at the center, the neutral-gas density does change when the magnetic field is varied. This variation of the neutral-gas profile happens because of the axial end losses along magnetic filed lines. Without such axial end losses, the neutral-gas density is determined by the plasma flux and does not change when the magnetic field is changed, as seen in Fig. 1.

The plasma radial density profile is shown in Fig. 3. It is seen in the figure that as the magnetic field intensity increases so that the plasma cross-field diffusion becomes smaller, the plasma radial density profile becomes nonmonotonic, having a local minimum at $x=0$. Along field lines the density is monotonically decreasing toward the boundaries, having a maximum at $z=0$. Therefore, in the $z-x$ plane the density has a saddle point at $(z,x)=(0,0)$. In the neighborhood of the middle plane at the center of the discharge, the plasma diffuses radially inward, toward the center of the discharge, rather than outward. Together with the plasma generated near that plane, the plasma that flows radially inward leaves the discharge axially along magnetic field lines.

VIII. DISCUSSION

We examined the effect of the magnetic-field intensity on the electron temperature and on the density profiles across magnetic field-lines of plasma and of neutral-gas in a gas discharge. Neutral-gas depletion, here defined as the ratio of neutral-gas density at the midplane to the neutral-gas density at the discharge boundary, was shown to be determined by the plasma particle flux outward of the discharge. Therefore, if this plasma particle flux is fixed, varying the magnetic field intensity does not affect the neutral-gas depletion. For a fixed plasma particle flux, the electron temperature decreases and the plasma density increases, when the magnetic field is increased. When there are plasma axial end losses along magnetic-field lines and neutral-gas depletion is significant, an increase in the magnetic field intensity was shown to result in a nonmonotonic plasma density profile across magnetic field lines. The plasma density has then a local minimum at the center of the discharge.

The results described in the paper were obtained following several assumptions about the dynamics of the plasma and of the neutral gas. Neutrals were assumed to move ballistically inward, all with the same velocity. They were assumed collisionless and to be coupled to the plasma through volume ionization and wall recombination only. We have shown in the past that the gas discharge steady-state crucially depends on the assumed neutral-gas dynamics.[26,28,30,34] It is interesting to examine the relation between the magnetic field and the neutral-gas depletion when the neutral-gas dynamics is different than the dynamics assumed here.

In future studies, we will extend our study to examine how the neutral-gas depletion and the gas-discharge vary when the magnetic field is varied for a fixed deposited power, instead of the fixed plasma particle flux assumed here.

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