Plasma lens and plume divergence in the Hall thruster

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The effect of magnetic field curvature on the plume divergence in the Hall thruster is analyzed. The two-dimensional plasma flow and electric field are determined self-consistently within the paraxial approximation in this plasma lens, a nearly axial electric field perpendicular to the curved magnetic field lines. The ion radial velocity along the thruster is described analytically. The authors suggest positioning the ionization layer near the zero of the magnetic field in a reversing-direction field configuration for a minimal beam divergence. They also show that an additional emitting electrode can reduce plume divergence. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2349827]

There is substantial interest in improving the performance of the Hall thruster[1–4] as a propulsion engine by reducing the divergence of the plume that could damage the solar panels and other parts of the satellite.[5] Ions in the Hall thruster are accelerated axially by an electric field that is perpendicular to an ideally radial applied magnetic field. The electric field should then be axial, causing no plume divergence. In reality, however, the magnetic field lines are curved, causing the electric field to have a small radial component that results in plume divergence. Indeed, that defocusing (and also focusing) effect of the magnetic field curvature has been recognized and investigated experimentally[6] and theoretically.[7] The effect of magnetic field curvature combined with a finite electron pressure has also been discussed.[7,8] In this letter we analyze the effect of the magnetic field curvature on the plume divergence in the Hall thruster at the limit of zero electron pressure. The configuration we analyze is the so-called plasma lens, a nearly axial electric field perpendicular to the curved nearly radial magnetic field lines, in which the two-dimensional plasma flow and electric potential distribution across the equipotential magnetic field lines should be determined self-consistently. Using the paraxial approximation we describe analytically the ion radial velocity along the thruster. We suggest positioning the ionization layer near the zero of the magnetic field in a reversing-direction field configuration for a minimal plume divergence. We also show how one can reduce plume divergence by the introduction of an additional biased electrode. Indeed, reduced plume divergence has been observed when a reversing-direction field[6] or an additional electrode has been used.[9]

Assuming the distance between its radial walls to be small, we model the cylindrical Hall thruster as planar, denoting the coordinate along the thruster by z, the radial coordinate by x, and the ignorable azimuthal coordinate by y. The magnetic field lines lie in the z-x plane, $\mathbf{B}=\hat{e}_z B_z(z,x)+\hat{e}_x B_x(z,x)$, so that we express the two components of the magnetic field with the scalar flux function $A(z,x)$: $\mathbf{B}=\nabla\times\mathbf{A}=-\hat{e}_y\times\nabla A$ where $\mathbf{A}=\hat{e}_y A(z,x)$. We analyze the acceleration region only, in which ions are assumed collisionless and of a negligible pressure, and ionization events are assumed rare. Since the canonical momentum, $m_i v_y+eA$, is constant along the ion trajectory (e, m_i, and $\mathbf{v}$ are the charge, mass, and velocity of the ion) we write the ion momentum equation in the z-x plane as

$$\mathbf{v}\cdot\nabla(\hat{e}_z v_z+\hat{e}_x v_x)=-\frac{e}{m_i}\nabla\left[\varphi-\frac{e(\Delta A)^2}{2m_i}\right]. \tag{1}$$

Here φ is the electric potential and ΔA is the change of A with respect to its value at the location at which $v_y=0$. In the Hall thruster the magnetic field is mostly in the radial direction, while the applied electric field is mostly in the axial direction. We employ the paraxial approximation, $\partial/\partial z \gg \partial/\partial x$, to expand the fields and the ion fluid velocity at the neighborhood of the plane $x=0$ (where we assume that $B_z=0=\partial\varphi/\partial x$) as $A(z,x)\cong A_0(z)+x^2A_2(z)$, $\varphi(z,x)\cong\varphi_0(z)+x^2\varphi_2(z)$, $v_z=v_{z0}(z)$, and $v_x=xv_{x1}(z)$. To lowest order the z component of Eq. (1) becomes an energy conservation relation of the form $v_{z0}^2(z)-v_{z0}^2(0)=U(z)\equiv(2e/m_i)[\varphi_0(0)-\varphi_0(z)]+(e/m_i)^2[A_0^2(z)-A_0^2(0)]$. The x component of the equation, divided by x, turns out to be

$$v_{z0}(z)\frac{dv_{x1}(z)}{dz}=f(z)\equiv-\frac{2e}{m_i}\left\{\varphi_2(z)-\frac{e}{m_i}[A_0(z)-A_0(0)]A_2(z)\right\}. \tag{2}$$

We are not interested in the vacuum electric potential, where no space charge implies that $\varphi_2=-\varphi_0''(z)/2$, but rather in the so-called plasma lens, in which the high electron mobility shorts the electric field along magnetic field lines. Then

$$\nabla\varphi\times\nabla A=0\Rightarrow\varphi_2(z)=\frac{A_2(z)}{A_0'(z)}\varphi_0'(z). \tag{3}$$

The effect of the electron pressure either along[8] or perpendicular[7] to the magnetic field lines is not considered here. We restrict ourselves further to the low current regime, typical to the Hall thruster, in which the self-magnetic field of the plasma is small relative to the curl-free magnetic field, so that $A_2(z)=-A_0''(z)/2$. Together with the expression for $\varphi_2(z)$ we write

$$f(z)\equiv\frac{e}{m_i}A_0''(z)\left\{\frac{\varphi_0'(z)}{A_0'(z)}-\frac{e}{m_i}[A_0(z)-A_0(0)]\right\}. \tag{4}$$

Thus the solution is determined by the values of the axial electric and radial magnetic fields on axis.

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We note in passing that the equivalent single-particle equations of motion, in which $x(z)$ is the particle trajectory and $v_x(z)=v_z(z)dx(z)/dz$, are $v_z^2(z)-v_z^2(0)=U(z)$ and

$$\frac{d^2x}{dz^2}+g_1(z)\frac{dx}{dz}+g_2(z)x=0, \tag{5}$$

where $g_1(z)\equiv d\ln v_z(z)/dz$ and $g_2(z)\equiv -f(z)/v_z^2(z)$. This second-order linear equation is written in a form equivalent to that in Ref. 10.

We assume that the plasma is quasineutral, so that the ion density equals the electron density. To lowest order in the paraxial approximation,

$$\frac{\Gamma_i}{v_{z0}(z)}=\frac{\Gamma_e}{\mu_z\varphi_0'(z)}. \tag{6}$$

Here Γ_i and Γ_e are the (positive) ion and (negative) electron particle flux densities in the z direction, μ_z is the electron mobility in the z direction, and electron pressure is neglected for the electron dynamics. Equation (6) determines $\varphi_0'(z)$ to be substituted in Eq. (4) for $f(z)$. Since in the Hall thruster the ions are unmagnetized we neglect the last two terms on the right-hand side of the definition of $U(z)$ and in Eq. (4). The ion radial velocity turns out to be

$$v_x(z,x)=\frac{ex}{m_i}\int_0^z\frac{\varphi_0'(z')}{v_{z0}(z')B_x}\frac{dB_x}{dz'}dz'=\frac{ex}{m_i\Gamma_i}\int_0^z\frac{\Gamma_e}{\mu_zB_x}\frac{dB_x}{dz'}dz', \tag{7}$$

since $B_x=-A_0'(z)$. This equation expresses the *focusing* (*defocusing*) effect of the plasma lens where $dB_x/dz>(<)0$. Using the unmagnetized form of $U(z)$ and Eq. (6) [in which the electron mobility is the cross-field mobility $\mu_z=\mu_\perp=e\nu/(m_e\omega_{ce}^2)$] and assuming $v_{z0}(0)=0$ we obtain

$$m_i\Gamma_iv_{z0}(z)=-\int_0^z\frac{m_e\omega_{ce}^2\Gamma_e}{\nu}dz'. \tag{8}$$

Here ω_{ce} is the electron cyclotron frequency, ν the collision frequency, and m_e the electron mass. The last equality expresses pressure balance; the ions gain momentum from the magnetic field pressure. The plume divergence due to magnetic field curvature, determined by the largest radial velocity acquired by the ions close to the walls, is

$$\frac{v_x(L,a)}{v_z(L)}\cong -a\frac{\int_0^L(\Gamma_eB_x/\nu)(\partial B_x/\partial z)dz}{\int_0^L(\Gamma_eB_x^2/\nu)dz}, \tag{9}$$

where $2a$ is the distance between the radial walls.

The expression for the plume divergence due to magnetic field curvature in Eq. (9) is general and does not depend on the specific form of the collision frequency ν. In the case of a constant Γ_e and ν that is a function of the magnetic field intensity only, v_x turns out to be a function of the change in magnetic field intensity along the ion trajectory only. If the Hall parameter ω_{ce}/ν is constant, as often assumed for Hall thrusters,[11] or if the collision frequency ν is constant the divergence angle becomes

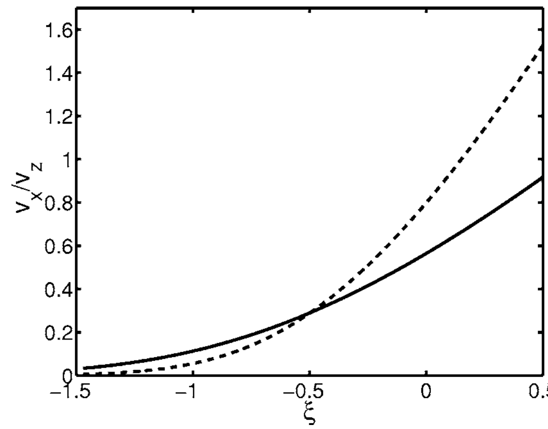

FIG. 1. Divergence angle as a function of the position of the ionization layer: constant Hall parameter (solid) and constant collision frequency (dashed).

$$\frac{v_x(L,a)}{v_z(L)}=\frac{a(B_i-B_f)}{(A_f-A_i)}=\frac{a(B_i-B_f)}{L\bar{B}},\quad\text{or}$$
$$\frac{v_x(L,a)}{v_z(L)}=\frac{a(B_i^2-B_f^2)}{2L\overline{B^2}}, \tag{10}$$

respectively. Here $\bar{B}$ and $\overline{B^2}$ are the average values of B and B^2 $(B\approx B_x)$, B_i and B_f and A_i and A_f are values of the magnetic field and of the flux function at the beginning and at the end of the acceleration channel. Although in the focusing region the radial electric field is *weaker*, the ions are *slower* there and, therefore, feel the focusing force *longer* [see Eq. (7)]. The result, expressed in Eq. (10), is that the plasma lens is to a lowest order symmetric. The focusing and defocusing effects cancel each other out if $B_i=B_f$. Ions that end their journey at the same magnetic field intensity as that at their starting point acquire no radial velocity. If, however, ions are born in the magnetized region $B_i\neq 0$ while the cathode is located outside that region $B_f=0$, the net effect is *divergence.* This important result shows that it is advantageous to locate the ionization layer where the magnetic field is zero. In order to have a finite magnetic field in the neighborhood of the anode for ionization enhancement, we propose to locate the ionization layer near the zero of the magnetic field in a reversing-direction magnetic field. We expect such a cusp configuration to help in both increasing the ionization and reducing the plume divergence. Indeed, employment of a reversing-direction field configuration has shown a reduced plume divergence.[6] In the past a cusped magnetic field combined with an electric field along an *axial* magnetic field has been used for ion acceleration.[12]

In a numerical example presented in Fig. 1, for a magnetic field intensity on axis of the form $B=B_m\exp[-(z-z_m)^2/L_1^2]$, shown are the divergence angles $v_x(\infty)/v_z(\infty)$ as functions of $\xi\equiv(z_i-z_m)/L_1$, where z_m and L_1 are constants and z_i the beginning of the acceleration region (the location of the ionization layer). The cathode is located at $z>2L_1$, where the magnetic field is negligible. These calculated angles are, for a constant Hall parameter, $(a/L_1)\times(2/\sqrt{\pi})\exp(-\xi^2)/[1-\mathrm{erf}(\xi)]$, and, for a constant collision frequency, $(a/L_1)(2\sqrt{2}/\sqrt{\pi})\exp(-2\xi^2)/[1-\mathrm{erf}(\sqrt{2}\xi)]$. In the numerical example a/L_1 is assumed to be 0.5. The results presented in Fig. 1 are a good approximation only when

v_x/v_z is small. For a specified magnetic field profile the collision frequency ν affects the divergence angle through the determination of the value of $\varphi_0'/v_{z0}=(\Gamma_e/\Gamma_i)m_e\omega_{ce}^2/e\nu$ at various field lines. A detailed discussion of this role of the collisionality will be presented in a future publication.

Plume divergence can also be reduced by increasing the electron flux density where $dB_x/dz>0$, thus increasing the focusing effect there. Let us assume an additional emitting electrode to be located at $z=L_e$, and that the electron flux density is Γ_{e1} in the region $0<z<L_e$, between the ionization layer and the additional electrode, and Γ_{e2} in the region $L_e<z<L$, between that electrode and the cathode. The radial velocity at the exit is zero if $\Gamma_{e1}\int_0^{L_e}(j_eB_x/\nu)(\partial B_x/\partial z)dz+\Gamma_{e2}\int_{L_e}^{L}(j_eB_x/\nu)(\partial B_x/\partial z)dz=0$. If the Hall parameter is constant and $B_f=0$,

$$\Gamma_{e1}(B_e-B_i)+\Gamma_{e2}B_e=0. \tag{11}$$

Here B_e is the magnetic field intensity at the position of the additional electrode. Employing relation (6) we find the potential at the additional electrode φ_e for Eq. (11) to hold:

$$\frac{\varphi_e}{\varphi_A}=1-\left[\frac{B_eA_e}{B_eA_e+(A_f-A_e)(B_e-B_i)}\right]^2. \tag{12}$$

Here A_e is the value of the flux function at the additional electrode and φ_A the applied voltage. We note that a reduced plume divergence has been observed when an additional electrode was introduced into the Hall thruster channel, although it was not biased according to Eq. (12).[9]

We have unfolded here the effect of the magnetic field curvature on the plume divergence in the Hall thruster. A more complete analysis should address the issue of ionization and should self-consistently calculate the location of the ionization layer and therefore also the actual length of the acceleration region.

For completeness we determine the regime of validity of the analysis here. The condition for the plasma to be quasineutral is $\epsilon_0\nabla^2\varphi/en_i\ll 1$. For $n_i\geqslant\Gamma_i/(2e\varphi_A/m_i)^{1/2}$ and $\nabla^2\varphi\simeq\varphi_A/L^2$ it becomes $\epsilon_0(2e/m_i)^{1/2}\varphi_A^{3/2}/L^2\ll e\Gamma_i$; the ion current is much larger than the space-charge limited current. The electron self-magnetic field is smaller than the applied field if the azimuthal current $j_{e\theta}\approx en_i\varphi_A/LB\approx e[\Gamma_i/(2e\varphi_A/m_i)^{1/2}]\varphi_A/LB\ll B/\mu_0L$. Together the ion flux should be

$$\epsilon_0\left(\frac{2e}{m_i}\right)^{1/2}\frac{\varphi_A^{3/2}}{L^2}\ll e\Gamma_i\ll\frac{B^2}{\mu_0}\left(\frac{2e}{m_i\varphi_A}\right)^{1/2}. \tag{13}$$

Multiplying these inequalities by the ion velocity at the thruster exit, we obtain that the electric-field energy density is smaller than the particle kinetic energy, which in turn, is smaller than the applied magnetic field energy density: $\epsilon_0(\varphi_A/L)^2/2\ll m_i\Gamma_i^2/2n_i\ll B^2/2\mu_0$. Another requirement was that the ions be unmagnetized, which, together with inequality (13), becomes $A_f\omega_{ci}L\ll\varphi_A\ll cA_f$, where ω_{ci} is the ion cyclotron frequency.

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