Enhanced Plasma Transport Due To Neutral Depletion

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(Received 22 April 2005; published 7 September 2005)

The dynamics of plasma and neutral gas in pressure balance are solved self-consistently to reveal the impact of neutral depletion. Analytical relations that determine the electron temperature, the rate of ionization, and the plasma density are derived. Because of the inherent coupling of ionization and transport, an increase of the energy invested in ionization can nonlinearly enhance the transport process. We show that such an enhancement of the plasma transport due to neutral depletion can result in an unexpected *decrease* of the plasma density when power is *increased,* despite the *increase* of the flux of generated plasma.

DOI: 10.1103/PhysRevLett.95.115002 PACS numbers: 52.20.−j, 52.50.Qt, 52.80.Pi

Plasmas in a pressure balance with a neutral gas are common in laboratory [1] and space [2]. Low-temperature laboratory plasmas in a pressure balance with a neutral gas are usually weakly ionized so that neutral-gas density is mostly unaffected by ionization. Useful analytical relations have been found for such plasmas within various diffusion models [3–6]. In the weakly ionized plasmas analyzed in the models particle balance is decoupled from energy balance and the electron temperature is found to be related to a single similarity variable, the product of the neutral-gas pressure and the plasma spatial extent [4]. The plasma density is determined by power balance and increases monotonically with deposited energy, as does plasma flux. Many plasmas in pressure balance, however, are not weakly ionized. In space plasmas in pressure balance with neutral gas, neutral depletion is often substantial. With the recent use of high power and low pressure gas, a significant neutral depletion became a major concern in low-temperature laboratory plasmas as well [7–9]. Nevertheless, analytical relations between the plasma and neutral parameters in the depleted-neutrals case, which could be of crucial importance in the study of laboratory and space plasmas, are not available. The difficulties in the analysis stem from the fact that when the neutral pressure is not uniform, the above-mentioned similarity variable is no longer well defined, and even an average value of that parameter is not predetermined anymore but rather varies with power. Moreover, particle balance and power balance become coupled and so do ionization and transport.

In this Letter we uncover the impact of neutral depletion on plasmas in pressure balance with neutral gas by solving self-consistently the plasma and neutral dynamics and deriving analytical relations. Since the results are of a general nature, not limited to weakly ionized plasmas, they are relevant to a variety of plasmas, ranging from low-temperature plasmas to interstellar gas [2]. We discover that the total number of neutrals replaces the above-mentioned similarity variable of the weakly ionized case as the parameter that determines the electron temperature. The classical linear diffusion equation [3] is replaced by a nonlinear diffusion equation for which we find an analytical solution in the form of Kepler's equation. It is shown that even a relatively small plasma density (1% of the neutral density) can alter the neutral density so that the plasma steady state is dramatically modified. Since the density of the plasma is determined by competition between ionization and decay through transport, an increase of the energy invested in ionization that increases the flux of generated plasma is expected to also increase the plasma density. One result of the inherent coupling of ionization and transport, however, is that a high enough ionization can nonlinearly enhance the transport process. We show that such an enhancement of the plasma transport due to neutral depletion can result in an unexpected *decrease* of the plasma density when power is *increased,* despite the *increase* of the flux of generated plasma. The possible existence of similar counterintuitive relations should be taken into account when measurements of plasma density are used to learn about laboratory or space plasma dynamics.

Let us assume a partially-ionized plasma slab the spatial extent of which is $2a$. The governing equations of the plasma and neutral dynamics are the continuity and momentum-balance equations. The continuity equations are

$$\frac{\partial \Gamma_i}{\partial x} = \beta N n; \qquad \frac{\partial \Gamma_N}{\partial x} = -\beta N n, \qquad (1)$$

where Γ_i and Γ_N are the plasma and gas fluxes, n and N are the plasma and the neutral-gas densities, and β is the ionization rate. The plasma is assumed unmagnetized along the x direction, and we restrict the analysis to the collisional regime. Therefore, both plasma and neutrals are pushed by gradients of pressure and exert on each other a drag force due to mutual collisions:

$$\frac{\partial(nT)}{\partial x} = F, \qquad \frac{\partial(NT_g)}{\partial x} = -F. \tag{2}$$

Here T is the electron temperature (assumed much larger than the ion temperature), T_g is the gas temperature, F $(-F)$ is the drag on the ions (neutrals) due to collisions with neutrals (ions) and points inward (outward), resulting in neutral pumping [7], balanced by the gradient of the plasma (neutral) pressure that exerts an outward (inward) force. Adding the two Eqs. (2), we obtain an equation that expresses a pressure balance between plasma and neutrals. Since $n(x = a) = 0$, we express that pressure balance as

$$NT_g + nT = N_w T_{g,w}, \tag{3}$$

where N_w and $T_{g,w}$ are the neutral density and temperature at $x = a$.

We first assume that the ion velocity v_i is much smaller than the neutral thermal velocity $v_T = \sqrt{8T_g/(\pi m)}$ (m is the neutral or ion mass). The drag force takes the form [6]

$$F = -\tfrac{4}{3} m\sigma_{iN} nN v_T v_i, \tag{4}$$

where σ_{iN} is the ion-neutral collision cross section. The opposite regime, in which $v_i \gg v_T$, is analyzed later. We also assume that v_i is larger than the neutral drift velocity.

Combining Eqs. (1) and (2) with the explicit forms for the neutral density (3) and for the force (4), we obtain the equation for the normalized plasma pressure $p_i \equiv nT/(N_W T_{g,w})$ as

$$\frac{\partial}{\partial \xi}\left[\frac{1}{(1-p_i)}\frac{\partial p_i}{\partial \xi}\right] + \alpha_L (1-p_i) p_i = 0, \tag{5}$$

where $\xi \equiv \int_0^x [T_{g,w}/T_g(x')](dx'/a)$ ($\xi = x/a$ if neutral temperature is uniform) and $\alpha_L \equiv 4m\sigma_{iN} v_T \beta a^2 N_W^2/(3T)$. In the case that $p_i \ll 1$, Eq. (5) is reduced to the familiar linear diffusion equation [3]. Equation (5) is first integrated to give $\partial p_i/\partial \xi = -\alpha_L^{1/2}(1-p_i)[p_i(0)^2 - p_i^2]^{1/2}$, an expression that allows us to express the total number of neutrals (per unit area), $N_T \equiv \int_0^a dxN$, as

$$N_T = \frac{N_w a}{\alpha_L^{1/2}} \int_0^1 \frac{dp_n}{(1-p_n^2)^{1/2}} = \frac{\pi}{2}\left(\frac{3T}{4m\sigma_{iN} v_T \beta}\right)^{1/2}, \tag{6}$$

where $p_n \equiv p_i/p_i(0)$. Relation (6) replaces the classical condition derived for the weakly ionized case [3,4]. It relates the total number of neutrals N_T to the electron temperature T. The parameter N_T is reduced to the similarity variable $N_w a$ in the weakly ionized case of uniform gas density [4].

We find additional relations by integrating Eq. (5) to obtain $\alpha_L^{1/2}\xi = \int_{p_n}^1 dp_n'/\{[1 - p_i(0)p_n'](1 - p_n'^2)^{1/2}\}$. Imposing the boundary condition $p_n = 0$ at $\xi = \xi_W \equiv \xi(x = a)$ provides us with the relation $\alpha_L^{1/2}\xi_W = \int_0^1 dp_n/\{[1 - p_i(0)p_n](1 - p_n^2)^{1/2}\}$. This equation can actually be integrated. By transforming it into $\alpha_L^{1/2}\xi = \int_\theta^{\pi/2} d\theta'/(1 - \sin\theta_0 \sin\theta')$, where $\sin\theta_0 \equiv p_i(0)$ and $\sin\theta \equiv p_n$, we obtain the relation $(\sin\theta - \sin\theta_0)/(\cos\theta\cos\theta_0) = \cot[\cos(\theta_0)\alpha_L^{1/2}\xi]$. Imposing the above boundary condition, this time in the form $\theta = 0$ at $\xi = \xi_W$, we obtain the relation

$$\theta_0 = \alpha_L^{1/2}\xi_W \cos\theta_0 - \frac{\pi}{2}. \tag{7}$$

Curiously, this algebraic plasma balance equation can be cast in the form of Kepler's equation [10]. Such an equation was recently used also to describe the dynamics of a quantum kicked rotor [11].

Using Eq. (7) we obtain the profile of the plasma density in the form

$$\frac{\sin\theta - \sin\theta_0}{\cos\theta\cos\theta_0} = \cot\left[\left(\theta_0 + \frac{\pi}{2}\right)\frac{\xi}{\xi_W}\right]. \tag{8}$$

Equations (7) and (8) are generalizations of the weakly ionized uniform neutral-density case to include neutral depletion. When neutral depletion is small, when $\theta_0 \ll \pi/2$ and $\xi_W \cong 1$, these relations yield $\alpha_L = (\pi/2)^2$, $\theta = (\pi/2)(1-\xi)$, and $p_n = \cos(\pi/2)\xi$; the specified value of α_L then determines the electron temperature, recovering the uniform neutral-density case.

Although neutral-gas heating could be significant [12], we restrict ourselves for simplicity to the uniform gas temperature $T_g = T_{gw}$ case, so that $\xi_W = 1$. We turn to an analysis of the effect of the power on the plasma steady state. The deposited power per unit area P equals the flux density of plasma particles Γ_i multiplied by the energy deposited in each such particle ε_T, $P = \varepsilon_T \Gamma_i(\xi = 1)$ [1]. From Eqs. (2) and (4) we obtain an expression for Γ_i and, by employing its value at the boundary, we obtain

$$P = \varepsilon_T\left(\frac{3\beta}{4\sigma_{iN} v_T}\right)^{1/2} v_s n_i(0). \tag{9}$$

Here $v_s = \sqrt{T/m}$. This last relation is of the same form as when the neutral density is uniform, although the neutral depletion may affect the values of the plasma parameters.

Equations (6), (7), and (9) determine the relations between the parameters of the neutral-depleted steady state, while Eq. (8) determines the spatial profile of the plasma density. These relations can be applied to a variety of plasmas. In space, for example, they could be used for the study of the relations between different phases in pressure equilibrium, such as the cold dense clouds embedded in the hot medium in the interstellar gas [13]. Similarly, they could be applied to determine the ratio between plasma and neutral pressures inside the ionized layer at the boundary of a molecular cloud [14], or in young stellar object jets [15]. Such applications to space plasmas may require modification of the model to account for the effects of gravity, magnetic field, or photoionization. As a concrete example, we choose laboratory neutral-depleted low-temperature plasmas.

We address two cases: the first case is a discharge in which N_T is specified and does not vary with power. This happens when the total amount of gas in a laboratory chamber is fixed. Since even when neutrals are depleted n is usually much smaller than N, this case corresponds to a fixed total number of gas particles, neutral plus ionized. In the second case the neutral density at the plasma boundary N_W is fixed while the power is increased. This happens, for example, when the edge of a discharge tube opens into a larger gas tube of a specified gas pressure, as is often the case in laboratory plasmas.

The numerical examples are for argon and the values of the atomic quantities are taken from the literature [1,6]. Also, $a = 5$ cm, the gas is at room temperature, and its pressure prior to the discharge is 10 mtorr. The calculated plasma and neutral-density profiles in both cases are shown in Fig. 1 for two power levels, low and high. When the power is low, the neutral density almost does not vary, while the plasma density has the familiar $\cos(\pi/2)\xi$ form. In both cases, however, when the power is high, the neutral and plasma densities are uniform across most of the discharge volume and sharply drop to zero (plasma) or sharply peak to a high density (neutrals) near the boundary, similar to what was found in measurements [8]. Note that in the first case the fixed total number of neutrals results in an *increase* of the total pressure with power. The increase of the total pressure, $N_W T_g$, is expressed at the discharge center by a large plasma pressure and near the wall by an increased neutral pressure.

The dependencies of the discharge parameters on power are shown in Fig. 2 (fixed N_T) and Fig. 3 (fixed N_W). Since, according to Eq. (6), the electron temperature is determined by the total number of gas particles, the electron temperature in the first case is fixed even if the deposited power is increased. While the constant electron temperature and the increase of the maximal density $n_i(0)$ and of the flux density Γ_i with power, are similar to the dependencies on power in the absence of neutral depletion, the flat density profile and the above-mentioned total pressure increase are clearly different.

The effect of neutral depletion is more pronounced in the second case. The specified constant neutral density at the plasma boundary N_W means a constant total pressure $N_W T_g$. As is shown in Fig. 3, the increase of power for a fixed total pressure is followed by an increase of plasma pressure at the expense of the neutral pressure. The decrease of N_T (the total number of gas particles) is followed by an increase of the electron temperature T. Since ε_T is a decreasing function of T, the increase of the electron temperature T with power results in an increase of the plasma flux Γ_i with power that is faster than linear. Interestingly, despite the fast increase of plasma production with power, the plasma density does not increase faster when the power is increased. Moreover, the behavior is nonmonotonic, and above a certain power the density *decreases* with the power *increase*. As is shown in the figure, this is true for both $n_i(0)$ and the total number of plasma particles, $\int_0^a dxn = (T_g/T)(N_w a - N_T)$ [following Eqs. (3) and (6)]. The decrease of plasma density with power exhibited in Fig. 3 is a result of deconfinement of the plasma by neutral depletion. The drag of the neutrals on the ions decreases when the neutrals are depleted. Thus, even though *more* ions are produced, they escape the volume *faster,* and therefore their density is *lower.* Often plasma dynamics is concluded from measurements of various plasma parameters. The possible existence of counterintuitive relations, demonstrated in the example, should be taken into account in interpreting these measurements.

For completion we briefly analyze the regime $v_i \gg v_T$ in which the collision term is nonlinear in v_i, a regime

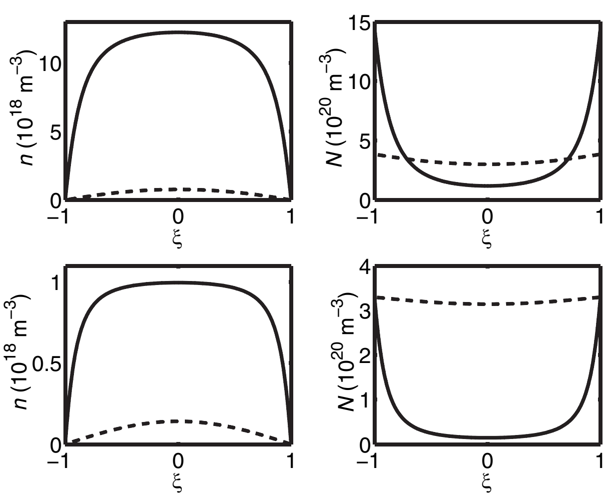

FIG. 1. Plasma and neutral-density profiles, fixed total neutral number—top ($P = 33$ kW/m^2 and 509 kW/m^2); fixed neutral density at the boundary—bottom ($P = 3$ kW/m^2 and 121 kW/m^2).

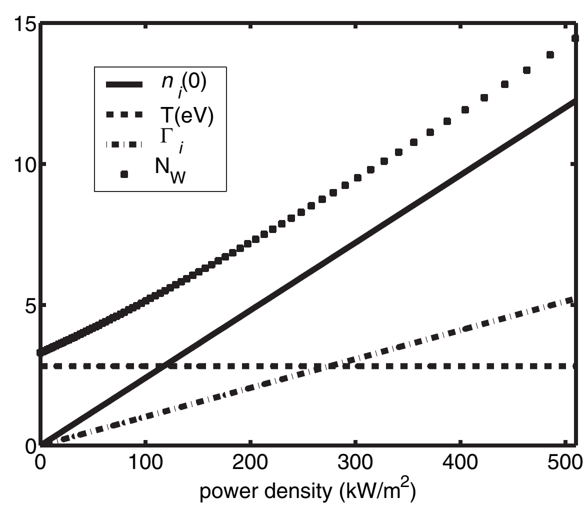

FIG. 2. Fixed total number of neutrals: $N_W(10^{20}\ \mathrm{m}^{-3})$, T(eV), $n_i(0)(10^{18}\ \mathrm{m}^{-3})$, $\Gamma_i(5\times 10^{21}\ \mathrm{m}^{-2}\,\mathrm{s}^{-1})$.

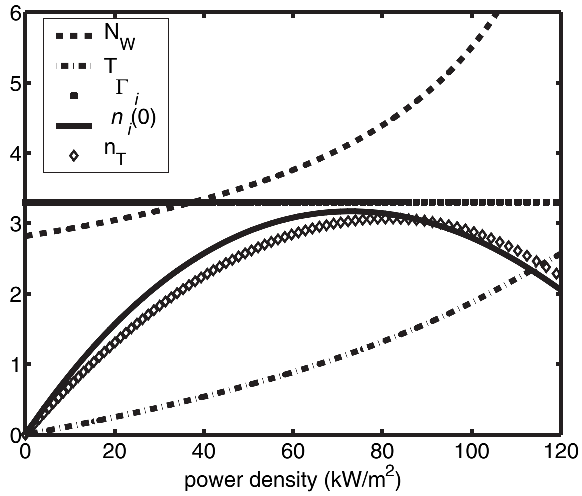

FIG. 3. Fixed neutral density at the boundary: $N_W(10^{20}\ \mathrm{m}^{-3})$, T(eV), $\Gamma_i(5\times 10^{21}\ \mathrm{m}^{-2}\,\mathrm{s}^{-1})$, $n_i(0)(5\times 10^{17}\ \mathrm{m}^{-3})$, $n_T(2\times 10^{16}\ \mathrm{m}^{-2})$.

explored by Godyak [5]. The force exerted on the ions by the neutrals is then

$$F = -m\sigma_{iN} n N v_i^2. \tag{10}$$

Equivalent to the previous analysis, the governing equation becomes $\partial/\partial x\sqrt{-(n/N)\partial/\partial x(nT)} = \sqrt{m\sigma_{iN}}\beta N n$. When pressure balance, Eq. (3), is employed, the equation in dimensionless form becomes

$$\frac{\partial}{\partial \xi}\sqrt{-\frac{p_i}{(1-p_i)}\frac{\partial p_i}{\partial \xi}} - \alpha_{NL}(1-p_i)p_i = 0, \tag{11}$$

where $\alpha_{NL} \equiv \beta\sqrt{m\sigma_{iN}(aN_W)^3/T}$. This equation is first integrated to give $-[p_i/(1-p_i)]\partial p_i/\partial\xi = \alpha_{NL}^{2/3}[p_i^3(0) - p_i^3]^{2/3}$, which is used to express the total number of neutrals as $(N_T/aN_w)\alpha_{NL}^{2/3} = \int_0^1 p_n(1-p_n^3)^{-2/3}dp_n = 2\pi/(3\sqrt{3})$. The last relation provides us with a relation between the total number of neutrals per unit area and the electron temperature,

$$N_T\left(\frac{m\sigma_{iN}\beta^2}{T}\right)^{1/3} = \frac{2\pi}{3\sqrt{3}}. \tag{12}$$

Again, the total number of neutrals is the parameter that determines the electron temperature.

We further integrate to obtain $\alpha_{NL}^{2/3}\xi = \int_{p_n}^1 dp_n' p_n'[1 - p_i(0)p_n']^{-1}(1-p_n'^3)^{-2/3}$, and, integrating the equation until the plasma boundary, we obtain that

$$\alpha_{NL}^{2/3}\xi_W = \int_0^1 \frac{p_n dp_n}{[1-p_i(0)p_n](1-p_n^3)^{2/3}}. \tag{13}$$

When neutrals are not depleted $p_i(0) \ll 1$, and we obtain the known result, $\alpha_{NL} = [2\pi/(3\sqrt{3})]^{3/2}$ [5,6] [which is identical to the relation (12) in which we substitute $N_T = N_W a$]. Similar to the previous regime [Eq. (9)], we obtain from power balance that $P = \varepsilon_T(\beta v_s^2/\sigma_{iN} v_T^3)^{1/3} \times (8/\pi)^{1/2} v_s n_i(0)$. Thus, the plasma behavior when the collision term is nonlinear is qualitatively similar to the behavior when the collision term is linear (analyzed in detail above).

In this Letter we have solved self-consistently the equations that describe the steady state of a plasma and a neutral gas in pressure balance as they often are in laboratory and in space. Relations that determine the electron temperature, the rate of ionization, and the plasma density were found analytically. The analytical results enabled us to examine the rich plasma behavior due to the neutral depletion. In particular, an unexpected nonmonotonic dependence of the plasma density on power was demonstrated.

One of us (A. F.) gratefully acknowledges the support of the CNRS during his research visit to Ecole Polytechnique. This research was partially supported by the Israel Science Foundation (Grant No. 59/99).

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