A Mechanism for Ion Acceleration Near the Anode of a Magnetically Insulated Ion Diode

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***Abstract*—We propose a theoretical model for the ion acceleration near the anode in the anode plasma of a magnetically insulated ion diode (MID). The applied diode voltage is screened by the anode plasma. We assume that as a result of the strong magnetization of the electrons and nonmagnetization of the ions in the anode plasma, a narrow space charge sheath is formed near the anode, of a width close to the electron Larmor radius. The potential drop that is formed across the sheath accelerates the ions born inside the sheath into the plasma. We solve a system of equations for the sheath and derive an approximate expression for the potential drop across the sheath. This expression exhibits the dependence of the ion energy on the magnetic field intensity and on the electron temperature and density. We also discuss an influence of the sheath width to the electron Larmor radius ratio as a parameter. The theoretical results are found to be in good agreement with the experimental measurements.**

***Index Terms*—Anode plasma, drift velocity, Hall current, ion acceleration, ion diode, magnetized electron, sheath.**

I. INTRODUCTION

STUDYING the properties of electrode plasmas in electrical discharges, in particular the plasma expansion, is highly important for understanding various key phenomena in such systems [1]. In this paper we attempt to explain part of the findings obtained in investigations of the anode plasma in a magnetically insulated ion diode (MID) [2]–[5]. These studies showed that a plasma 1–2 mm wide is formed near the anode surface within first 30 ns of the 100-ns-long voltage pulse. The plasma is seen to be fully penetrated by the magnetic field ($\approx$7 kG) externally applied into the anode-cathode gap parallel to the anode. Ions are extracted from the plasma into the anode-cathode gap and accelerated in the $\approx$0.3 MV applied voltage toward the cathode. The applied voltage is screened by the plasma. Throughout the applied voltage pulse ions are seen to move from the anode surface into the plasma with a kinetic energy of a few tens of eV [3], a kinetic energy much lower than the energy they later acquire while they cross the gap, but higher than the electron temperature, 7–10 eV. The are ions found to acquire this energy of a few tens of eV within $\approx$30 μm from the anode surface [5].

Recently, it was suggested that the ions acquire this energy through the expansion of a relatively dense plasma that is formed in a region a few-tens of μm wide near the anode [6], into the rest of the plasma. In this model, the electron temperature in the dense plasma was assumed to be higher than in the plasma. During the expansion, the electron thermal kinetic energy is converted into ion-directed kinetic energy.

In this paper we explore a different mechanism for the ion acceleration in the plasma. As in the previous model [6], we assume also here that ions that are born near the anode surface are accelerated toward the anode plasma by an electric field that is formed in a narrow sheath that is adjacent to the anode surface. However, here we assume that the electrons are drawn by the sheath electric field toward the anode surface, rather than being pushed by the pressure gradient away from the anode surface, as we assumed in the previous model [6]. We note that the voltage that is formed across the sheath is not related to the large applied voltage across the anode-cathode gap, a voltage that, as mentioned above, is screened by the plasma. The assumptions of this second mechanism are also consistent with the measurements [2]–[5]. Moreover, as we will show in Section III, the scaling of the ion energies with the magnetic field intensity, that is predicted by the present model, is also consistent with the experimental results [4]. The mechanism we explore here is similar to that described by Zharinov [7] and Morozov [8] for the plasma acceleration with a closed Hall current.

In Section II we present the model and in Section III we solve the model equations and compare the solutions to the experimental results.

II. THE MODEL

We first describe qualitatively the mechanism of the ion acceleration. Fig. 1 is a schematic of the anode plasma configuration. A planar anode lies in the y-z plane. The plasma is immersed in an external magnetic field that is parallel to the z direction. For the parameters mentioned above, a magnetic field of $\approx$7 kG, ion energies of few tens of eV, and electron temperature from 7 to 10 eV, the plasma width from 1 to 2 mm is smaller than the Larmor radii of the various plasma ions, and much larger than the electron Larmor radius r_e, which is $\approx$10 μm. We therefore treat the ions in the plasma as unmagnetized and the electrons as magnetized.

Following Zharinov [7], we assume that the magnetic field causes a sheath to be formed near the anode surface, in which a relatively high electric field normal to the anode surface (the x direction) prevails. This electric field is much larger than the electric field in the rest of the plasma (but much weaker than the applied electric field in the gap). The sheath

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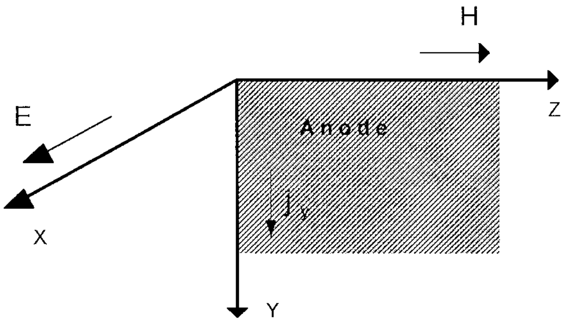

Fig. 1. Schematic of the experimental configuration.

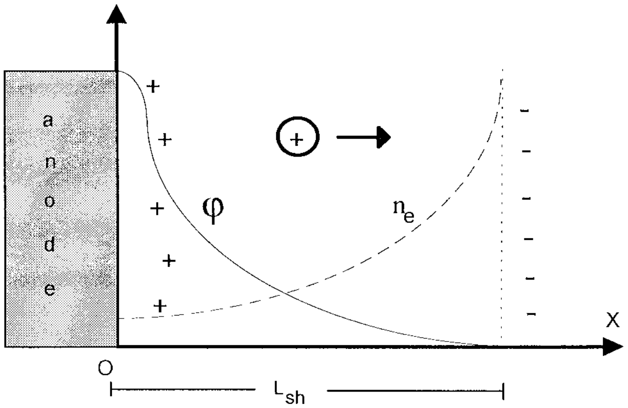

Fig. 2. Schematic of the near-anode electrical sheath and the potential and electron density profiles.

width $L_{\rm sh}$ is assumed to be comparable to the electron Larmor radius r_e ($\approx$10 μm). This is consistent with the experimental measurements mentioned above [3], that indicate that the ion acceleration in the plasma occurs within 30 μm from the anode. Though it is marginal, we also treat the electrons in the sheath as magnetized. An electron drift (Hall current) is formed in the y direction, normal to both the electric and the magnetic fields. The mobility of the electrons across the sheath is small, thus causing the plasma to be charged negatively with respect to the surface. The potential difference across the sheath depends on the plasma parameters, such as density and temperature, and on the magnetic field intensity. Ions accelerate as they cross the sheath where they acquire the energies that are observed inside the plasma.

The motions of both the ions and the electrons are assumed to be determined by the electric field, thus the ions and electrons move in opposite directions (this is contrary to the assumption made in [6] that due to the electron pressure both ions and electrons move in the same direction). A schematic of the sheath is shown in Fig. 2.

We assume that all the quantities only depend on x. The continuity equations for the ions and the electrons are then

$$dj_{ix}/dx = ZeS_i(x) \tag{1a}$$

and

$$dj_{ex}/dx = -eS_e(x) \tag{1b}$$

respectively, where $S_i(x)$ and $S_e(x)$ are the ion and electron source functions, j_i and j_e are the ion and electron current densities, $-e$ is the elementary charge and Ze is the ion charge. Integrating (1) we obtain

$$j_{ix}(x) = j_{ix}(0) + Ze\int_0^x S_i(x')\,dx' \tag{2a}$$

$$j_{ex}(x) = j_{ex}(L_{\rm sh}) - e\int_{L_{\rm sh}}^x S_e(x')\,dx'. \tag{2b}$$

The anode surface is located at $x = 0$, and the boundary between the sheath and the plasma is at $x = L_{\rm sh}$.

We now turn to the electron momentum equations. We assume that $L_{\rm sh}$ is order of the electron Larmor radius so that we can neglect the electron inertia. A similar assumption is made in analyzing the magnetron [9]. We also neglect the electron pressure. The electron motion in the magnetic field is thus determined by the electric field and by collisions. We obtain the following expressions for the components of the electron current:

$$j_{ey} = en_e v_{ey} = en_e \frac{cE_x}{H} \tag{3a}$$

and

$$j_{ex} = j_{ey}/\beta_e = en_e \frac{cE_x}{\beta_e H} \tag{3b}$$

where E_x is the electric field in the x-direction, H is the intensity of the magnetic field, c is the velocity of light in vacuum, v_{ey} is the electron drift velocity in the y direction, n_e, is the electron density, and $\beta_e \equiv \omega_e/\nu = \omega_e\tau_e$ is the Hall parameter. Also, $\omega_e \equiv eH/cm_e$ is the electron cyclotron frequency, where m_e is the electron mass, and τ_e is the electron-ion collision time, which is assumed to be [10]

$$\tau_e = \frac{4\cdot 10^4 \cdot T_e^{3/2}}{Z\cdot n_e}. \tag{4}$$

Here, T_e is the electron temperature in eV, n_e is expressed in cm^{-3}, and τ_e is obtained in seconds. It is seen in (3b) that, as was said above, the electrons move toward the anode, in a direction opposite the direction of the electric field.

Because the ions are collisionless and unmagnetized in sheath, we can write the Poisson equation with (3) as [6]

$$\frac{d^2\varphi}{dx^2} = -4\pi e\int_0^x \frac{S_i(x')\,dx'}{(\varphi(x')-\varphi(x))^{1/2}}\left(\frac{ZM}{2e}\right)^{1/2} + 4\pi e\frac{H\beta_e}{c} \times\left(-\frac{d\varphi}{dx}\right)^{-1}\left[-e\int_{L_{\rm sh}}^x S_e(x')\,dx' + j_{ex}(L_{\rm sh})\right]. \tag{5}$$

Here x' is the coordinate where the ion was born, φ is the electrical potential, and M is the ion mass and the relation $E_x = -\frac{d\varphi}{dx}$ was used.

We now make several approximations: we assume that

$$j_{ex}(L_{\rm sh}) \ll e\int_{L_{\rm sh}}^x S_e(x')\,dx'$$

and

$$j_{ix}(0) \ll Ze\int_0^x S_i(x')\,dx'.$$

We assume that ionization is dominant and, consequently, $ZS_i(x) = S_e(x) = S(x)$. These two assumptions imply that the ion and electron fluxes exiting the region are equal

$$j_{ix}(L_{\rm sh}) = j_{ex}(0). \tag{6}$$

The function $S(x)$ is not know. We, therefore, are not able to solve accurately for $\varphi(x)$ and we derive an approximate solution. First we replace $\varphi(x')$ in integral (5) by $\varphi(x=0) \equiv \varphi_a$, the potential at the anode surface. Following the above approximations, Poisson's equation becomes:

$$\begin{aligned}\frac{d^2\varphi}{dx^2} &= -4\pi e\left(\frac{M}{2Ze(\varphi_a-\varphi)}\right)^{1/2}\int_0^x S(x')\,dx' \\ &\quad + 4\pi e\frac{H\beta_e}{c}\left(\frac{d\varphi}{dx}\right)^{-1}\int_{L_{\rm sh}}^x S(x')\,dx'. \end{aligned}\tag{7}$$

For simplicity, we further assume that the ion and the electron currents are constant across the sheath, and have the equal average value

$$j \equiv e\int_0^{L_{\rm sh}} \frac{S(x')\,dx'}{2}. \tag{8}$$

This is only an approximation, since the ion current is zero at the anode while the electron current is zero at the plasma boundary. With approximation (8), (7) becomes

$$\frac{d^2\varphi}{dx^2} = -8\pi j\left[\left(\frac{M}{2Ze(\varphi_a-\varphi)}\right)^{1/2} + \frac{H\beta_e}{c}\left(\frac{d\varphi}{dx}\right)^{-1}\right]. \tag{9}$$

Integration of (9) gives

$$\begin{aligned}&\left(\frac{d\varphi}{dx}\right)^2 - \left(\frac{d\varphi}{dx}\right)^2(x=0) \\ &\quad = -16\pi j\left[-\left(\frac{2M}{Ze}(\varphi_a-\varphi)\right)^{1/2} + \frac{H\beta_e}{c}x\right]\end{aligned} \tag{10}$$

Again, for simplicity, we assume that at the anode surface $(x=0)$ and at the external sheath boundary $(x=L_{\rm sh})$ the electric field is small, i.e., $\frac{d\varphi}{dx} \approx 0$ and $\varphi(x=L_{\rm sh}) \equiv \varphi(L_{\rm sh})$. Thus, from (10) we obtain the relation

$$\varphi_{\rm sh} \equiv \varphi(0) - \varphi(L_{\rm sh}) = \frac{Ze}{2M}\left(\frac{\beta_e H L_{\rm sh}}{c}\right)^2 \tag{11}$$

or

$$\frac{e\varphi_{\rm sh}}{T_e} = Z\beta_e^2\frac{mL_{\rm sh}^2}{Mr_e^2}. \tag{12}$$

As mentioned above, following [7], for simplicity we make the further assumption that $L_{\rm sh} = k_r r_e$ where k_r is coefficient of proportionality. Equation (12) becomes

$$\varphi_{\rm sh} = k_r^2 Z\beta_e^2\frac{mT_e}{Me}. \tag{13}$$

In order to expose the explicit dependence of the sheath potential on the various parameters, we use expression (4) for the collision time. The anode sheath potential drop becomes

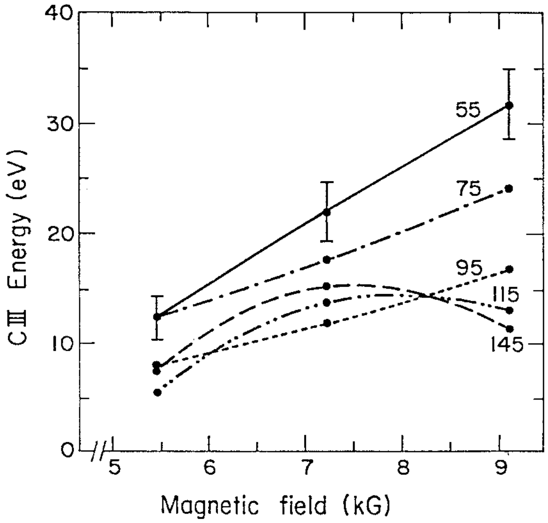

Fig. 3. Experimental data of the ion energy versus magnetic field intensity (taken from [3]).

$$\varphi_{\rm sh} = k_r^2\frac{2.7\cdot 10^{20}H^2T_e^4}{ZA_i n_e^2} \tag{14}$$

where A_i is the atomic number, H is expressed in Gauss, and $\varphi_{\rm sh}$ obtained in volts.

III. Results and Discussion

Examination of expression (14) shows that the anode potential drop $\varphi_{\rm sh}$ and, consequently, the ion velocity are strongly dependent on the plasma parameters (T_e and n_e). The potential $\varphi_{\rm sh}$ varies as T_e^4 and as n_e^{-2}.

We note that the anode plasma in the experiment is composed of several ions. However, hydrogen is ionized slower than carbon and it is likely that the protons are mostly born in the plasma and not in the sheath, as is demonstrated in [5]. We, therefore, assume that all the plasma ions in the sheath are carbon ions.

In our model all the ions are at the same-charge state. The ion energy $eZ\varphi_{\rm sh}$ calculated using (14) turns out to be independent of Z. In the experiment, both CII and CIII were present. For such a plasma, that is composed of ions of more than one charge-state, our model only gives an average energy. This calculated average energy, that we further discuss below, is smaller than the CIII energy and larger than the CII energy that we would have calculated had we used a model that takes into account the presence of two charge states. Incidentally, the measured CII and CIII energies are found to be similar (see Fig. 7 in [3]), probably because CII is ionized into CIII only at the plasma edge of the sheath. However, the two simplifications in the model, the assumption of only one-charge-state ions and the assumption that all the ions are accelerated across the full sheath voltage, do limit the accuracy of our calculations.

Fig. 3 (taken from [3]) shows the measured CIII energy as a function of the intensity of the applied magnetic field for five different times after the application of the voltage. The

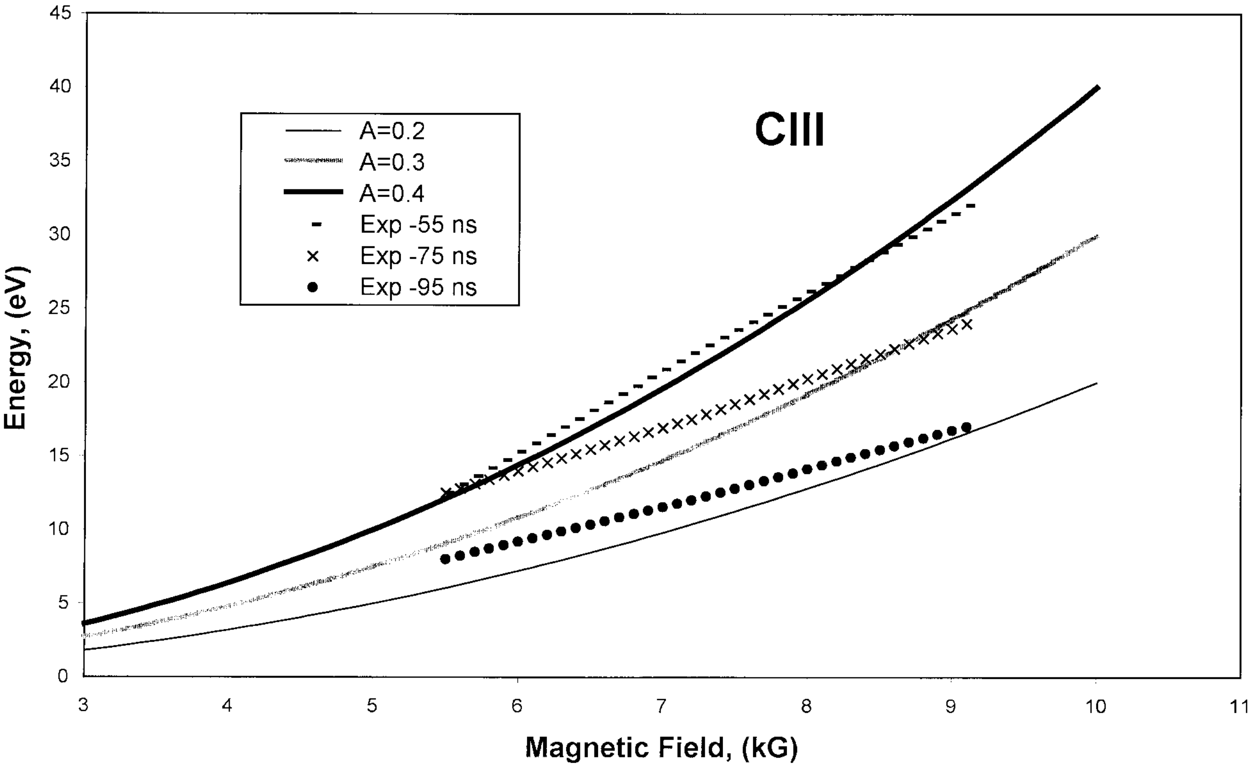

Fig. 4. Comparison of the dependence of the CIII energy in a CIII plasma on the magnetic field intensity predicted by our model with that obtained by the measurements. The dotted curves (Exp-55 ns, Exp-75 ns and Exp-95 ns) are the measured results also shown in Fig. 3. The calculated results are for different values of $A = 0.2$, 0.3, and 0.4.

experimental measurements are shown (in form of approximation lines) again in Fig. 4 for the three earlier times in Fig. 3. Fig. 4 also show CIII energy as a function of the intensity of the magnetic field calculated by (14), which is presented in the form

$$eZ\varphi_{\mathrm{sh}} = A \cdot H^2 \tag{15}$$

where $A = 2.25 \cdot 10^{25}\frac{k_r^2 T_e^4}{n_e^2}$, and the magnetic field is in kG, T_e in eV, and n_e in cm^{-3}. The calculated results are presented for three values of A $(0.2, 0.3$, and $0.4)$. It easy to see from Fig. 4 a good agreement with the experimental data for those values of A. The influence of the coefficient A corresponds to the influence of the combination of the electron temperature T_e, the density n_e and the coefficient k_r. For example, when $n_e = 1.5 \cdot 10^{15}$ cm^{-3} and $k_r = 1$ the electron temperature $T_e = 11.9$ eV, 13.2 eV, and 14.2 eV for $A = 0.2, 0.3$, and 0.4, respectively, when $n_e = 2 \cdot 10^{15}$ cm^{-3} and $k_r = 3$ the electron temperature $T_e = 7.9$ eV, 8.8 eV and 9.4 eV for $A = 0.2, 0.3$, and 0.4 respectively, and when $n_e = 1 \cdot 10^{15}$ cm^{-3} and $k_r = 2$ the electron temperature $T_e = 6.7$, 7.4, and 7.9 eV for $A = 0.2, 0.3$, and 0.4, respectively.

Plotted in Fig. 4 is the ion energy $Ze\varphi_{\mathrm{sh}}$ versus H, where Z for CIII is two. As mentioned above, the potential φ_{sh} is calculated by using (15), assuming that the plasma is only composed of CIII and therefore $A_i = 12$. We see in the Fig. 4 that by an appropriate choice of the electron density and temperature, which are approximately the measured values, our model predicts the measured ion energy. For example, $k_r = 1$ and $n_e = 1 \cdot 10^{15}$ cm^{-3}, a calculation with $T_e = 9.7$ eV (that corresponds to $A = 0.2$) yields approximately the experimental curve at $t = 95$ ns, and a calculation with $T_e = 11.5$ eV (that corresponds to $A = 0.4$) yields a curve that is similar to the experimental curve at $t = 55$ ns. This suggests the possibility that the observed decrease of ion energies in time is a result of the decrease in time of the electron temperature and increase of the electron density. We note that the values of T_e and n_e, for which there is an agreement between the measured and the calculated ion energes depend on the sheath width to electron Larmor radius ratio k_r (for $k_r = 3$ and $n_e = 1 \cdot 10^{15}$ cm^{-3} the electron temperature decreases up to ~5.6 eV). For T_e and n_e to be in range of measured values k_r should be of order unity.

In the model presented here ions that are born near the anode are drawn toward the plasma. The plasma quasineutrality is nevertheless preserved in the experiment, despite the fact that the electrons that are born near the anode do not follow the ions, but rather move to the anode. The plasma may remain quasineutral due to the following reasons. First, in the experiment an ion current is drawn from the plasma into the diode gap and toward the cathode, reducing the accumulation of a positive charge in the plasma. Second, the source of the plasma ions is not only the ions born in the sheath, but also ionization in the plasma, an ionization that provides the plasma with both ions and electrons. Finally, the large diamagnetic electron current in the plasma [11] may add net negative charges to the plasma that neutralize part of the ions charge. We note that the sheath model presented here is time-independent. This seems to be a good approximation since the ion transit time across the sheath is about 1 ns, much shorter than the voltage pulse duration.

It is clear from the model that if the electron density and temperature are constant the ion energy increases monotonically with the increase of the magnetic field. However, a nonmonotonic dependence of the CIII energy on the magnetic field intensity and the existence of a maximum were observed in the experiments at the end of the discharge pulse ($t \sim 100$ ns, Fig. 3). In the frame of the Hall model this observed nonmonotonic dependence may be due to the plasma density and electron temperature variation with the magnetic field intensity, more sensitive after the discharge pulse.

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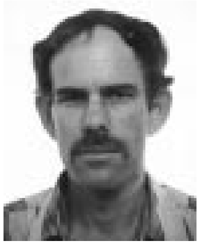

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