A vacuum sheath propagation along a cathode

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The velocity of propagation of a vacuum sheath along the cathode is calculated. A regime of parameters is identified in which, surprisingly, this velocity is lower for higher currents. © *1996 American Institute of Physics.* [S1070-664X(96)00208-X]

I. INTRODUCTION

Emission of electrons from an electrode that is immersed in a plasma is a basic process common to many devices. When the electron current drawn from the electrode is large, a sheath is formed along the electrode that may evolve into a vacuum gap. The evolution of the sheath determines the energy of the emitted electrons that propagate in the plasma, and therefore affects the overall resistance of the plasma. In the plasma opening switch (POS)[1,2] the sheath evolution might be crucial in determining the conduction time and the load power delivery. Plasma erosion, plasma pushing by the magnetic field pressure, and magnetic insulation of the electrons govern the sheath evolution. Because of the complexity of the phenomena, most analytical treatments of the sheath evolution are either one dimensional (1D)[1–4] or stationary two dimensional (2D).[5,6] The evolution in time of the 2-D sheath is not well understood. Mendel[7] analyzed this 2-D evolution at the stage at which the current is so large that part of the electrons in the sheath that becomes a vacuum gap is magnetically insulated. During this stage the voltage between the cathode and the plasma is several kV. Thus, a large flux of electrons (hundreds of kA) with moderate energy (keV) flow in the plasma. This stage lasts for a short time only. When the sheath reaches the plasma–vacuum boundary the voltage between the electrodes across the plasma becomes several MV. In Mendel's analysis the velocity of sheath propagation was shown to be $(18c^2 v_A)^{1/3}$, where c is the velocity of light in vacuum and v_A is Alfvén velocity. As could be intuitively anticipated, for a larger v_A the sheath propagates with a higher velocity. In this paper we show that when v_A is larger than $\frac{3}{2}c\epsilon^{3/4}$ (ϵ is the mass ratio), the velocity of the sheath is smaller for a larger v_A. This counterintuitive decreasing of the sheath velocity with the current increase is a result of the physical mechanism by which the plasma carries a current.

II. THE MODEL

Figures 1(a)–1(c) are schematics of a current-conducting plasma in the POS. The plasma fills a space of a finite axial extent between the two conductors of a transmission line. Power flows from the generator (to the left of the plasma in the figures) towards the load (to the right of the plasma). When a voltage appears between the electrodes, electrons are emitted from the cathode at the generator side of the cathode–plasma boundary. These electrons participate in the current conduction by the plasma. The plasma near the cathode in the region through which the current flows becomes tenuous, either because of erosion or because of being pushed by the magnetic field pressure away from the cathode. We assume that a vacuum gap is formed across which the electrons emitted from the cathode pass during their motion into the plasma.[1,2] At this stage [shown in Fig. 1(a)] the cathode–plasma boundary is therefore composed of two regions, region 1 of unperturbed plasma, and region 2 in which electrons are emitted and cross the vacuum gap. The vacuum gap widens and propagates axially. At the second stage the electrons that are emitted from the cathode at the generator side of the plasma, where the gap size is larger, become magnetically insulated. This part of the cathode–plasma boundary is denoted as region 3 in Fig. 1. Regions 2 and 3 of the boundary compose the sheath, which in this paper is assumed to be a vacuum sheath, out of which the plasma has been pushed. Region 3 propagates to the right as does region 2. It is this stage, shown in Fig. 1(b), during which the cathode–plasma boundary is composed of the three regions, that is treated in this paper. At the third stage region 2 reaches the load-side edge of the plasma, and the sheath (regions 2 and 3) then occupies the whole cathode–plasma boundary. This stage is shown in Fig. 1(c). This is the termination of the conduction phase of the POS operation. A voltage appears at the load and the switch begins to open. Finally, all the emitted electrons become magnetically insulated and no longer flow through the plasma. Only region 3 exists then at the cathode–plasma boundary.

In simplified 1-D models the sheath is assumed to evolve in time while remaining axially uniform. The whole sheath is considered first as region 2 and later as region 3. In order to find the conduction time, we take into account in this paper the 2-D nature of the sheath and calculate the velocity of propagation of regions 2 and 3 along the cathode.

In order to describe the evolution of the magnetic field flux in the sheath at the cathode–plasma boundary, we apply Faraday's law to the dashed rectangle in Fig. 1(b). Faraday's law takes the form

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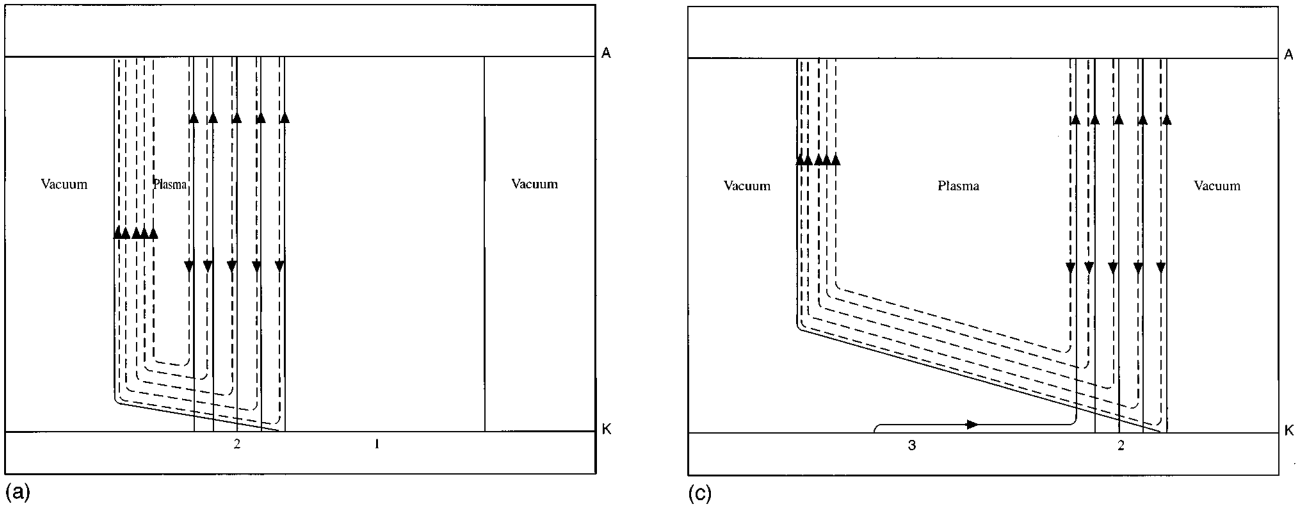

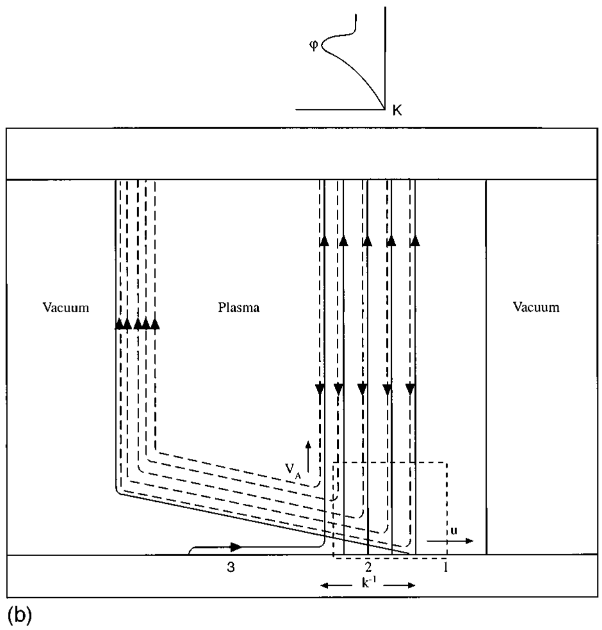

FIG. 1. Schematics of the vacuum sheath evolution. The sheath is composed of regions 2 and 3 located between the cathode and the plasma. The electrons emitted from the cathode in region 2 (denoted by solid lines) move ballistically in the plasma, while the plasma electrons (denoted by dotted lines) generate a return current and then flow along the plasma boundary. The width of the current layer at the plasma–vacuum boundary is enlarged in the figures. The emitted electrons are first accelerated by the electric field and then decelerate while moving into the plasma. The sketch between (a) and (b) shows the nonmonotonic dependence of the electric potential on the distance from the cathode and φ denotes the value of the potential at the peak of the potential plot. Region 1 is an unperturbed plasma. In region 3 the electrons do not cross the gap since they are magnetically insulated. (a) The initial stage—no region of magnetic insulation (region 3). (b) The second stage—the sheath propagates axially. The dashed rectangle is used for applying Faraday's law. This is the stage treated in the paper. (c) The third stage—the sheath has reached the load-side of the plasma.

$$-\frac{1}{c}\frac{\partial}{\partial t}\int_x^\infty B(x',t)y_0(x',t)dx' = \varphi(x,y_0(x),t)-\frac{B(x,t)^2}{8\pi ne}. \quad (1)$$

Here x is the coordinate parallel to the cathode in the direction of the power flow and y is the coordinate perpendicular to the cathode, B is the z (and only) component of the magnetic field (it is negative in our notation), φ is the electric potential difference across the sheath evaluated at y_0, n is the electron plasma density, and e is the elementary charge. We assume that the magnetic field is present only inside the sheath of a size $y_0(x,t)$, and that the magnetic field in the sheath is a function of x and t only. Therefore, the left-hand side (lhs) of Eq. (1) is the change-in-time of the magnetic field flux through the dashed rectangle. The right-hand side (rhs) is the integral along the sides of the dashed rectangle of the parallel electric field. This integral (the loop voltage) equals the potential difference between the cathode and the plasma along the left side of the rectangle, because the electric fields parallel to the three other sides of the rectangle are zero. Inside the plasma the electric field is zero and therefore it is zero along the upper and the right sides of the rectangle. At the cathode the parallel electric field is zero, and therefore there is also no contribution to the loop voltage from the lower side of the rectangle.

We note that, as shown in the sketch at the top of Fig. 1(b), the potential does not vary monotonically as a function of the distance from the cathode. The electrons emitted from the cathode are first accelerated by the electric field during their motion in the sheath. They are then decelerated while they move into the plasma across the potential hill at the plasma boundary. The potential hill exists because the boundary between an unmagnetized plasma and a vacuum that is permeated by a magnetic field is positively charged.[8] The resulting electric field pushes the plasma ions away from the cathode. This electric field also causes the plasma electrons to drift along the plasma boundary and to form a diamagnetic current. The unmagnetized plasma–vacuum boundary was discussed in detail in Ref. 9 for the case that an electron beam is injected into the plasma and the height of the potential hill was shown to equal the second term on the rhs of Eq. (1). The associated electric field that pushes the plasma ions away from the cathode decelerates the electrons emitted from the cathode after they have been accelerated in the sheath. Thus, while the first term on the rhs of Eq. (1) expresses the electron acceleration in the sheath, the second term expresses the deceleration across the potential hill. The sum of the two terms on the rhs of Eq. (1) is the loop voltage along the sides of the dashed rectangle in Fig. 1(b). Equation (1) was used by Mendel in his analysis,[7] with only the first term on the rhs. The novel phenomenon described in this paper results from the inclusion of the second term.

In Fig. 1 the electrons emitted from the cathode (denoted by solid lines) are shown moving as a beam ballistically across the plasma. The plasma electrons (denoted by dotted lines) are generating a return current in the plasma and a diamagnetic current along the plasma–vacuum boundary. As mentioned above, this diamagnetic current is driven by the potential hill that is expressed in the second term on the rhs of Eq. (1). We note that the present analysis is valid also in the case that there is no return current but rather the electrons emitted from the cathode provide the diamagnetic current. We also note that there might be a sheath at the anode–plasma boundary, and a voltage across it. We do not address the anode sheath here.

We assume that the sheath size is determined by the magnetic field pressure

$$\frac{\partial y_0}{\partial t} = -\frac{B}{(4\pi n M/Z)^{1/2} c}, \tag{2}$$

and that the electron flow in the gap is governed by the Child–Langmuir law

$$-j = \frac{1}{9\pi}\left(\frac{2e}{m}\right)^{1/2} \frac{\varphi^{3/2}}{y_0^2}. \tag{3}$$

Here Z and M are the ion charge number and mass, m is the electron mass, and j is the current density in the y direction. We also use Ampère's law,

$$-\frac{4\pi}{c} j = \frac{\partial B}{\partial x}, \tag{4}$$

neglecting the displacement current. Equations (1)–(4) are the governing equations. Equation (3) describes the sheath evolution in region 2 only, in which the electrons are unmagnetized.

In order to simplify the analysis, we introduce the following dimensionless quantities:

$$\tilde{y}_0 \equiv \frac{\omega_{pe}}{c} y_0, \tag{5a}$$

$$\tilde{x} \equiv \frac{\omega_{pi}}{2c} x, \tag{5b}$$

$$\tau \equiv \tfrac{3}{2}\epsilon^{1/4}\omega_{pi} t, \tag{5c}$$

$$\tilde{\varphi} \equiv \frac{8e}{9mc^2\epsilon^{1/2}} \varphi, \tag{5d}$$

and

$$b \equiv \frac{B}{B_0}. \tag{5e}$$

Here $\omega_{pi} \equiv (4\pi n e^2 Z/M)^{1/2}$ is the ion plasma frequency, $c/\omega_{pe} \equiv \epsilon^{1/2} c/\omega_{pi}$ is the electron skin depth ($\epsilon \equiv Zm/M$), and B_0 is the largest (negative) value reached by the magnetic field in the transmission line.

Equation (1), written for the above dimensionless quantities, becomes

$$p\frac{\partial}{\partial \tau}\int_{\tilde{x}}^{\infty} b(\tilde{x}',\tau)\tilde{y}_0(\tilde{x}',\tau)d\tilde{x}' = \tilde{\varphi}(\tilde{x},\tau) - p^2 b^2(\tilde{x},\tau). \tag{6}$$

Equation (2) becomes

$$\frac{\partial \tilde{y}_0}{\partial \tau} = pb, \tag{7}$$

and Eqs. (3) and (4) are combined to form

$$-p\frac{\partial b}{\partial \tilde{x}} = \frac{\tilde{\varphi}^{3/2}}{\tilde{y}_0^2}. \tag{8}$$

In Eqs. (6)–(8) the single parameter p is

$$p \equiv \frac{2}{3}\frac{v_{\mathrm{A}}}{c\epsilon^{3/4}}, \tag{9}$$

where $v_{\mathrm{A}} \equiv -B_0/(4\pi n M/Z)^{1/2}$. Similarly to the equations in Mendel's analysis,[7] Eqs. (6)–(8) have solutions of a traveling wave form

$$\tilde{y}_0 = \tilde{y}_m \exp[-\tilde{k}(\tilde{x} - \tilde{u}\tau)], \tag{10a}$$

$$b = \exp[-\tilde{k}(\tilde{x} - \tilde{u}\tau)], \tag{10b}$$

and

$$\tilde{\varphi} = \tilde{\varphi}_m \exp[-2\tilde{k}(\tilde{x} - \tilde{u}\tau)], \tag{10c}$$

for $\tilde{x} - \tilde{u}\tau > 0$. Region 3 in which the electrons are magnetically insulated is located at $\tilde{x} < \tilde{u}\tau$. By choosing a traveling wave solution we describe the evolution during the period of time at which the generator current and the associated magnetic field have reached constant values. However, we believe that the behavior in the general case is similar. Region 2 of the sheath extends in our solution to a constant-in-time axial length $\tilde{k}^{-1}$. Region 2 and the boundary between regions 2 and 3 propagate axially with a constant velocity $\tilde{u}$. We next find the values of $\tilde{k}$ and $\tilde{u}$ and the values of $\tilde{y}_m$ and $\tilde{\varphi}_m$ as functions of the parameter p.

Equations (6)–(8) become

$$\frac{p^2}{\tilde{k}} = \tilde{\varphi}_m - p^2, \tag{11a}$$

$$\tilde{k}\tilde{u}\tilde{y}_m = p, \tag{11b}$$

and

$$p\tilde{k} = \frac{\tilde{\varphi}_m^{3/2}}{y_m^2}. \tag{11c}$$

For a given p these are three equations for the unknown $\tilde{\varphi}_m$, $\tilde{y}_m$, $\tilde{k}$, and $\tilde{u}$. An additional equation is obtained from the

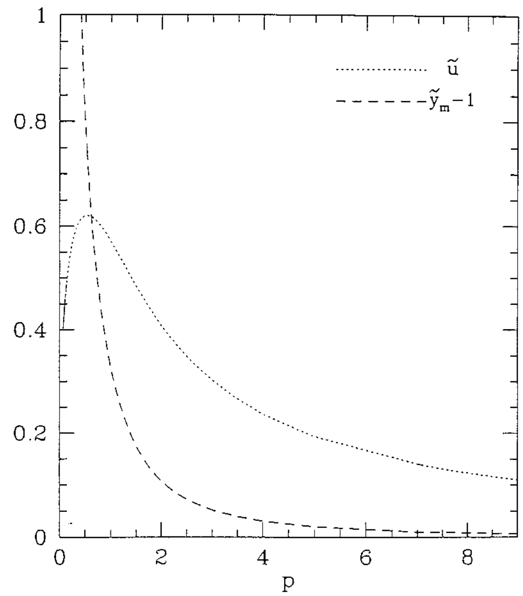

FIG. 2. The normalized velocity of sheath propagation $\tilde{u}$ and sheath size $\tilde{y}_m$ as a function of the parameter p.

magnetic insulation of the electrons. The gap size at $\tilde{x}-\tilde{u}\tau=0$ equals the electron Larmor radius. In the dimensionless coordinates this implies that

$$p\tilde{y}_m=\tilde{\varphi}_m^{1/2}. \tag{12}$$

It is convenient to combine the equations to form

$$p^2\tilde{y}_m(\tilde{y}_m^2-1)=1. \tag{13}$$

III. THE VACUUM SHEATH EVOLUTION

Figure 2 shows $\tilde{u}$ and $\tilde{y}_m$ as a function of p. At the limit in which

$$p\ll 1, \tag{14}$$

the sheath size is

$$\tilde{y}_m=\frac{1}{p^{2/3}}. \tag{15}$$

In that case, the sheath

$$\tilde{y}_m\gg 1 \tag{16}$$

and is much broader than the electron skin depth. The other quantities are

$$\tilde{k}=p^{4/3}, \tag{17a}$$

$$\tilde{\varphi}=p^{2/3}, \tag{17b}$$

$$\tilde{u}=p^{1/3}. \tag{17c}$$

Equations (15) and (17) describe the case analyzed by Mendel,[7] and inequality (14) defines the regime of validity of this case. The potential difference between the cathode and the plasma [the rhs of Eq. (11a)] is $p^{2/3}$.

At the opposite limit

$$p\gg 1. \tag{18}$$

In that case the sheath size is comparable to the electron skin depth,

$$\tilde{y}_m\cong(1+\delta), \tag{19}$$

where

$$\delta\equiv\frac{1}{2p^2},$$

and

$$\tilde{k}=p^2, \tag{20a}$$

$$\tilde{\varphi}=p^2, \tag{20b}$$

$$\tilde{u}=\frac{1}{p}. \tag{20c}$$

We note that in this regime the potential difference between the cathode and the plasma [the rhs of Eq. (11a)] is unity. While in the regime described by Mendel[7] $\tilde{u}$ is an increasing function of p, in the new regime explored here $\tilde{u}$ is a decreasing function of p. These two regimes are presented in Fig. 2.

With the definitions (5), let us write these results in dimensional units. The dimensional quantities are $k=\tilde{k}\omega_{pi}/2c$, $y_m=\tilde{y}_m c/\omega_{pe}$, $\varphi_m=9\tilde{\varphi}_m mc^2\epsilon^{1/2}/8e$, and $u=3\tilde{u}c\epsilon^{1/4}$.

In the first regime

$$\frac{2}{3}\frac{v_\mathrm{A}}{c\epsilon^{3/4}}\ll 1. \tag{21}$$

The sheath size is

$$y_m=\frac{c}{\omega_{pe}}\epsilon^{1/2}\left(\frac{3c}{2v_\mathrm{A}}\right)^{2/3}, \tag{22a}$$

the inverse length of the sheath is

$$k=\frac{\omega_{pi}}{2c\epsilon}\left(\frac{2v_\mathrm{A}}{3c}\right)^{4/3}, \tag{22b}$$

the electric potential is

$$\varphi_m=\frac{9mc^2}{8e}\left(\frac{2v_\mathrm{A}}{3c}\right)^{2/3}, \tag{22c}$$

and the sheath velocity is

$$u=(18c^2v_\mathrm{A})^{1/3}. \tag{22d}$$

The energy that the electrons emitted from the cathode have when they move into the plasma is obtained from the rhs of Eq. (11a). In the regime defined by Eq. (21) this energy is

$$\frac{9mc^2}{8}\left(\frac{2v_\mathrm{A}}{3c}\right)^{2/3}. \tag{23}$$

In the second regime

$$\frac{2}{3}\frac{v_\mathrm{A}}{c\epsilon^{3/4}}\gg 1. \tag{24}$$

Then the sheath size is

$$y_m\cong\frac{c}{\omega_{pe}}, \tag{25a}$$

and its inverse length

$$k=\frac{\omega_{pe}}{2c}\left(\frac{2v_{\mathrm{A}e}}{3c}\right)^2, \tag{25b}$$

where

$$v_{Ae} \equiv \frac{v_A}{\epsilon^{1/2}},$$

the electric potential is

$$\varphi_m = \frac{B_0^2}{8\pi ne}, \tag{25c}$$

and the sheath velocity is

$$u = \frac{9c^2\epsilon}{2v_A}. \tag{25d}$$

The velocity of propagation of the sheath along the cathode is lower for higher currents. The energy that the electrons emitted from the cathode have when they move into the plasma, in the regime defined by Eq. (24), is

$$\frac{9mc^2}{8}\,\epsilon^{1/2}. \tag{26}$$

The energy (26) is larger than the energy (23). The energy (26) is the maximal energy the electrons emitted from the cathode have inside the plasma. It is clear though that during their motion across the potential hill they may have a higher energy.

IV. DISCUSSION

Several conditions have to be satisfied in order for the model to be valid. We require that

$$ky_m \ll 1, \tag{27a}$$

$$u \gg v_A, \tag{27b}$$

and

$$v_{Ae} \ll 1. \tag{27c}$$

In order for (27) to be satisfied, p should be much smaller than $2/(3\epsilon^{1/4})$. A more stringent condition follows the assumption that the electrons emitted from the cathode move as a beam ballistically in the plasma. In that case the beam current is neutralized by the plasma return current, as seen in Fig. 2. The current neutralization is possible only if the beam density is lower than the plasma density, or if the velocity of the beam electrons v_b is larger than the velocity of the plasma electrons v_e. In our units the requirement turns out to be

$$v_b = \frac{3}{2}\,c\epsilon^{1/4} \gg v_e = \frac{v_A p^2}{2}. \tag{28}$$

In order for that to be satisfied,

$$p \ll \frac{2^{1/3}}{\epsilon^{1/6}}. \tag{29}$$

The interesting regime, in which u is a decreasing function of p, is therefore

$$1 \ll p \ll \frac{2^{1/3}}{\epsilon^{1/6}}. \tag{30}$$

For a hydrogen plasma this regime is for p between 1 and 4.4, while for a carbon plasma with $Z=2$, p is between 1 and 6. If the electron density is 10^{15} cm^{-3}, the magnetic field at the cathode, for which inequality (30) is satisfied, is approximately between 6 and 28 kG for the hydrogen plasma, and between 4 and 24 kG for the carbon plasma. The regime described by inequality (14) is valid only for weaker magnetic fields.

An additional requirement in order for the plasma to remain unmagnetized is that the sheath velocity u is larger than the velocity of the Hall-induced penetration[10] v_H $[\equiv v_A c/(\omega_{pi} l)$, l is a characteristic plasma length]. This requirement imposes the condition

$$p \ll \frac{\sqrt{2}}{\epsilon^{1/4}} \left(\frac{l\omega_{pi}}{c} \right)^{1/2}. \tag{31}$$

A necessary condition in order for both the left inequality in (30) and inequality (31) to be satisfied is that

$$l \gg \frac{c}{2\omega_{pe}}. \tag{32}$$

One could argue that the emitted electrons do not necessarily move ballistically in the plasma, nor is a plasma return current set up. However, as we mentioned in Sec. II, our model describes the sheath evolution also in the case that the emitted electrons flow along the plasma boundary and form the diamagnetic current. The upper limit in inequality (30) is in fact too strict.

In summary, we presented here a 2-D time-dependent model of a vacuum sheath evolution along a cathode. We identified a regime of parameters in which the velocity of sheath propagation is lower for higher currents. We note that the assumption that the sheath propagates by forming a vacuum gap is not necessarily true. Currently under investigation is the propagation of the sheath along the cathode in a quasineutral plasma that is induced by plasma erosion, and that is followed only later by a vacuum gap formation.

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