On the conduction of a current in a plasma-filled diode

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Two issues in plasma-filled diodes are addressed: electrostatic transient effects during the rise time of the current, and current neutralization in the bulk of the plasma. For the first issue an analytical method is used to recover some features of the diode behavior recently demonstrated in simulations [Phys. Fluids B **4**, 3608 (1992)]. The potential and the electron flow are shown to be oscillatory at the initial phase of current rise, before ions start to move. The possibility of electron trapping in the potential hill is discussed. For the second issue an equilibrium is constructed that describes a plasma of dimensions larger than the electron skin depth. A beam of charged particles moves ballistically into the plasma and the plasma electrons generate a return current that neutralizes the beam current. When the plasma electrons reach the plasma boundary they bend into a skin layer and conduct the current along the plasma boundary. © *1995 American Institute of Physics.*

I. INTRODUCTION

The complicated behavior of current conduction in plasma-filled diodes (PFD) that is reflected in numerical simulations[1–4] results from the simultaneous presence of several phenomena. Examples are the formation of space-charge-limited flows, plasma oscillations, electron trapping, potential hill formation, ion erosion, electron beam–plasma interaction, current neutralization, and the establishment of plasma diamagnetism. In order to better understand the physics of the plasma-filled diode, we focus on two issues and describe them analytically. We believe that it is useful to relate detailed numerical simulations to simple analytical pictures.

Both issues dealt with in this paper are concerned with the early stage of the current conduction, when ions are still immobile. The first issue is the time-dependent electrostatic evolution of the electron current at the low impedance phase. We develop an analytic model to describe the time-dependent electron dynamics. The formalism is similar to Lagrangian formalisms that were employed to study nonlinear plasma oscillations in other systems.[5,6] The calculated electron velocity and density, and the electric potential are shown to be spatially and temporally oscillatory. Similar plasma oscillations were recently demonstrated in numerical simulations.[1] The oscillations are of interest, since it has been suggested that they lead to the formation of a large potential hill in the plasma. When electron trajectories cross each other our analytical method cannot be used. Particle reflection and trapping are therefore not described here. We identify, however, conditions for the occurrence of particle reflection.

The second issue is the neutralization of the electron beam current by the plasma electrons. During the current conduction high-energy electrons that are emitted from the cathode cross the plasma. It is not clear whether the diode current is conducted by these beam electrons or whether the plasma electrons generate a return current that neutralizes the beam current. In the latter case the diode current is conducted by a diamagnetic current in a skin layer along the plasma boundary. We present an analytic description of a steady-state current conduction in the plasma. An electron beam moves ballistically into the plasma. The plasma electrons move in a direction opposite to the beam electrons so that the net current in the bulk of the plasma is zero. When the plasma electrons reach the plasma boundary they bend into a skin layer, carrying the current along the plasma boundary. The plasma electron flow, the electric potential, and the magnetic field are calculated self-consistently, under the assumption that the electron skin depth is much smaller than the plasma dimensions and that the beam density is much smaller than the plasma density. Further study is required to find out which dynamic evolution of the diode results in such a steady state.

In Sec. II we describe the model for plasma oscillations. In Sec. III we give numerical examples and compare our results with the results in Ref. 1. In Sec. IV we describe the equations for the steady state and in Sec. V we give an example.

II. PLASMA OSCILLATIONS

In this section we describe a PFD by a time-dependent one-dimensional (1-D) electrostatic model. We assume that the time is so short that the ions do not respond and are immobile. Therefore, there is no plasma erosion or gap formation. The current is carried by the plasma electrons, which are evacuated toward the anode and by the electrons emitted from the cathode. We neglect magnetic forces due to both external and self-fields. The dynamics of the electrons is, therefore, determined by

$$\frac{dv}{dt}=-\frac{e}{m}E. \tag{1}$$

In our 1-D geometry v is the velocity in the x direction normal to the electrodes, E is the electric field in the x direction, and $-e$ and m are the electron charge and mass.

Using Gauss' law, we observe that the electric field that acts on the electron along its orbit satisfies

$$\frac{dE}{dt}=-4\pi j_c(t)+4\pi Zen_i[x(t)]v. \tag{2}$$

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This is a key equation that enables us to pursue this analysis. The change in time of the electric field is determined by the change of the net charge between the cathode and the particle. The change in time of the net charge is a result of the current at the cathode and the ion charge that the electron crosses.

In writing Eq. (2) we also used the assumption that the electric field at the cathode does not change in time, and we explicitly assume that $E=0$ at the cathode. A space-charge-limited flow is established once the electric field at the cathode is large enough to generate a dense cathode plasma. The instantaneous vanishing of E at the cathode follows the assumption that the space-charge-limited flow is instantaneously formed. The dynamics might be somewhat different had we taken into account the evolution toward a space-charge-limited flow. However, the same assumption of an instantaneous formation of a space-charge-limited flow was also made in the simulations.[1] An accompanying result of the instantaneous formation of a space-charge-limited flow is that the cathode current equals the circuit current.

An important requirement for the validity of Eq. (2) is that there are no trajectory crossings of the electrons. As we mentioned above, we assume that the plasma electrons move ahead of the emitted electrons. Also, there are no crossings of trajectories between the emitted electrons. The second picture, presented in Sec. IV, is very different. There the beam electrons move across the plasma electrons, which are not evacuated. No attempt is being made here to reconcile these two pictures. In the first picture, described in this section, we make the same assumptions made in the simulations.[1] Therefore, the results of our analytical model could help in understanding the results of the simulations. The relevance of both the simulations and the analytical model to experiments depends to a large extent on the validity of the abovementioned assumptions common to both.

Combining Eqs. (1) and (2), we obtain

$$\frac{d^2v}{dt^2}=\frac{4\pi e}{m}\,|j_c(t)|-\frac{4\pi e^2}{m}\,Zn_i[x(t)]v. \tag{3}$$

The nature of the electron dynamics is a result of Eq. (3). The cathode current accelerates the electrons but the ion charge provides a restoring force that generates oscillations with the plasma frequency. The presence of the positive ion background makes the dynamics of the beam electrons different from that described by the Child–Langmuir law.

Equivalently to Eq. (3), we may write

$$\frac{d^3x}{dt^3}=\frac{4\pi e}{m}\,|j_c(t)|-\frac{4\pi e^2}{m}\,Z\,\frac{d}{dt}\int_0^{x(t)}n_i(x)dx, \tag{4}$$

which is integrated to

$$\frac{d^2x}{dt^2}=\frac{4\pi e}{m}\int_{t_i}^{t}dt'|j_c(t')|-\frac{4\pi e^2}{m}\,Z\int_0^{x(t)}n_i(x)dx. \tag{5}$$

We denote by t_i the time at which the electron enters the diode. We used the condition $E(x=0,t)=0$ in deriving Eq. (5). The electron accelerates due to the accumulated charge: electron charge through the cathode current and ion charge due to the electron motion.

We now assume for simplicity that the ion density is uniform,

$$n_i(x)=n_{i0}. \tag{6}$$

Equation (3) takes the form

$$\frac{d^2v}{dt^2}+\omega_p^2v=\omega_p^2v_c(t), \tag{7}$$

where

$$\omega_p^2\equiv\frac{4\pi n_{i0}e^2}{m}\,Z \tag{8}$$

and

$$v_c(t)\equiv\frac{|j_c(t)|}{Zen_{i0}}. \tag{9}$$

Equation (7) describes an harmonic oscillator driven by the external "force," the cathode current. The solution of this equation that satisfies the conditions

$$v(t_i)=\frac{dv}{dt}\,(t_i)=0, \tag{10}$$

is

$$v(t,t_i)=\omega_p\int_{t_i}^{t}dt'\;v_c(t')\sin\,\omega_p(t-t'). \tag{11}$$

Similarly, Eq. (5) becomes

$$\frac{d^2x}{dt^2}+\omega_p^2x=\omega_p^2[x_c(t)-x_c(t_i)], \tag{12}$$

where

$$x_c(t)=\int_0^t dt'\;v_c(t'), \tag{13}$$

and thus

$$x(t,t_i)=\omega_p\int_{t_i}^{t}dt'[x_c(t')-x_c(t_i)]\sin\,\omega_p(t-t'). \tag{14}$$

Using Eq. (1), the electric field is found to be

$$E(t,t_i)=-\frac{m}{e}\,\omega_p^2\int_{t_i}^{t}dt'\;v_c(t')\cos\,\omega_p(t-t').$$

We may write also an expression for the potential:

$$\varphi(x,t)=-\int_0^x dx'\;E(x',t)$$
$$=-\int_t^{t_i(x,t)}dt_i'\;\frac{\partial x'}{\partial t_i'}\,E[x'(t,t_i'),t]. \tag{15}$$

From Eq. (14), we find that

$$\frac{\partial x}{\partial t_i}=-\omega_p\int_{t_i}^{t}dt'\;\frac{\partial x_c(t_i)}{\partial t_i}\,\sin\,\omega_p(t-t'), \tag{16}$$

and therefore

$$\frac{\partial x}{\partial t_i}=-v_c(t_i)\cos\omega_p(t-t')\Big|_{t_i}^{t}$$

$$=-v_c(t_i)[1-\cos\omega_p(t-t_i)]. \quad (17)$$

From Eq. (17) it is clear that as long as v_c is positive, $\partial x/\partial t_i$ cannot be positive and therefore there are no trajectory crossings. Swanekamp *et al.*[1] have obtained trajectory crossings and particle reflections in their simulations, followed by particle trapping and building of a potential hill. It is important to compare in detail the simulations and the analysis and to find out the cause of the difference in the results.

If the voltage is specified the above equations may be formulated as a set of integral equations, similar to the way the Pierce diode problem is formulated.[6] For simplicity, we restrict ourselves to the low impedance phase of the PFD. At this phase the circuit current is determined by the external circuit. As we mentioned above, the cathode current $j_c(t)$ equals the circuit current. We assume, therefore, that the cathode current is specified and solve for the potential distribution, and for the density and velocity of the electron fluid. We choose the current to rise linearly in time and to be of the form

$$v_c(t)=v_0t/\tau, \quad (18)$$

similar to the form studied by Swanekamp *et al.*[1] It follows that

$$x_c(t)=v_0t^2/2\tau. \quad (19)$$

From Eqs. (11) and (14), we obtain that the electron velocity and location are

$$v(t,t_i)=\frac{v_0}{\tau}\left(t-t_i\cos\omega_p(t-t_i)-\frac{\sin\omega_p(t-t_i)}{\omega_p}\right) \quad (20)$$

and

$$x(t,t_i)=\frac{v_0}{\tau}\left(\frac{(t^2-t_i^2)}{2}-\frac{t_i}{\omega_p}\sin\omega_p(t-t_i)+\frac{1}{\omega_p^2}[\cos\omega_p(t-t_i)-1]\right). \quad (21)$$

The electric field is found to be

$$E(t,t_i)=-\frac{m\omega_p^2}{e\tau}v_0\int_{t_i}^{t}dt'\,t'\cos\omega_p(t-t')$$

$$=-\frac{mv_0}{e\tau}[\omega_pt_i\sin\omega_p(t-t_i)+1-\cos\omega_p(t-t')]. \quad (22)$$

Using Eqs. (15), (21), and (22), we find that the potential is

$$\varphi(x,t)=-\frac{mv_0^2}{e\tau^2}\int_t^{t_i(x,t)}dt_i'\,t_i'[1-\cos\omega_p(t-t_i')]\times[\omega_pt_i'\sin\omega_p(t-t_i')+1-\cos\omega_p(t-t_i')]. \quad (23)$$

Performing the integration, we find that

$$\frac{e\varphi}{mv_0^2/2}=\frac{-2}{(\omega_p\tau)^2}\left(\frac{3}{4}[(\omega_pt_i)^2-(\omega_pt)^2]+4\omega_pt_i\sin\omega_p(t-t_i)-\frac{33}{8}\cos\omega_p(t-t_i)+3\frac{7}{8}+\frac{1}{4}\cos 2\omega_p(t-t_i)-\frac{5}{4}(\omega_pt)^2+(\omega_pt_i)^2\times\cos\omega_p(t-t_i)+(\omega_pt_i)^2\frac{\cos 2\omega_p(t-t_i)}{4}\right). \quad (24)$$

The periodic potential and flow are expected to be only transient. Ion motion and the self-magnetic field will cause trajectory crossings and eventually the disappearance of the oscillations. The simulations do indeed show how later trajectory crossings destroy the periodic structure of the potential and the flow. Since the oscillation period is very short, on the order of ω_p^{-1}, it is difficult to detect the oscillations experimentally. Yet they are important because they may affect the later evolution of the diode.

III. PLASMA OSCILLATIONS—NUMERICAL EXAMPLES

We turn now to several numerical examples. We assume that

$$n_{i0}=10^{12}\ \text{cm}^{-3},\quad v_0=3.1\times10^9\ \text{cm/s},\quad \tau=10^{-9}\ \text{s}, \quad (25)$$

and that the plasma is composed of C^{++}. Therefore,

$$Z=2$$

and

$$\omega_p=7.9\times10^{10}\ \text{s}^{-1}. \quad (26)$$

In Figs. 1(a)–1(c) the electron velocity is shown as a function of x [found from Eqs. (20) and (21)] and in Figs. 2(a)–2(c), the electrostatic potential is shown as a function of x [found from Eqs. (21) and (24)]. When the figures are compared to the figures in Ref. 1, it is seen that at $t=1$ ns the results are very similar. At the later times the results are different. The electron trajectories that we find do not cross each other, and therefore both velocity and potential exhibit an oscillatory structure. In the simulations[1] electron trajectories cross and at these later times the structure ceases being oscillatory.

IV. STEADY STATE

Contrary to the 1-D time-dependent model of the previous section, in this section we consider a two-dimensional (2-D) stationary picture. Magnetic fields are taken into account here. The analysis is somewhat standard, but we feel that the application to the description of the flow in the PFD configuration is useful. We assume that a beam of charged particles of a density n_b and velocity $\mathbf{v}_b$ moves ballistically into a plasma of a density n. The equations that govern the dynamics of the plasma electrons are the continuity equation,

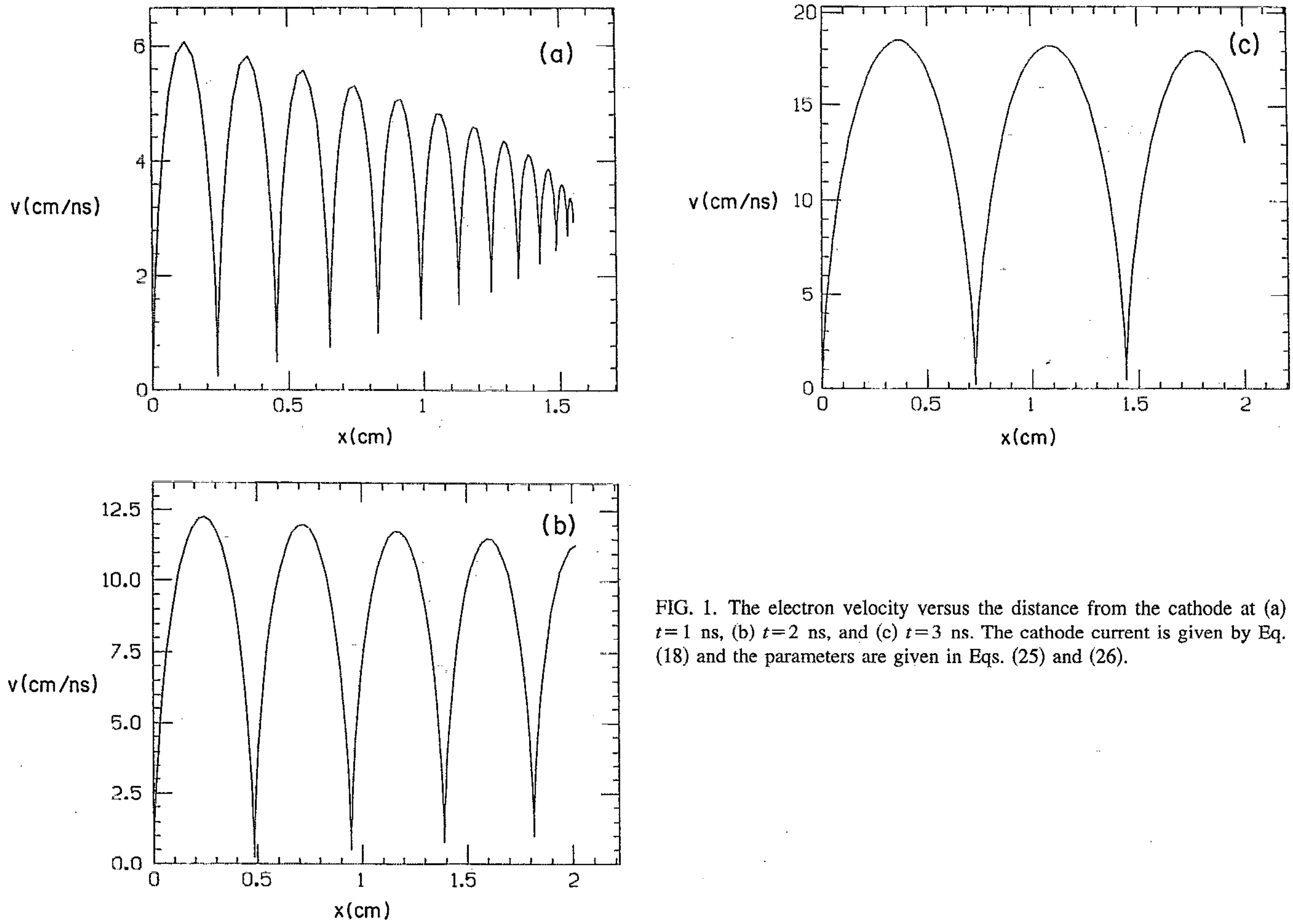

FIG. 1. The electron velocity versus the distance from the cathode at (a) $t=1$ ns, (b) $t=2$ ns, and (c) $t=3$ ns. The cathode current is given by Eq. (18) and the parameters are given in Eqs. (25) and (26).

$$\frac{\partial n}{\partial t}+\nabla\cdot n\mathbf{v}=0; \tag{27}$$

and the equation of motion,

$$m\frac{d}{dt}\mathbf{v}=-e\left(\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c}\right). \tag{28}$$

Here n and $\mathbf{v}$ are the electron density and velocity, $\mathbf{E}$ and $\mathbf{B}$ are the electric and the magnetic fields, c is the velocity of light in vacuum, and

$$\frac{d}{dt}\equiv\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla \tag{29}$$

is the convective derivative. The fields are governed by Faraday's law,

$$-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=\nabla\times\mathbf{E}, \tag{30}$$

by Ampère's law,

$$\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}-\frac{4\pi}{c}e(n\mathbf{v}+n_b\mathbf{v}_b)=\nabla\times\mathbf{B}, \tag{31}$$

and by Gauss' law,

$$\nabla\cdot\mathbf{E}=4\pi e(n_i-n-n_b). \tag{32}$$

Combining Eqs. (28) and (30), we obtain that

$$\frac{\partial}{\partial t}\boldsymbol{\omega}=\nabla\times(\mathbf{v}\times\boldsymbol{\omega}), \tag{33}$$

where

$$\boldsymbol{\omega}\equiv\nabla\times\mathbf{v}-\frac{e\mathbf{B}}{mc}, \tag{34}$$

the generalized vorticity, is frozen into the electron fluid.

We now restrict ourselves to a 2-D geometry, in which

$$\boldsymbol{\omega}\equiv\hat{e}_z\omega(x,y);\quad \mathbf{B}=\hat{e}_zB(x,y);\quad \frac{\partial}{\partial z}=0. \tag{35}$$

Equations (27), (33), and (35) yield

$$\frac{d}{dt}\left(\frac{\omega}{n}\right)=0. \tag{36}$$

We further assume that the external current has been established and that the plasma currents have been induced, so that the displacement current is neglected. Therefore

$$\nabla\cdot n\mathbf{v}=0, \tag{37}$$

and we define streamfunctions I and Ψ, so that

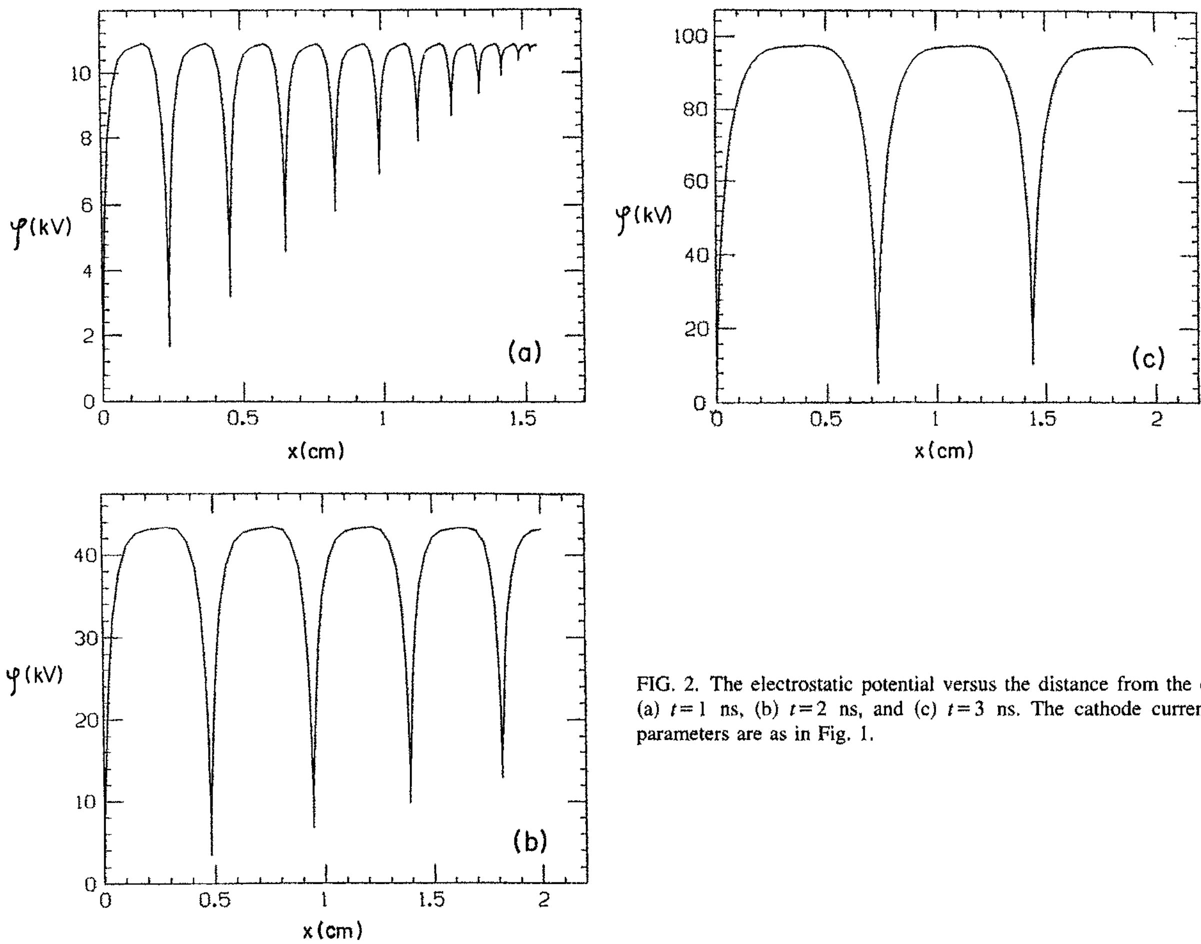

FIG. 2. The electrostatic potential versus the distance from the cathode at (a) $t=1$ ns, (b) $t=2$ ns, and (c) $t=3$ ns. The cathode current and the parameters are as in Fig. 1.

$$\left(\frac{4\pi e^2}{mc^2}\right) n_b \mathbf{v}_b = \hat{e}_z \times \nabla I \tag{38}$$

and

$$\left(\frac{4\pi e^2}{mc^2}\right) n\mathbf{v} = \hat{e}_z \times \nabla \Psi. \tag{39}$$

Let us examine steady-state solutions. Equations (36) and (39) yield

$$\nabla \Psi \times \nabla \left(\frac{\omega}{n}\right) = 0. \tag{40}$$

The ratio of generalized vorticity to density has to be constant along the streamlines. The general solution of Eq. (40) is

$$\frac{\omega}{n} = F(\Psi), \tag{41}$$

where F is an arbitrary function. Using Eqs. (38) and (39), and neglecting the displacement current, we write Eq. (31) as

$$\Psi + I = \frac{eB}{mc}. \tag{42}$$

Equation (41) becomes

$$\frac{c^2}{\omega_p^2}\left[\frac{1}{\bar{n}}\Delta\Psi + \nabla\Psi\cdot\nabla\left(\frac{1}{\bar{n}}\right)\right] - \Psi - I = nF(\Psi). \tag{43}$$

Here

$$\bar{n} \equiv n/n_0, \tag{44}$$

where n_0 is a characteristic plasma density and

$$\omega_p^2 \equiv \frac{4\pi n_0 e^2}{m}, \tag{45}$$

is the square of the characteristic plasma frequency.

Let us examine a plasma that is initially at rest and unmagnetized. The initial generalized vorticity is therefore zero,

$$\omega = 0. \tag{46}$$

We examine solutions in which ω remains zero. Thus

$$\frac{c^2}{\omega_p^2}\left[\frac{1}{\bar{n}}\Delta\Psi + \nabla\Psi\cdot\nabla\left(\frac{1}{\bar{n}}\right)\right] - \Psi - I = 0 \tag{47}$$

is the governing equation. We now assume that the beam is not magnetized and propagates in the x direction only. Therefore

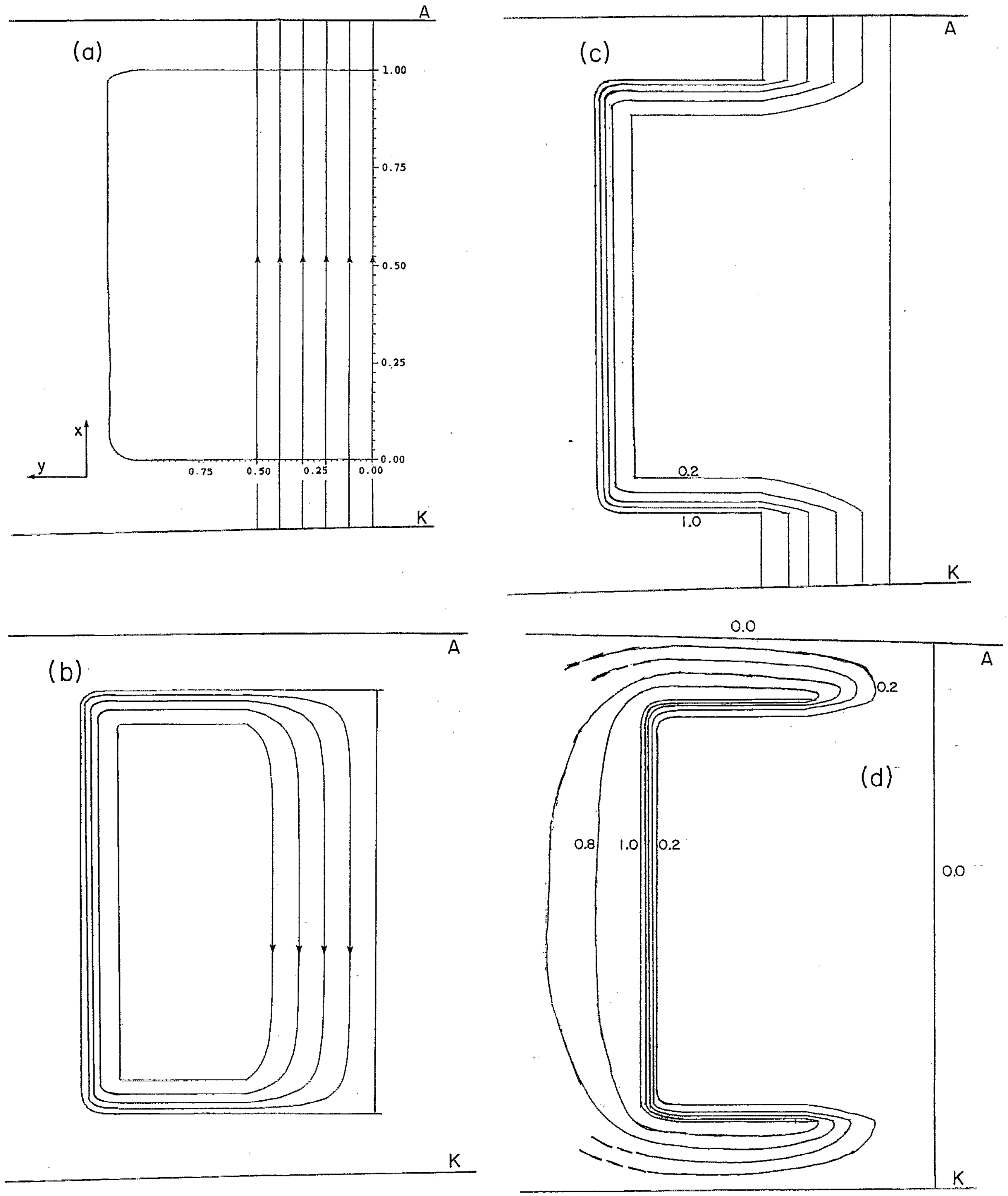

FIG. 3. A steady current conduction by a plasma located between two electrodes, where x is normalized to l, $y_0/l=0.5$, and $c/\omega_p l=0.05$. Shown are the contour lines of (a) the beam streamfunction, given in Eq. (73), (b) the plasma streamfunction, (c) the magnetic field, and (d) the electrostatic potential.

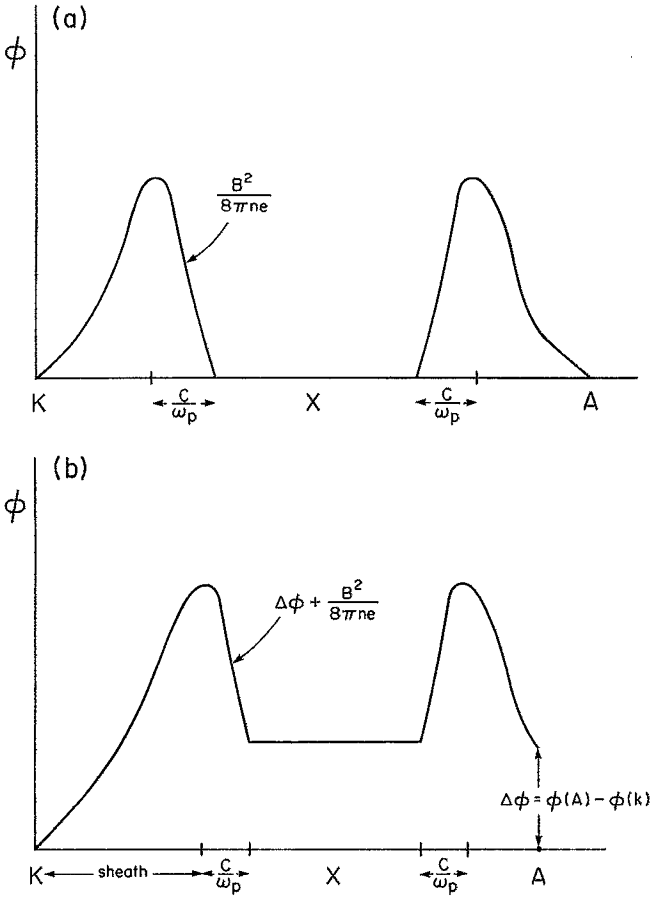

FIG. 4. A schematic of the electrostatic potential versus x at $y \neq 0$ when (a) the two electrodes are grounded, and (b) there is a nonzero voltage between the electrodes.

$$I = I(y). \tag{48}$$

On the other hand, we look for solutions in which the plasma is bounded in the x direction. Therefore

$$n = n(x); \quad \frac{\partial}{\partial x} \gg \frac{\partial}{\partial y}. \tag{49}$$

Equation (47) becomes

$$\frac{c^2}{\omega_p^2}\left[\frac{1}{\bar{n}}\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial\Psi}{\partial x}\frac{\partial}{\partial x}\left(\frac{1}{\bar{n}}\right)\right] - \Psi - I = 0. \tag{50}$$

Equations (50) and (42) are combined to

$$\frac{c^2}{\omega_p^2}\frac{\partial}{\partial x}\left(\frac{1}{\bar{n}}\frac{\partial B}{\partial x}\right) = B \tag{51}$$

or

$$\frac{c^2}{\omega_p^2}\frac{\partial^2}{\partial x^2}\left(\frac{1}{\bar{n}}\frac{\partial B}{\partial x}\right) - \bar{n}\left(\frac{1}{\bar{n}}\frac{\partial B}{\partial x}\right) = 0. \tag{52}$$

Let us express the magnetic field as

$$B(x,y) = \left(\frac{mc}{e}\right) I(y) b(x), \tag{53}$$

and therefore

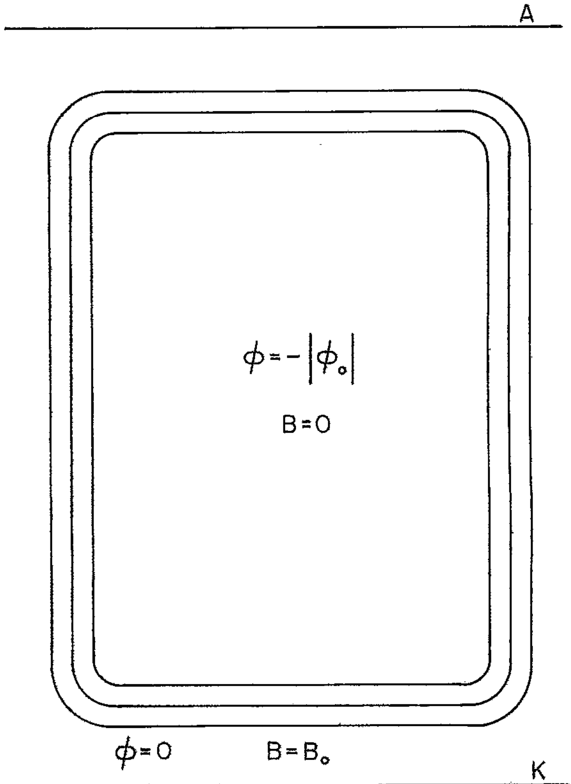

FIG. 5. A schematic of the contour lines of the electrostatic potential in an unmagnetized plasma surrounded by a vacuum that is permeated by the magnetic field.

$$\Psi(x,y) = I(y)[b(x) - 1]. \tag{54}$$

Since in a steady state

$$\mathbf{E} = -\nabla\phi, \tag{55}$$

we multiply the time-dependent equation (28) by $\mathbf{v}$, and obtain

$$\mathbf{v}\cdot\nabla\left(\frac{mv^2}{2} - e\phi\right) = 0, \tag{56}$$

which expresses the conservation of the total energy along the electron trajectory. Using (39) we obtain a formal equation for the electrostatic potential,

$$\frac{m}{2}\left(\frac{c^2}{\omega_p^2\bar{n}}\right)^2 |\nabla\Psi|^2 - e\phi = G(\Psi), \tag{57}$$

where G is an arbitrary function. Since initially $\phi = 0 = \mathbf{v}$, we choose $G(\Psi) = 0$ and

$$\phi = \frac{m}{2e}\left(\frac{c^2}{\omega_p^2\bar{n}}\right)^2 |\nabla\Psi|^2. \tag{58}$$

We discuss two cases: a uniform plasma density and a nonuniform plasma density, and the issue of quasineutrality.

A. Uniform plasma density

Let us now choose a uniform density plasma,

$$\bar{n} = 1. \tag{59}$$

We assume that the plasma is bounded at $x=0$ and at $x=l$, so that

$$b(x=0,y)=1=b(x=l,y). \tag{60}$$

The solution of Eq. (52) with the boundary conditions (60) is

$$B(x,y)\cong\frac{mc}{e}\,I(y)\left[\exp\left(-\frac{\omega_p x}{c}\right)+\exp\left(-\frac{\omega_p}{c}\,(l-x)\right)\right]. \tag{61}$$

We assumed that

$$\frac{\omega_p l}{c}\gg 1. \tag{62}$$

The streamfunction is

$$\Psi(x,y)=I(y)\left[\exp\left(-\frac{\omega_p x}{c}\right)+\exp\left(-\frac{\omega_p}{c}\,(l-x)\right)-1\right]. \tag{63}$$

Therefore, the velocity of the plasma electrons is

$$v_x=-\frac{c^2}{\omega_p^2}\frac{\partial I}{\partial y}\left[\exp\left(-\frac{\omega_p x}{c}\right)+\exp\left(-\frac{\omega_p}{c}\,(l-x)\right)-1\right], \tag{64a}$$

$$v_y=-\frac{c}{\omega_p}\,I(y)\left[\exp\left(-\frac{\omega_p x}{c}\right)-\exp\left(-\frac{\omega_p}{c}\,(l-x)\right)\right]. \tag{64b}$$

The electrostatic potential is approximately

$$\phi(x,y)\cong\frac{B^2(x,y)}{8\pi n e}. \tag{65}$$

B. Nonuniform plasma density

Let us now assume that near the plasma boundary the density is

$$\bar{n}=x/x_0. \tag{66}$$

Solving Eq. (52), we find that

$$B(x)\sim\frac{\partial A_i}{\partial x}\left[x\left(\frac{\omega_p^2}{c^2 x_0}\right)^{1/3}\right], \tag{67}$$

where A_i is the Airy function. The thickness of the current layer,

$$\delta=\left(\frac{x_0 c^2}{\omega_p^2}\right)^{1/3}, \tag{68}$$

is the location where

$$\frac{c^2}{\omega_p^2(x)}=\frac{x_0 mc^2}{4\pi n_0 x e^2} \tag{69}$$

is equal to x^2.

C. Quasineutrality

We require that

$$\frac{\nabla\cdot\mathbf{E}}{4\pi n_0 e}\ll 1. \tag{70}$$

For that,

$$\frac{\phi}{(c^2/\omega_p^2)4\pi n_0 e}\ll 1, \tag{71}$$

or using Eq. (65), we obtain the standard condition,

$$\omega_c\ll\omega_p, \tag{72}$$

where ω_c is the electron cyclotron frequency.

V. STEADY STATE—AN EXAMPLE

We specify the streamfunction of the beam current to be

$$I(y)=\begin{cases} I_0 y/y_0, & y\leq y_0,\\ I_0, & y\geq y_0,\end{cases} \tag{73}$$

for $y\geq 0$. The steady state we write here is extended to $y\leq 0$ in two different ways to describe two different configurations. The first configuration is of a rectangular plasma column that is much longer in the z direction than in the x and y direction. The plane $y=0$ is a symmetry plane. In this configuration,

$$I(y)=-I(-y),$$
$$B(x,y)=-B(x,-y), \tag{74}$$

and

$$\phi(x,y)=\phi(x,-y).$$

A second configuration that our description is relevant to, is of a hollow cylindrical plasma, in which the plasma radial dimension is much smaller than the radius of the plasma. In this case

$$I(y\leq 0)=0,\quad B(x,y\leq 0)=0, \tag{75}$$

and

$$\phi(x,y\leq 0)=0.$$

For the example we chose a plasma of a uniform density, with the parameters

$$y_0/l=0.5$$

and

$$\frac{c}{\omega_p l}=0.05. \tag{76}$$

Figure 3(a) shows the beam streamfunction $I(y)$. Figure 3(b) shows the contour lines of the plasma streamfunction $\Psi(x,y)$. Figure 3(c) shows the contour lines of the magnetic field, the lines along which the current flows. Figure 3(d) shows the contour lines of the electrostatic potential $\phi(x,y)$. The quantities in the figures are given by Eqs. (73), (63), (61), and (65). At the plasma boundaries in the y direction, and in the vacuum outside the plasma, the contour levels are drawn only schematically. Figures 4(a) and 4(b) are schematics of the potential $\phi(x,y_1)$, where $y_1\neq 0$. In Fig. 4(a) the electrodes are grounded, while in Fig. 4(b), $\phi(A)\neq\phi(K)$. In contrast to the PFD, Fig. 5 describes a simpler configuration.

It shows a schematic of the contour lines of $\phi(x,y)$ when an unmagnetized plasma is surrounded by a vacuum permeated by the magnetic field.

VI. SUMMARY

Here we have presented two pictures of current conduction through a plasma. The first picture was of nonlinear plasma oscillations. Less simplified analysis should show the result of various effects not included in our model. We hope that the study of trajectory crossings, particle trapping, and potential hill formation will benefit from our simple analysis. The second picture was of a steady 2-D current flow. In this picture a plasma return current neutralizes the external current. It would be interesting to see if a time-dependent evolution actually results in the stationary picture we have presented. Numerical simulations explore the role of ion motion and of various instabilities. For understanding the complex pictures seen in simulations, simple pictures such as those presented here, could be helpful.

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