Fast decay of plasma return currents due to whistler waves

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The evolution of the return current induced by a charged particle beam in a magnetized plasma is studied. The beam current is perpendicular to the background magnetic field. The return current is shown to depart from the beam along the background magnetic field with a whistler rather than a diffusion or an Alfvén velocity. In a plasma bounded by two conductors the return current oscillates with the whistler period. Analytical expressions for the evolution of the magnetic field and of the plasma return current are derived for a beam with a finite width and with various rise time dependences. When the whistler time is shorter than the rise time of the beam current, the plasma return current does not grow beyond the whistler time.

I. INTRODUCTION

The generation and the decay of plasma return currents due to the injection of a charged particle beam is a long studied subject.[1-4] The plasma return current forms in a few plasma periods and it decays in the magnetic field diffusion time $t_D=(4\pi L^2/c^2\eta_c)$ (c is the light velocity, L is the characteristic length of the system, and η_c is the collisional resistivity).

The decay of the plasma return current induced by the injection of a charged particle beam into a magnetized plasma, was studied by Berk and Pearlstein.[1] Assuming that there are no variations along the background magnetic field, they showed that when the beam propagates perpendicular to the background magnetic field, the decay of the return currents is determined by the ion dynamics. Fast magnetosonic waves propagate across the background magnetic field and the resulting characteristic decay time is $t_A=L/V_A$ (V_A is the Alfvén velocity). For plasmas of low collisionality this time is much shorter than t_D, and thus the plasma return current decreases faster and the self-magnetic field of the beam appears faster than when the magnetic field evolves due to diffusion only. When the beam propagates along the background magnetic field the decay of the plasma return current is not influenced by the ion dynamics and the decay time is t_D.

The penetration of an external magnetic field into a magnetized plasma along the background magnetic field has recently been analyzed.[5] The external magnetic field was assumed to be generated in the vacuum adjacent to a plasma, by currents that flow outside the plasma. It was shown that if the background magnetic field has a component normal to the plasma–vacuum boundary, the magnetic field in the vacuum propagates into the plasma along the background magnetic field on the whistler time scale rather than on the diffusion time scale. The whistler time scale is $t_W=4\pi L^2/c^2\eta_H$ where $\eta_H=B_b/nec$ (B_b is the background magnetic field, n the plasma density). The whistler propagation is governed by the electron rather than by the ion dynamics. The plasma pushing becomes relevant only when the magnetic field propagates over a distance of the order of the ion skin depth.

The whistler propagation has also been suggested[6] as the mechanism responsible for the observed fast penetration into a charge-neutralized ion beam of a magnetic field across which the beam propagates. The two-dimensional linearized problem has been solved in the rest frame of the beam and a whistler propagation of the magnetic field has been demonstrated. There also seems to be direct experimental evidence of this whistler propagation of the external magnetic field into a charged-neutralized ion beam.[7]

In the present paper we study, as was studied in Ref. 1, the decay of the plasma return current induced by the injection of a charged particle beam into a magnetized beam. However, contrary to Ref. 1, we allow variations along the background magnetic field. Similarly to the problems treated in Refs. 5 and 6, we show that, due to these variations, the electron, rather than the ion, dynamics is dominant, as long as $L<c/\omega_{pi}$ (the ion skin depth). The magnetic field then propagates along the background magnetic field as a whistler wave, with a velocity higher than the Alfvén velocity. The decay of the return current is on the whistler time scale t_W.

Charged particle beams are injected into magnetized plasmas of a characteristic length shorter than the ion skin depth, in several plasma devices. In a magnetically insulated ion diode some of the electrons emitted from the cathode flow through the magnetized anode plasma.[8] In the current-toggled plasma opening switch, electrons emitted from the cathode are injected into the plasma that is immersed in the magnetic field of the slow field coil.[9] In certain schemes for ion-driven inertial confinement fusion, a charged-neutralized ion beam is focused by a solenoidal magnetic lens.[10] The present paper may be relevant to the evolution of the plasma return current in such devices.

We consider a one-dimensional (1-D) model problem. A charged beam of a specified current propagates in the plasma across a background magnetic field. This configuration is similar to the configuration analyzed in the 1-D model problem of Ref. 1, but while in Ref. 1 the variations were normal to the background magnetic field, we allow variations along the background magnetic field only. The simplified 1-D geometry enables us to describe analytically the evolution of the plasma return current for various rise times and spatial distributions of the specified beam current. Concurrently with our preliminary research,[11] the two-dimensional (2-D) evolution, that is also affected by the finite length of the

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beam and by the nonuniformity of the background magnetic field, has been studied numerically by Oliver and Sudan.[12]

In Sec. II we briefly review the model for the whistler wave magnetic field propagation in plasmas due to the Hall field.[5] In Sec. III the 1-D model for the plasma response to a charged beam is presented. In Sec. IV the return current and the magnetic field are calculated for the case of an infinitely narrow beam of a current that rises either as a step function in time or linearly in time. In Sec. V the return current and the magnetic field are calculated for a beam of a finite width. Finally, Sec. VI is dedicated to conclusions.

II. WHISTLER WAVE PROPAGATION OF THE MAGNETIC FIELD IN PLASMAS

We here briefly review the model presented in Ref. 5 for the fast magnetic field propagation due to the Hall field in a magnetized plasma of low collisionality.

The time scale for the process under consideration is assumed to be longer than the electron cyclotron period and shorter than the ion cyclotron period, thus the ions are assumed to be immobile, and the magnetic field propagation is caused by the electrons. The displacement current is ignored. The governing equations are Faraday's law,

$$\nabla\times\mathbf{E}=-\frac{1}{c}\,\partial_t\mathbf{B}, \tag{1}$$

Ampère's law,

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\,\mathbf{J}, \tag{2}$$

and Ohm's law with the Hall field,

$$\mathbf{E}=\eta_c\mathbf{J}+\frac{\mathbf{J}\times\mathbf{B}}{enc}. \tag{3}$$

Here $\mathbf{E}$, $\mathbf{B}$, and $\mathbf{J}$ are the electric field, the magnetic field, and the current density, respectively. A magnetized plasma with a slab geometry is considered. All quantities are assumed to depend only on the x coordinate, parallel to a background magnetic field $\mathbf{B}_b=B_x\hat{x}$. A magnetic field $\mathbf{B}=B_0\hat{y}$ is switched on as a step function in time at $t=0$ at the vacuum–plasma boundary that is located at $x=0$ (i.e., one face of the plasma slab). The plasma fills a region that is bounded on the other face of the slab at $x=L$ by a conductor (the case of a semi-infinite plasma was also considered in Ref. 5). The problem turns out to be linear irrespective of the relative intensities of the background and the propagating magnetic fields. The equation for the propagating magnetic field $B\equiv B_z+iB_y$ is a complex diffusion equation,

$$\partial_t B=\frac{c^2}{4\pi}\,\eta\partial_x^2 B, \tag{4}$$

where $\eta\equiv\eta_c+i\eta_{\rm H}$ ($\eta_{\rm H}\equiv B_x/nec$). It has been shown[5] that for $\eta_c/\eta_H\ll 1$ the magnetic field propagates with the whistler characteristic time $t_W=4\pi L^2/c^2\eta_{\rm H}$ rather than with the much longer diffusion time $t_D=4\pi L^2/c^2\eta_c$. Because of the analogous roles of η_c and of $\eta_{\rm H}$ in the definitions of the diffusion and the whistler times, $\eta_{\rm H}$ is named "Hall resistivity," although one should remember that the Hall field does not cause any dissipation. Furthermore, the fast evolution can also occur when $\eta_c=0$ and the dissipation then is zero.

For later purposes we here write the solution of Eq. (4) for the magnetic field in a finite plasma slab,

$$B=iB_0\left\{1-\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{(n+\frac{1}{2})}\sin\left[\left(n+\frac{1}{2}\right)\frac{\pi x}{L}\right]\times\exp\left(-\frac{(n+\frac{1}{2})^2\pi c^2}{4L^2}(\eta_c+i\eta_{\rm H})t\right)\right\}. \tag{5}$$

III. ONE-DIMENSIONAL BEAM MODEL

In this section we derive general expressions for the evolution of the plasma return current, that is induced by a charged beam of a specified current density. In the next sections we use these expressions for some particular cases. As in the previous section, the governing equations are Faraday's law [Eq. (1)], Ampère's law,

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\,(\mathbf{J}_p+\mathbf{J}_b) \tag{6}$$

(where now the current density $\mathbf{J}=\mathbf{J}_p+\mathbf{J}_b$ is the sum of the beam current density $\mathbf{J}_b$ and the plasma current density $\mathbf{J}_p$) and the equation for the plasma current density

$$\mathbf{E}=\eta_c\mathbf{J}_p+\frac{\mathbf{J}_p\times\mathbf{B}}{enc}. \tag{7}$$

Also as in the previous section, we assume that the plasma is immersed in a uniform background magnetic field $\mathbf{B}_b=B_x\hat{x}$, and that all quantities depend only on the x coordinate, along the background magnetic field. The beam of a current $I_b(t)=I_0Y(t)$ flows in the z direction, and its current density is $\mathbf{J}_b=J_b\hat{z}$. The beam density is assumed to be much smaller than the plasma density n.

We define $A\equiv A_z+iA_y$ (where A stands for E, J, or B). Equation (7) is rewritten as

$$E=(\eta_c+i\eta_{\rm H})J_p. \tag{8}$$

Taking the derivative of Eq. (6) with respect to time, and using Eqs. (1) and (8), we obtain the following equation for the electric field

$$\frac{\partial^2 E}{\partial x^2}=\frac{4\pi}{c^2}\,\partial_t\left(J_b+\frac{E}{\eta_c+i\eta_{\rm H}}\right). \tag{9}$$

We denote by $A(s)$ the Laplace transform of $A(t)$, where again A stands for E, J, or B. The equation for $E(x,s)$ is

$$\frac{\partial^2 E(x,s)}{\partial x^2}=\frac{4\pi s}{c^2}\left(J_b(x,s)+\frac{E(x,s)}{\eta_c+i\eta_{\rm H}}\right). \tag{10}$$

We assumed that $E(x,t=0)=0$, since $J_b(x,t<0)=0$. If the plasma is located between $x=-L$ and $x=L$, the plasma return current is

$$I(t)=L^{-1}\left(\int_{-L}^{L}\frac{E(x,s)}{\eta_c+i\eta_{\rm H}}\,dx\right). \tag{11}$$

where $L^{-1}[A(s)]$ is the inverse Laplace transform of $A(s)$. Following Eq. (6), the plasma return current can also be written as

$$I(t)=\left(\frac{ic}{4\pi}\right)B|_{-L}^{L}-I_0Y(t). \tag{12}$$

We choose to address the problem in which the plasma is bounded by conductors at $x=\pm L$, and therefore the boundary conditions are

$$E(x=\pm L,t)=0. \tag{13}$$

Equation (10), with a specified beam current density, and the boundary conditions (13) determine $E(x,s)$. Once $E(x,s)$ is found, the plasma current density can be found from Eq. (8). In order to find the magnetic field, one has to solve Eq. (6), with the specified beam current density and the calculated plasma current density, and, in addition, to require that the magnetic field flux between the two conducting boundaries be zero. The vanishing of the magnetic field flux at $t>0$ results from the vanishing of the magnetic field flux at $t=0$ and from the conservation of the magnetic field flux between the two conducting boundaries resultings from Eq. (13).

For simplicity, we consider beams of current densities that are symmetrical with respect to the plane at $x=0$. Both Eq. (10) and the boundary conditions (13) are then symmetrical with respect to this plane. As a result, the electric field and the plasma current density turn out to be symmetrical as well, while the magnetic field turns out to be antisymmetrical.

In the next sections we consider some particular beam current densities.

IV. INFINITELY NARROW BEAM

In this section the beam current is assumed to be localized at $x=0$, i.e.,

$$J_b(x,t)=I_0Y(t)\delta(x). \tag{14}$$

Integrating Eq. (10) from $x=-\epsilon$ to $x=\epsilon$ we obtain:

$$\partial_x E(x,s)|_{-\epsilon}^{\epsilon}=\frac{4\pi s}{c^2}I_0Y(s) \tag{15}$$

for $\epsilon\rightarrow 0$, while the equation for $|x|>0$ is

$$\frac{\partial^2 E(x,s)}{\partial x^2}=\frac{4\pi s}{c^2}\left(\frac{E(x,s)}{\eta_c+i\eta_{\mathrm{H}}}\right). \tag{16}$$

We solve Eq. (10) with the beam current density (14) and the boundary conditions (13). Alternatively, we may solve Eq. (16) for $x>0$ and for $x<0$ with the boundary conditions (13), and then match the two solutions at $x=0$, using the jump condition (15). We find that

$$E(x,s)=-\frac{2\pi sI_0Y(s)}{c^2k\cosh(kL)}\sinh[k(L-|x|)], \tag{17}$$

where

$$k\equiv\sqrt{\frac{4\pi s}{c^2(\eta_c+i\eta_{\mathrm{H}})}}.$$

As expected, since Eq. (10) and the boundary conditions (13) are symmetrical with respect to the plane at $x=0$, so is the electric field (17). From Eq. (11) we obtain

$$I(t)=L^{-1}\left[I_0Y(s)\left(1-\frac{1}{\cosh(kL)}\right)\right]. \tag{18}$$

For the time dependence let us examine two cases, a beam current switched on as a step function in time, i.e., $Y(t)=\Theta(t)$ (the Heaviside function), and a beam current that is linearly rising in time, i.e., $Y(t)=t/t_0$.

A. Beam current switched on as a step function in time

The Laplace transform of $Y(t)=\Theta(t)$ is

$$Y(s)=\frac{1}{s}. \tag{19}$$

Using Eqs. (18) and (19), we find that the plasma return current is[13]

$$I(t)=-\frac{2I_0}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+\frac{1}{2})}\times\exp\left(-\frac{(n+\frac{1}{2})^2\pi c^2}{4L^2}(\eta_c+i\eta_{\mathrm{H}})t\right). \tag{20}$$

The evolution of the magnetic field in the plasma due to the infinitely narrow beam is identical to the evolution of the magnetic field in the plasma due to a magnetic field that rises at the plasma–vacuum boundary at $x=0$. This identity holds for any time-dependent beam current $I_b(t)$ and any time-dependent imposed magnetic field $B_0(t)$ at $x=0$, as long as $I_b(t)=-cB_0(t)/2\pi$. Indeed, using Eqs. (2) and (5) we can calculate the return current in the plasma that is induced by a magnetic field that is switched on as a step function in time at the plasma boundary. As expected, the calculated current turns out to be equal to the current expressed in Eq. (20), the return current due to the infinitely narrow beam. The evolution of the magnetic field in the plasma, as a result of a beam described by Eq. (14) that is switched on as a step function in time, is also governed by Eq. (5). Figure 1 shows the magnetic field in the plasma versus x/L due to a current of a narrow beam that is switched on at $x=0$ as a step function in time at $t=0$. Since the beam is narrow (its width is much narrower than L), the evolution of the magnetic field is described approximately by Eq. (5), except at the region in the vicinity of $x=0$. In Sec. V we will discuss the effects of the finite width of the beam. It is seen in the figure that the velocity of the magnetic field propagation is much higher than the diffusion velocity. The return current has not only a z component (the only component it has when the collisional resistivity is dominant), but also a y component.

At the early time shown in Fig. 1, $t=0.01t_W$, the magnetic field has not yet reached the conducting boundary at $x=L$. Therefore, the presence of the conductors has not yet affected the evolution of the magnetic field. At later times,

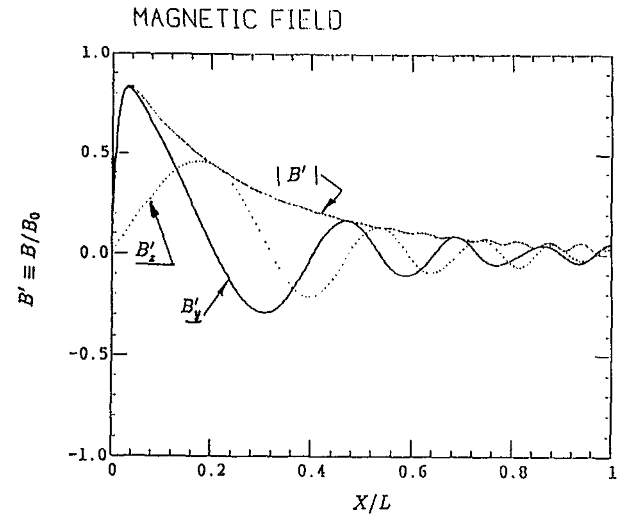

FIG. 1. The magnetic field, its magnitude and its z and y components, as a function of x/L at $t/t_D=0.0005$. The magnetic field is induced by a narrow beam $\alpha L=100$ that is switched on as a step function in time at $t=0$ [Eq. (34)]. Here $\eta_H/\eta_c=20$.

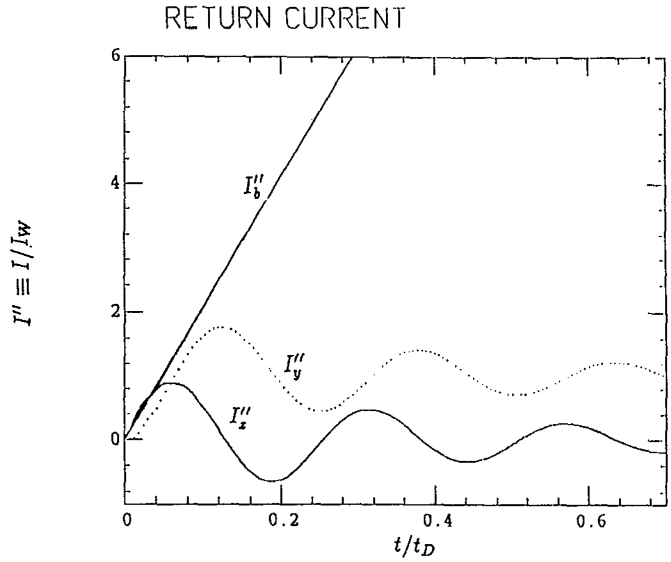

FIG. 2. A linearly rising in time beam current and the z and y components of the plasma return current as a function of time. The beam is infinitely narrow [Eq. (22)]. Here $\eta_H/\eta_c=10$.

reflections from the conducting boundaries generate standing waves. When there are no collisions, the magnetic field is periodic in time with a period $t'_W = 32L^2/c^2\eta_H$. This period is the time it takes for the fundamental, slowest, mode to propagate to the boundary at $x=L$ and to be reflected backwards to $x=0$. Due to a finite collisionality, the amplitude of the oscillations decreases in time on the resistive time scale.

It can easily be seen from Eq. (20) that, since $(2/\pi)\Sigma_{n=1}^{\infty}[(-1)^n/(n+\frac{1}{2})]=1$, the initial plasma return current cancels the beam current exactly, and therefore $I(0)=-I_0$. This instantaneous current neutralization is valid only if the rise time is much longer than the electron plasma period. Otherwise, the electron inertia has to be taken into account. It will be shown in Sec. V that an instantaneous current neutralization occurs for beams of arbitrary time dependence and width, as long as the electron inertia may be neglected.

B. Beam current linearly rising in time

In this case $Y(t)=t/t_0$, and therefore

$$Y(s)=\frac{1}{t_0 s^2}. \tag{21}$$

Using Laplace transform properties, we find the plasma return current in this case as the integral in time of the plasma return current expressed in Eq. (20) multiplied by $1/t_0$. The calculated plasma return current is

$$I(t)=\frac{8I_0L^2}{t_0\pi^2c^2(\eta_c+i\eta_H)}\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+\frac{1}{2})^3}\times\exp\left(-\frac{(n+\frac{1}{2})^2\pi c^2}{4L^2}(\eta_c+i\eta_H)t\right)-\frac{\pi^3}{4}\right]. \tag{22}$$

A beam current that is linearly rising in time and the resulting plasma return current [Eq. (22)] are shown in Fig. 2. As seen in the figure, for times $t<t_W=0.1t_D$ the plasma return current is approximately equal to the beam current. At $t\cong t_W$ the z component of the plasma return current reaches its maximal value

$$I_z\cong I_W\equiv\frac{I_0L}{2V_Wt_0}, \tag{23}$$

where $V_W\equiv c^2\eta_H/4\pi L$. At later times, when $t>t_W$, the return current does not grow further, even though the beam current does. The return current then oscillates with an amplitude that equals I_W. Therefore, the whistler mechanism determines a time t_W, beyond which the plasma return current stops growing. The plasma return current at times $t\gg t_W$ is much smaller than the beam current at these times, and is approximately equal to the beam current that flows at $t=t_W$. Although this maximal current was found here for a beam current linearly rising in time, a similar maximal current is expected for any monotonically increasing in time beam current.

The maximal return current calculated here, I_W, should be compared to the maximal return current calculated in Ref. 1 for a beam of the same current density, as discussed here, but when variations were allowed only perpendicular to the background magnetic field. It has been shown there that, as a result of a fast magnetosonic wave, the return current reaches a maximal value $I_A=I_0L/V_At_0$ (V_A is the Alfvén velocity) at $t=t_A$, and does not grow further. We assume that the distance from the beam to the conducting boundaries perpendicular to the background magnetic field is L as well. If $L<c/\omega_{pi}$, the whistler time t_W is shorter than the Alfvén time t_A. Therefore, if variations are allowed both perpendicular to and along the background magnetic field, the dominant mechanism is the whistler propagation of the re-

turn current along the field, that we have described here. The maximal plasma return current is not I_A, but rather the smaller current I_W.

V. A FINITE WIDTH BEAM

We here assume a beam of a finite width, a current I_0 and a current density

$$J_b(x,t)=-I_0Y(t)\alpha\frac{\sinh[\alpha(L-|x|)]}{2[1-\cosh(\alpha L)]}. \tag{24}$$

We note that, by defining the dimensionless parameters $t'\equiv[(tc^2\eta_c)/4\pi L^2]$, $x'\equiv x/L$, and $E'\equiv(E/\alpha I_0\eta_c)$, we obtain from Eq. (9)

$$\partial_{x'}^2E'=-\frac{1}{2}\frac{\sinh[\alpha L(1-|x'|)]}{1-\cosh(\alpha L)}\partial_{t'}Y(t')+\partial_{t'}\frac{E'}{1+i(\eta_H/\eta_c)}. \tag{25}$$

In this form of the equation there are two characteristic parameters: αL and (η_H/η_c). Laplace transforming Eqs. (9) and (24), we obtain an equation for $E(x,s)$:

$$\left(\partial_x^2-\frac{4\pi s}{c(\eta_c+i\eta_H)}\right)E(x,s)=\left(-\frac{2\alpha\sinh[\alpha(L-|x|)]}{1-\cosh(\alpha L)}\right)\frac{2\pi s}{c^2}Y(s)I_0. \tag{26}$$

The solution of the last equation, with the boundary conditions (13), is

$$E(x,s)=\frac{2I_0\pi sY(s)\alpha}{c^2(\alpha^2-k^2)}\left(\frac{\sinh[\alpha(L-|x|)]}{1-\cosh(\alpha L)}+\frac{\alpha}{k}\frac{\cosh(\alpha L)}{1-\cosh(\alpha L)}\frac{\sinh[k(L-|x|)]}{\cosh(kL)}\right). \tag{27}$$

We notice that, in the limit $\alpha L\rightarrow\infty$, the expression (27) is reduced to the expression (17). Following Eqs. (8), (24), and (27), the total current density is

$$J(x,s)=\frac{2\pi sY(s)\alpha I_0}{c^2\eta(\alpha^2-k^2)}\left(-\frac{\sinh[\alpha(L-|x|)]}{1-\cosh(\alpha L)}\frac{\alpha^2}{k^2}+\frac{\alpha}{k}\frac{\cosh(\alpha L)}{1-\cosh(\alpha L)}\frac{\sinh[k(L-|x|)]}{\cosh(kL)}\right). \tag{28}$$

Using Eq. (6), we find that the magnetic field is

$$B=-i\frac{4\pi}{c}\int_0^x J\,dx'. \tag{29}$$

Following Eqs. (28) and (29), the Laplace transform of the magnetic field is calculated and found to be

$$B(x,s)=\mp\frac{iB_0}{c(\alpha^2-k^2)}\alpha^2Y(s)\left(\frac{\cosh[\alpha(L-|x|)]}{1-\cosh(\alpha L)}-\frac{\cosh(\alpha L)}{1-\cosh(\alpha L)}\frac{\cosh[k(L-|x|)]}{\cosh(kL)}\right), \tag{30}$$

where $B_0\equiv2\pi I_0/c$ and the plus and minus signs correspond to $x>0$ and $x<0$, respectively. The Laplace transform of the plasma return current is

$$I(s)=-\frac{I_0Y(s)}{1-\cosh(\alpha L)}\left(-\frac{\cosh(\alpha L)}{\cosh(kL)}\frac{\alpha^2}{\alpha^2-k^2}+\frac{k^2}{\alpha^2-k^2}+\cosh(\alpha L)\right). \tag{31}$$

The remaining return current, the difference between the beam current and the plasma return current, flows in the conductors as a surface current I_S. This surface current is

$$I_S(x=\mp L,s)=\pm\frac{icB(x=\mp L,s)}{4\pi}. \tag{32}$$

Using properties of Laplace transforms, we may calculate $A(t)$ at the limit $t\rightarrow0$, by making the inverse Laplace transformation of $sA(s)$, at the limit $s\rightarrow\infty$, where A is E, J, or B. Since

$$sJ(x,s)\approx\frac{2\pi s^2Y(s)\cosh(\alpha L)}{c^2[1-\cosh(\alpha L)]k^3}\rightarrow0, \tag{33}$$

for all $Y(s)$ such that $Y(s)=\vartheta(1/\sqrt{s})$ and $s\rightarrow\infty$, the total current density $J(s)=J_b(s)+J_p(s)$ is zero and the beam current is initially neutralized. Therefore, the plasma responds to a fast increase of the beam current by a fast formation of a return current. Initially, the electric field is large, while the magnetic field and the total current are small.

Let us find the magnetic field and the plasma return current for the case of a beam current switched on as a step function in time. We perform the inverse Laplace transformation of $B(x,s)$ and of $I(s)$ for $Y(s)=1/s$. The calculation of the magnetic field is made easier by writing $\alpha^2/(\alpha^2-k^2)$ as $[k^2/(\alpha^2-k^2)+1]$. The magnetic field is found to be

$$B(x,t)=\pm iB_0\left[\frac{\cosh[\alpha(L-|x|)]-\cosh(\alpha L)}{1-\cosh(\alpha L)}+\frac{2\cosh(\alpha L)}{\pi[1-\cosh(\alpha L)]}\times\sum_{n=0}^{\infty}\frac{(\alpha L)^2\sin[(n+\frac{1}{2})\pi(x/L)]}{(n+\frac{1}{2})[(n+\frac{1}{2})^2\pi^2+(\alpha L)^2]}\times\exp\left(-\frac{(n+\frac{1}{2})^2\pi c^2}{4L^2}(\eta_c+i\eta_H)t\right)\right]. \tag{34}$$

The plasma return current is

$$I(t)=\frac{2I_0\cosh(\alpha L)}{\pi[1-\cosh(\alpha L)]}\cdot\sum_{n=0}^{\infty}\frac{(-1)^n(\alpha L)^2}{(n+\frac{1}{2})[(n+\frac{1}{2})^2\pi^2+(\alpha L)^2]}$$

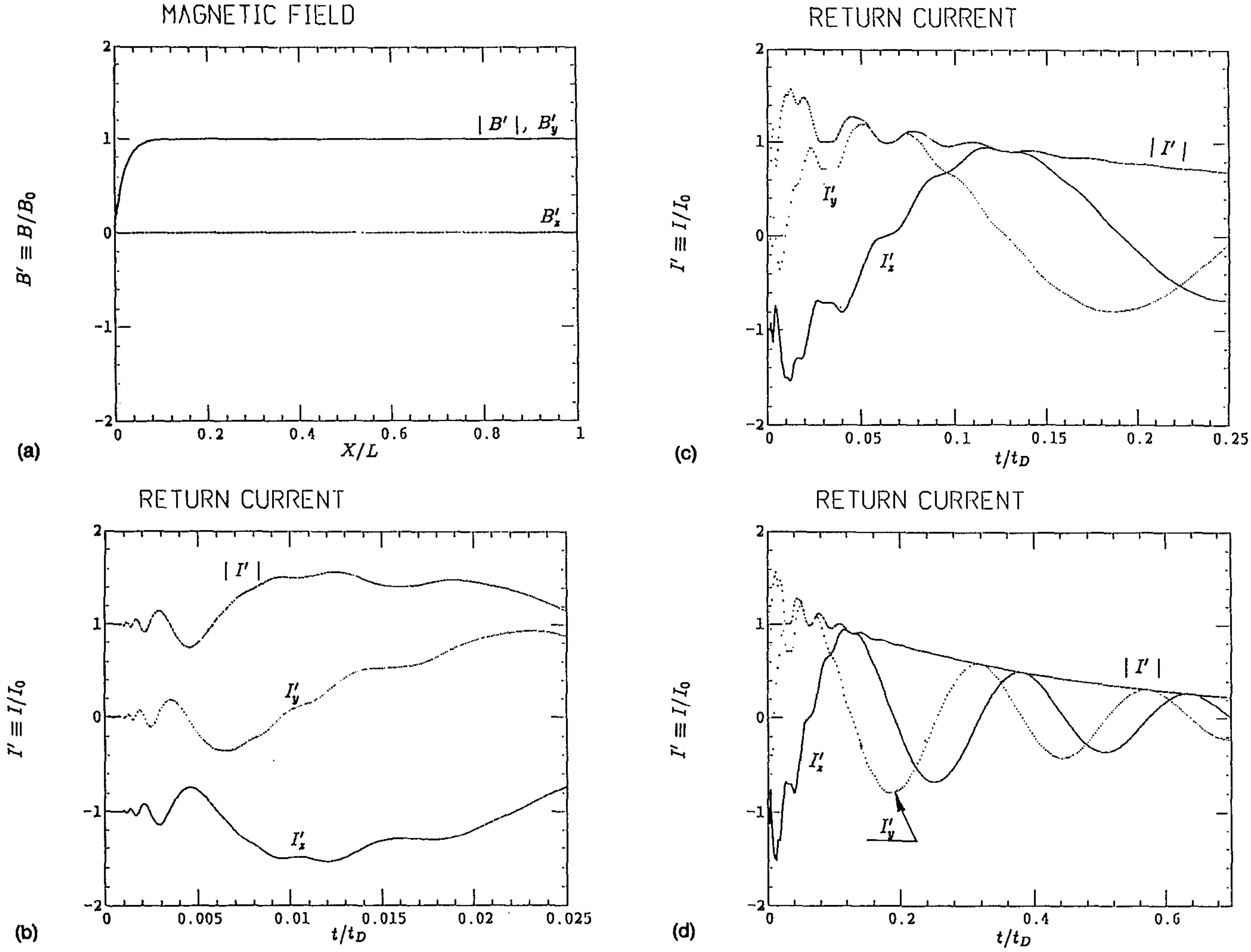

FIG. 3. The magnetic field due to the current of a narrow beam $\alpha L=50$ and the induced plasma return current. The beam current is switched on as a step function in time at $t=0$ and $\eta_H/\eta_c=10$. (a) The magnitude and the z and y components of the magnetic field due to the beam current only as a function of x/L. (b) The magnitude and the z and y components of the plasma return current as a function of time for $0<t/t_D<0.025$, (c) $0<t/t_D<0.25$, (d) $0<t/t_D<0.7$. See Eq. (35).

$$\times\exp\left(-\frac{(n+\frac{1}{2})^2\pi c^2}{4L^2}(\eta_c+i\eta_H)t\right). \tag{35}$$

At the limit $\alpha L\to\infty$ expression (34) for the magnetic field and expression (35) for the plasma return current are reduced to expressions (5) and (20) for the magnetic field and for the plasma return current in the case of an infinitely narrow beam current. We have previously shown [Eq. (33)] that, for arbitrary time dependence, the beam current is initially neutralized. In the Appendix we show directly from the expressions (34) and (35) that $B(x,t=0)=0$ and that the initial total current is zero for this particular time dependence of the beam current, that is switched on as a step function in time.

We notice that some of the features of the magnetic field evolution in the case of the infinitely narrow beam current also appear here. These features include the evolution of the magnetic field on the whistler time scale and, for $\eta_c=0$, the periodicity in time of the magnetic field and of the plasma return current with a period $t'_W=32L^2/c^2\eta_H$. The effect of the finite width of the beam can be seen by comparing the narrow beam limit ($\alpha L\to\infty$) with the wide beam limit ($\alpha L\to 0$). In the narrow beam limit the amplitude of the modes in Eqs. (34) and (35) is proportional to $1/(n+\frac{1}{2})$, while in the wide beam limit the amplitude is proportional to $1/(n+\frac{1}{2})^3$. Therefore, when the beam is narrow the amplitude of the modes decreases slower as a function of the mode number. As a result, when the beam is narrow more modes in the series (34) and (35) are dominant. The magnetic field and the return current then propagate with shorter wavelengths and higher frequencies. This effect of the width of the beam is demonstrated in Figs. 3 and 4. In the figures the plasma return current is shown as a function of time [Eq. (35)] for $\eta_H/\eta_c=10$. In Fig. 3 the beam is narrow, $\alpha L=50$, while in Fig. 4 the beam is wide, $\alpha L=0.1$. Fast oscillations exist when the beam is narrow and they are much less apparent when the beam is wide. The beam in Fig. 1, for which the magnetic field is shown, is narrow, $\alpha L=100$.

For the case discussed here, of a beam of a finite width, we do not write the explicit expressions for the fields and the currents when the beam current is linearly rising in time.

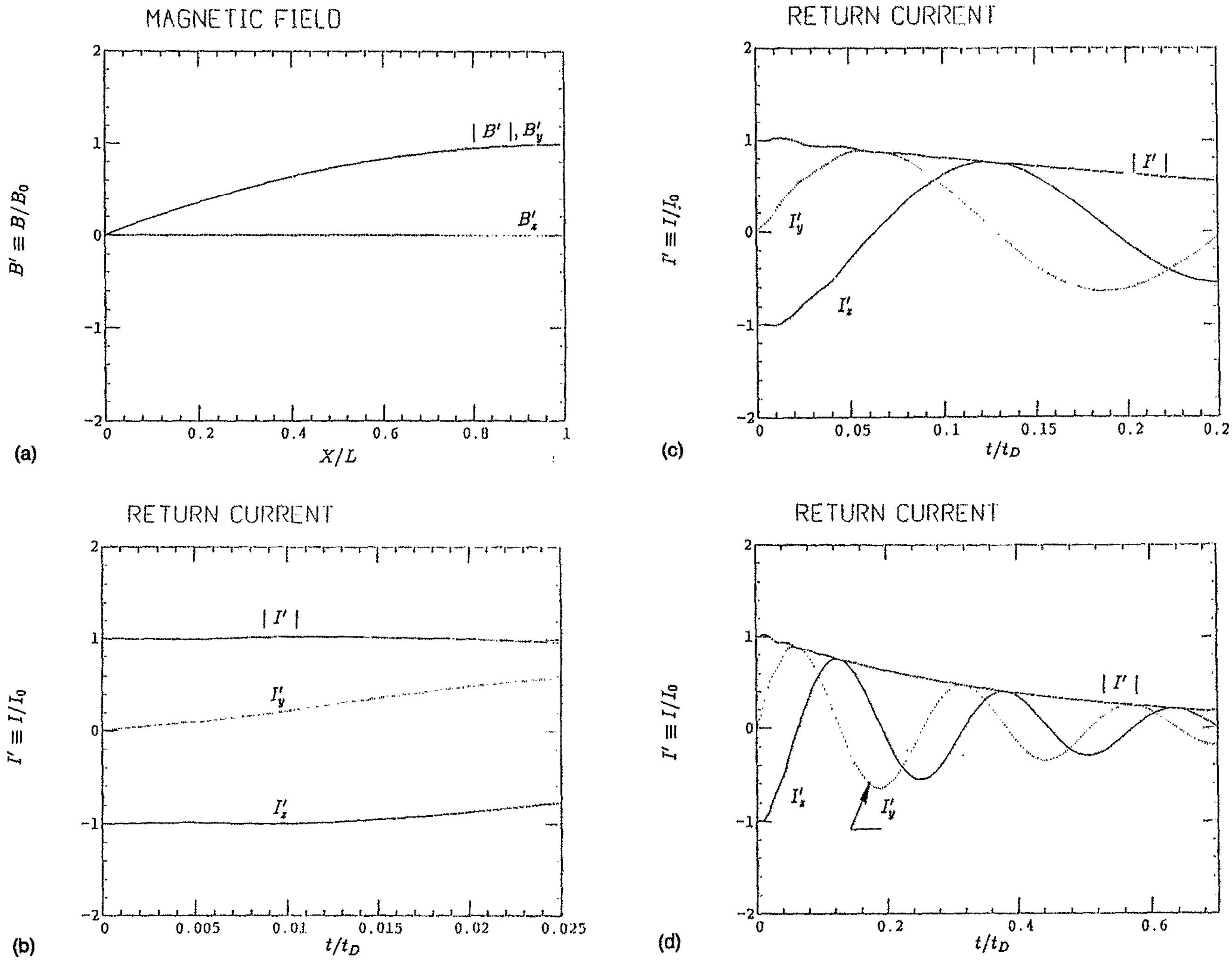

FIG. 4. The magnetic field due to the current of a wide beam $\alpha L=0.1$ and the induced plasma return current. The beam current is switched on as a step function in time at $t=0$ and $\eta_H/\eta_c=10$. (a) The magnitude and the z and y components of the magnetic field due to the beam current only as a function of x/L. (b) The magnitude and the z and y components of the plasma return current as a function of time for $0<t/t_D<0.025$, (c) $0<t/t_D<0.25$, (d) $0<t/t_D<0.7$. See Eq. (35).

However, we mention that these expressions could be found by use of an integration in time of the expressions [Eqs. (34) and (35)] for the fields and the currents when the beam current is switched on as a step function in time. If $Y(t)=t/t_0$, it is clear that the amplitude of the mode is reduced by the factors $[1/(n+\frac{1}{2})^2]$ and $[L^2/(c^2 t_0 \eta_H)]$ relative to the amplitude described above. As a result of the first factor, the higher modes are less dominant and the more gradual increase of the beam current results in a smoother magnetic field and plasma return current. The second factor makes the amplitude of the plasma return current inversely proportional to the current rise time t_0.

VI. CONCLUSIONS

In this paper we have studied the effect of the whistler waves on the evolution of the plasma return current induced by a beam current in a magnetized plasma. We have shown, in a simplified 1-D model, that if variations are allowed along the background magnetic field, and if the characteristic length along this field is smaller than the ion skin depth, the whistler mechanism is dominant in the decay of the plasma return current. The whistler velocity is then higher than the Alfvén velocity, and is also higher than the usually low diffusion velocity. If the plasma is bounded by conductors, the return current in the plasma exhibits whistler oscillations in time. The oscillations are damped on the diffusion time. When the beam current is monotonically rising in time, the plasma return current stops growing beyond the whistler time, thus remains small relative to the beam current that continues to grow. A finite width and a finite rise time of the beam current reduce the amplitudes of the high modes with high frequencies, thus smoothing in time, and in space, the plasma return current.

Even though the main physical process is captured by our 1-D model, some processes require at least 2-D modeling. These are, for example, the finite length of the beam and the nonuniformity of the background magnetic field.

APPENDIX: INITIAL VALUES OF THE MAGNETIC FIELD AND OF THE CURRENT

We show here that the magnetic field expressed in Eq. (34) satisfies $B(x,t=0)=0$. This is easily seen by noticing that the Fourier series of

$$\left(\frac{\cosh[\alpha(L-x)]}{\cosh(\alpha L)}-1\right)$$

expanded between $x=0$ and $x=L$ is

$$\int_0^L dx\left(\frac{\cosh(\alpha(L-x)}{\cosh(\alpha L)}-1\right)\sin\left((n+\tfrac{1}{2})\frac{\pi}{L}x\right)$$

$$=-\frac{1}{(n+\frac{1}{2})(\pi/L)}+\frac{1}{2}\frac{1}{[(n+\frac{1}{2})(\pi/L)+i\alpha]}$$

$$+\frac{1}{2}\frac{1}{[(n+\frac{1}{2})(\pi/L)-i\alpha]}$$

$$=-\frac{\alpha L}{(n+\frac{1}{2})\pi[(n+\frac{1}{2})^2\pi+(\alpha L)^2]}.$$

Therefore, $B(x,t=0)=0$.

Since the magnetic field is zero at $t=0$, it is obvious from Ampère's law that the current density and the current are zero at $t=0$. We show that this is so also by showing that the plasma return current expressed in Eq. (35) equals $-I_0$ at $t=0$. The plasma return current at $t=0$ can be calculated by performing the summation of the series. We assume that $2\alpha L/\pi<1$ and obtain

$$I_0\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{(n+\frac{1}{2})^3\pi^2\{1+[\alpha L/\pi(n+\frac{1}{2})]^2\}}$$

$$=I_0\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{k+n}(\alpha L)^2}{(n+\frac{1}{2})^3\pi^3}\left(\frac{\alpha L}{\pi(n+\frac{1}{2})}\right)^{2k}$$

$$=I_0\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{k+n}(\alpha L)^{2k+2}2^{2k+4}}{(2n+1)^{2k+3}\pi^{2k+3}}. \quad \text{(A1)}$$

On the other hand,

$$\frac{1-\cosh(\alpha L)}{\cosh(\alpha L)}=\sum_{k=0}^{\infty}\frac{(-1)^kE_k(\alpha L)^{2k}}{2k!}-1$$

$$=\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha L)^{2k}(-1)^{k+n}2^{2k+2}}{\pi^{2k+1}(2n+1)^{2k+1}}-1,$$

where E_k are the Euler numbers. The term $k=0$ of the last sum is

$$\sum_{n=0}^{\infty}\frac{4(-1)^n}{\pi(2n+1)}=1.$$

Therefore,

$$I_0\frac{[1-\cosh(\alpha L)]}{\cosh(\alpha L)}$$

$$=I_0\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha L)^{2k+2}(-1)^{k+n+1}2^{2k+4}}{\pi^{2k+3}(2n+1)^{2k+3}}.$$

This expression is identical to Eq. (A1). We therefore obtain that $[I(t=0)/I_0]=-1$. The case in which $2\alpha L/\pi>1$ can be analyzed in a similar fashion.

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