Magnetic field penetration due to the Hall field in (almost) collisionless plasmas*

A. Fruchtman† and K. Gomberoff
Department of Physics, Weizmann Institute of Science, Rehovot 76100, Israel

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The fast penetration of magnetic fields into plasmas due to the Hall field is described. The penetration occurs in nonuniform plasmas of a characteristic length smaller than the ion skin depth, it is much faster than the ion motion, and its rate is independent of the resistivity. Some previous results are described: a shock penetration of the magnetic field accompanied by a large energy dissipation, and field expulsion from an initially magnetized plasma. It is then shown how the Hall field can enhance the penetration into a plasma surrounded by vacuum. Finally, it is demonstrated how the evolution of the magnetic field in a plasma that conducts current between electrodes depends crucially on its evolution near the electrodes, when a realistic density profile is taken into account.

I. INTRODUCTION

The pushing of a plasma by the magnetic field that permeates the vacuum near the plasma was one of the first topics to be studied in plasma physics.[1] If the collisionality of the plasma is low, the plasma is pushed by the magnetic pressure with a characteristic velocity $V_A=B/(4\pi nM)^{1/2}$ (B is the intensity of the magnetic field, n is the plasma density, and M is the ion mass). On the other hand, if the resistivity of the plasma is high enough, the dominant process is the penetration of the magnetic field into the plasma[2] with the characteristic velocity $V_D=c^2\eta/(4\pi L)$ (η is the resistivity, L the characteristic length, and c the velocity of light in vacuum). In this paper we describe a mechanism for fast penetration of the magnetic field into the plasma that is independent of the resistivity. The characteristic velocity of this penetration is $V_c=c^2 \times(B/nec)/(4\pi L)$ ($-e$ is the electron charge). The fast evolution of the magnetic field results from the Hall electric field, and the velocity V_c is proportional to the Hall "resistivity" $B/(nec)$. The length L in the expression for the velocity V_c is either the characteristic length of a density gradient,[3] the radius of curvature of the magnetic field,[4] or the length of the magnetic-field gradient in the whistler wave propagation.[5] Equivalently to the magnetic Reynolds number, a characteristic parameter R could be defined,[6] which is the ratio of the plasma pushing velocity V_A to the penetration velocity V_c:

$$R\equiv V_A/V_c=L/(c/\omega_{pi}). \quad (1)$$

When L is smaller than the ion skin depth c/ω_{pi}, the field penetration is the dominant process. Since we assume quasineutrality, the scalelength L should be larger than the electron skin depth c/ω_{pe}. The dominance of these penetration mechanisms for scale lengths between the electron and the ion skin depths, makes them relevant to important space and laboratory plasmas. Examples include the plasma opening switch (POS)[7–9] and plasma beams which cross magnetic fields.[10] Various aspects of the penetration due to plasma nonuniformity (either a density gradient or magnetic-field curvature) are being intensively studied currently.[3,4,6,11–20] We have also explored the propagation of the magnetic field along a background magnetic field as a whistler wave in a one-dimensional (1-D) geometry.[5,21,22] Armale and Rostoker have recently analyzed the whistler mechanism in a two-dimensional (2-D) geometry that corresponds to a plasma beam.[23] For lack of space we will not describe here this interesting evolution as a whistler wave. Rather, we will focus on the penetration due to plasma nonuniformity. We start by showing that the picture of collisionless pushing of a plasma by the magnetic field as a specular reflection of ions and electrons, is not valid for a nonuniform plasma of a scalelength shorter than the ion skin depth.

The pushing of a collisionless plasma by a magnetic field was described in an early paper by Rosenbluth.[1] This pushing, while current flows from the plasma to an electrode, was recently studied by Mendel.[24] The plasma boundary is pushed with velocity V_A. Figure 1 shows the plasma pushing in the rest frame of the plasma boundary. In this frame the plasma particles are moving toward the boundary with the velocity V_A. The plasma is separated from the vacuum by a non-neutral sheath of width c/ω_{pe}. The plasma ions that move towards the sheath are reflected by a potential hill of height $B_0^2/8\pi ne$. The ions are essentially unmagnetized since their Larmor radius is much larger than c/ω_{pe}. An obvious necessary condition for the 1-D picture to hold is, therefore, that the characteristic scalelength is larger than c/ω_{pe}. Inspection of the electron dynamics, though, shows that the condition should be much more stringent. The electrons are accelerated during their motion inside the sheath and acquire a large perpendicular velocity $V_{Ae}\equiv V_A(M/m)^{1/2}$. The time the ions or electrons spend in the sheath is approximately $(c/\omega_{pe})/V_A$, which is the hybrid electron–ion cyclotron period $c(Mm)^{1/2}/eB_0$ (m is the electron mass). The perpendicular displacement of the electrons, while they are

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†Invited speaker.

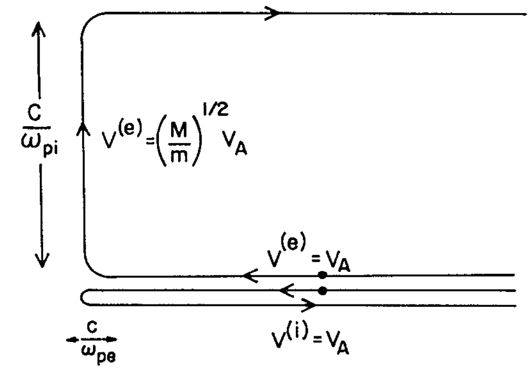

FIG. 1. Pushing of a collisionless plasma by a magnetic field.

reflected by the magnetic field, is approximately the product of this time and the perpendicular velocity and is therefore c/ω_{pi}. Thus, in order for the 1-D picture to be valid, the characteristic scalelength must be much larger than c/ω_{pi}. The 1-D picture of specular reflection is invalid for nonuniform plasmas in which the characteristic length is smaller than c/ω_{pi}. Into such nonuniform plasmas the magnetic field can penetrate.

In Sec. II we describe the model and show how plasma nonuniformity can lead to fast magnetic-field evolution. The important role of collisions is also demonstrated. Despite the recent progress[15–18] in the study of the low collisionality case (when electron inertia is dominant), the collisional case (when the resistivity is dominant) will be the subject of most of this paper. We emphasize that even in the collisional case the rate of penetration is determined by the plasma nonuniformity and not by the resistivity.

In Sec. III we describe the penetration of a magnetic field into a plasma surrounded by vacuum. This geometry has relevance to plasma beams. In Sec. IV we briefly review our previous studies of magnetic-field penetration into the POS. In those previous studies the plasma density was assumed to have a simple profile. Section V is devoted to study the effect of a realistic density profile on the field evolution. We suggest that this modified density may result in a dramatically different behavior of the magnetic field. In particular, if the density profile is more realistic, the difference between the penetrations for opposite polarities of the switch is not as drastic as in a uniform density plasma. We conclude in Sec. VI by commenting on the low collisionality regime.

II. THE MODEL

We start with the electron continuity equation

$$\frac{\partial n}{\partial t}+\nabla\cdot n\mathbf{v}=0, \tag{2}$$

the electron momentum equation

$$\frac{m}{e}\frac{d}{dt}\mathbf{v}=-E-\frac{\mathbf{v}\times\mathbf{B}}{c}-\frac{\nabla p_e}{en}+\eta\mathbf{j}, \tag{3}$$

and Faraday's law

$$-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=\nabla\times\mathbf{E}. \tag{4}$$

Here, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mathbf{v}$ and p_e are the electron flow velocity and (the assumed isotropic) pressure, and $\mathbf{j}$ is the current density. Also, $d/dt\equiv\partial/\partial t+\mathbf{v}\cdot\nabla$.

We will refer repeatedly to a 2-D geometry, in which the magnetic field has one component only in the ignorable coordinate, and in which the parallel flow is zero. In a rectangular geometry, in which $\partial/\partial y=0$, and $\mathbf{B}=\hat{e}_yB_y(z,x)$, Eqs. (2)–(4) become

$$n\frac{d}{dt}\left(\frac{\Omega_y+\omega_y}{n}\right)=\frac{1}{m}\left\{\frac{1}{n},p_e\right\}+\frac{e\eta}{m}(\nabla\times\mathbf{j})_y. \tag{5}$$

Here $\boldsymbol{\omega}\equiv\nabla\times\mathbf{v}$ is the vorticity, and $\boldsymbol{\Omega}\equiv e\mathbf{B}/mc$. Similarly, in a cylindrical geometry, in which $\partial/\partial\theta=0$ and $\mathbf{B}=\hat{e}_\theta B_\theta(z,r)$, Eqs. (2)–(4) become

$$nr\frac{d}{dt}\left(\frac{\Omega_\theta+\omega_\theta}{nr}\right)=\frac{1}{m}\left\{\frac{1}{n},p_e\right\}+\frac{e\eta}{m}(\nabla\times\mathbf{j})_\theta. \tag{6}$$

The Poisson brackets in Eq. (5) are

$$\{f,g\}\equiv\frac{\partial f}{\partial z}\frac{\partial g}{\partial x}-\frac{\partial g}{\partial z}\frac{\partial f}{\partial x}, \tag{7}$$

while in Eq. (6) the derivative, with respect to x, is replaced by a derivative, with respect to r. Here we make two observations, with respect to Eqs. (5) and (6). The first observation is concerned with the freezing of the flux into the electron fluid, a freezing that is expressed in Eqs. (5) and (6). Because of this freezing a fast evolution of the magnetic field ensues, if the electron trajectory is along varying n in slab geometry or along varying nr^2 in a cylindrical geometry. Compression of the magnetic flux along the electron trajectory causes this fast evolution. The second observation, on the other hand, is that the magnetic field cannot penetrate into a cold unmagnetized plasma if the resistivity is zero.

Equations (2)–(4) are to be complemented with the momentum equation for the ions, an appropriate heat equation for the electrons and Ampère's law. In the regime of EMHD (electron magnetohydrodynamics),[11] the time scale is so fast that the ions are immobile. Therefore,

$$\mathbf{j}=-en\mathbf{v}. \tag{8}$$

We also neglect the displacement current in Ampère's law

$$\frac{4\pi}{c}\mathbf{j}=\nabla\times\mathbf{B}. \tag{9}$$

From Eq. (9) and also from the equivalent assumption of quasineutrality, it follows that $\partial n/\partial t=0$. We neglect the electron inertia at this stage. Equations (1)–(3), (8) and (9) become

$$\frac{\partial B}{\partial t}=\frac{c^2\eta}{4\pi}\nabla^2\mathbf{B}-\frac{c}{4\pi e}\nabla\times\left(\frac{1}{n}(\nabla\times\mathbf{B})\times\mathbf{B}\right)$$
$$+\frac{c}{e}\nabla\left(\frac{1}{n}\right)\times\nabla p_e. \quad (10)$$

This equation describes the evolution of the magnetic field that is determined by the electron dynamics. The neglect of the electron inertia results in the neglect of the vorticity in Eqs. (5) and (6).

III. PENETRATION INTO A PLASMA BEAM

In this section we describe how a density gradient can cause fast magnetic-field penetration into a plasma beam. We assume again that the magnetic field has one component only in the ignorable coordinate y. Equation (10) is then reduced to

$$\frac{\partial B_y}{\partial t}=\frac{c^2\eta}{4\pi}\nabla^2 B_y+\frac{c}{e}\left\{\frac{1}{n},\frac{B_y^2}{8\pi}+p_e\right\}. \quad (11)$$

Let us use the right-hand coordinates $1/n(z,x)$ and $g(z,x)$, where $\nabla(1/n)\cdot\nabla g=0$ and $|\nabla g|=1$. Equation (11) becomes

$$\frac{\partial B_y}{\partial t}-\frac{c}{e}\left(\frac{B_y}{4\pi}+\frac{\partial p_e}{\partial B_y}\right)\left|\nabla\left(\frac{1}{n}\right)\right|\frac{\partial B_y}{\partial g}=\frac{c^2\eta}{4\pi}\nabla^2 B_y. \quad (12)$$

This is correct for the particular case in which $p_e=p_e(B_y)$, which will be described later. We also used $\{1/n,g\}=|\nabla(1/n)|$. When the resistivity is zero the equation is hyperbolic. The equidensity contours are projections of the characteristics in the (z,x,t) space onto the (z,x) plane. The magnetic field propagates along equidensity contours with the velocity

$$v_c=\frac{c}{e}\left(\frac{B_y}{4\pi}+\frac{\partial p_e}{\partial B_y}\right)\left|\nabla\left(\frac{1}{n}\right)\right|. \quad (13)$$

This is the explicit form of the velocity V_c mentioned in the Introduction. The fast evolution is driven by the density gradient. Across the equidensity contours the magnetic field propagates with the usual velocity of resistive diffusion.

Consider the plasma plotted in Fig. 2(a). Plotted are the equidensity contour lines in the (z,x) plane. The plasma is assumed infinite in the y direction, normal to the figure plane. The magnetic field lies in the y direction, and is expected to penetrate into the plasma on the diffusion time $4\pi b^2/c^2\eta$. If, however, the density has large gradients, the penetration is enhanced. The plasma in Fig. 2(b) has a large density gradient on its right side. The magnetic field penetrates into the region of maximum density on the diffusion time. This is followed by fast propagation with velocity $cB/(4\pi enb)$ along the equidensity contours. The penetration time is, therefore, the sum of the two times, and equals (for a cold plasma)

$$\tau\cong\frac{4\pi d^2}{c^2\eta}+\frac{4\pi enba}{cB}. \quad (14)$$

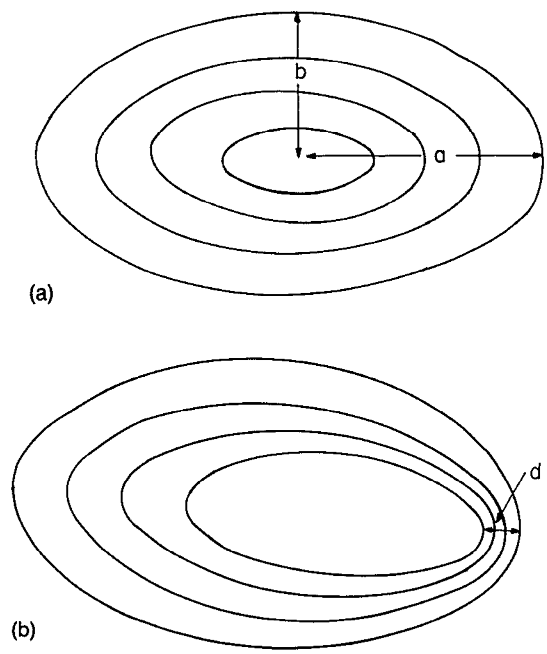

FIG. 2. Density contour levels of plasmas: (a) The magnetic field penetrates on the resistive diffusion time. (b) A fast diffusion on the right side is followed by a fast penetration along the equidensity contours.

The penetration velocity is enhanced relative to the diffusion velocity if $4\pi enba/cB\ll 4\pi b^2/c^2\eta$, i.e., $\eta(enc/B)\ll b/a$. Also, d should be much smaller than b. This is the condition in this geometry for the Hall "resistivity" $B/(enc)$ to dominate the penetration.

The geometry described in Fig. 2 has relevance to magnetic-field penetration into the plasma beams. In one mode of beam propagation the magnetic field penetrates very fast into the plasma.[10] The actual geometry of such a system is 3D. We assumed that the beam is thick in the direction parallel to the magnetic field, and therefore approximated the geometry as 2D. This enabled us to find a mechanism for enhanced penetration. The 3-D geometry of the beam could also be approximated as a 2-D geometry of an infinitely long cylinder. In such a geometry the magnetic field in the vacuum has a component normal to the plasma surface. As was recently shown, this allows an evolution on the whistler time scale.[23]

IV. PENETRATION INTO A PLASMA-SIMPLIFIED DENSITY PROFILE

We next examine the penetration of the magnetic field into a plasma that conducts current between two electrodes. The possibility that this mechanism causes fast penetration of the magnetic field into the plasma in the POS has recently attracted much attention. The POS is composed of a hollow cylindrical plasma that fills the gap between two cylindrical concentric conductors of the transmission line.[7] The current of the transmission line flows between the two cylindrical conductors through the plasma. The current and its associated magnetic field evolve in the plasma, according to

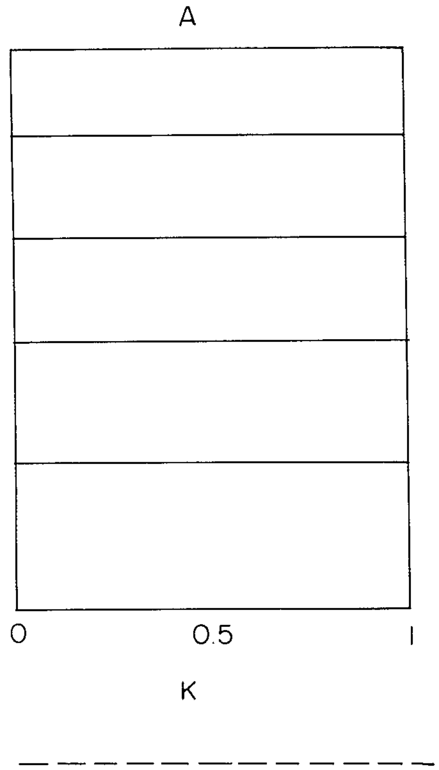

FIG. 3. The nr^2 contour levels of a hollow cylindrical plasma of uniform density.

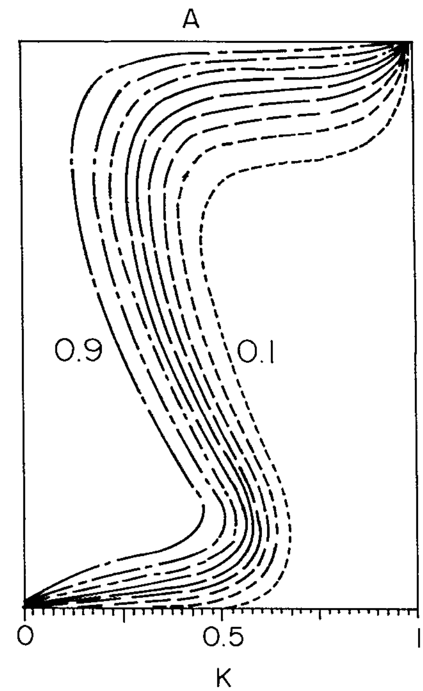

FIG. 4. The rB_θ contour levels during the shock penetration. The nr^2 contour levels are as in Fig. 3.

$$\frac{\partial b}{\partial t}=\frac{c^2\eta}{4\pi}\Delta^* b+\frac{crb}{4\pi e}\left\{\frac{1}{nr^2},b\right\}+\frac{cr}{e}\left\{\frac{1}{n},p_e\right\}. \tag{15}$$

Here, $\Delta^* b\equiv\partial^2 b/\partial r^2-(1/r)(\partial b/\partial r)+\partial^2 b/\partial z^2$. This equation follows Eq. (10) and the assumptions that the magnetic field has an azimuthal component only, and that $\partial/\partial\theta=0$. Here, $b\equiv rB_\theta$. We now use the right-hand coordinates $1/nr^2$ and $g(z,r)$, where $\nabla(1/nr^2)\cdot\nabla g=0$, and $|\nabla g|=1$. For a cold plasma Eq. (15) becomes

$$\frac{\partial b}{\partial t}-\frac{crb}{4\pi e}\left|\nabla\left(\frac{1}{nr^2}\right)\right|\frac{\partial b}{\partial g}=\frac{c^2\eta}{4\pi}\Delta^* b. \tag{16}$$

When the resistivity is zero the equation is hyperbolic and the nr^2 contour lines are projections of the characteristics onto the (z,r) plane. The magnetic field propagates along the nr^2 contour lines with the velocity

$$v_c=\frac{crb}{4\pi e}\left|\nabla\left(\frac{1}{nr^2}\right)\right|. \tag{17}$$

If the density has radial dependence only, the nr^2 contour lines are parallel to the z axis. We solved in detail the case of uniform density plasma.[4,12,13] The magnetic field propagates into the plasma along the characteristics (the nr^2 contour lines) from the vacuum on the generator side when the switch polarity is negative (the cathode is at the inner conductor) and from the vacuum on the load side when the switch polarity is positive (the cathode is at the outer conductor). Figure 3 shows the nr^2 contour levels for a plasma of uniform density. Figure 4 shows the $-b$ contour lines at a certain time during the penetration when the polarity is negative. The plots are the solution of Eq. (16) for the uniform density case, which simplifies to

$$\frac{\partial b}{\partial t}+\frac{cb}{4\pi enr^2}\frac{\partial b}{\partial z}=\frac{c^2\eta}{4\pi}\left(\frac{\partial^2 b}{\partial r^2}-\frac{1}{r}\frac{\partial b}{\partial r}+\frac{\partial^2 b}{\partial z^2}\right). \tag{18}$$

Far enough from the electrodes the r derivatives could be neglected and the equation is approximated as Burgers equation. The magnetic field propagates parallel to the z axis.

We should emphasize that the direction of propagation of the wave is not the direction of propagation of the magnetic-field energy. The magnetic-field energy does not flow axially in Fig. 4 along the nr^2 contours. Rather, it flows radially outward, convected by the electron flow. The magnetic-field energy changes in the bulk of the plasma because of the plasma nonuniformity. In the cylindrical geometry it is because of the radial dependence. The radial flow of magnetic-field energy is not uniform, and this causes fast changes of the magnetic field in the plasma. These changes appear formally as waves along the nr^2 contours.

It can easily be shown that since the magnetic-field flux has to be conserved, magnetic-field energy has to be dissipated during the magnetic-field penetration into the plasma. We have shown that half of the energy that is penetrating the plasma is dissipated. In the calculation we assumed that our model equations are valid even at the electrode boundary, and that the electrodes are perfect conductors. A third of the power that flows from the vacuum into the plasma near the cathode was shown to be dissipated. A quarter of the two-thirds magnetic-field energy that flows radially into the plasma is dissipated in the shock front. This makes the dissipated energy half of the total penetrating magnetic-field energy.[12]

We found the distribution of the electron thermal energy by solving an electron heat equation, together with the equation for the magnetic field

$$\frac{\partial \epsilon}{\partial t}+\nabla\cdot\frac{5}{3}\epsilon\mathbf{v}=\mathbf{E}\cdot\mathbf{j}, \qquad (19)$$

where ϵ is the electron thermal energy, $p_e=\frac{2}{3}\epsilon$. The analysis of the power flow shows that the radial energy flow is composed of one-third electron thermal energy flux and two-thirds magnetic-field energy flux. As we mentioned above, a quarter of these 2/3 magnetic-field energy flux is dissipated and converted into electron thermal energy.[13] The solution of the heat equation (19) shows that a half of the third electron thermal energy flux is deposited in the plasma. As a result, the energy in the shock downstream is equally divided between magnetic-field energy and electron thermal energy.[13]

This equipartition of energy can be shown in a different way. We used Eq. (6) neglecting electron inertia and pressure, and showed a direct relation between the deviation from the frozen-in law and the energy dissipated per particle along the particle path in the shock solution:

$$\left[\frac{B}{nr}\right]=\frac{8\pi}{rB_0}\int_{-\infty}^{\infty}\frac{dt}{n}\,\eta\, j^2. \qquad (20)$$

From this relation it also follows that the energy at each position in the shock downstream is equally divided between magnetic-field energy and electron thermal energy.[13]

V. PENETRATION INTO A PLASMA—A REALISTIC DENSITY PROFILE

The choice of nr^2 contours that end at the plasma-vacuum axial boundaries has a dramatic effect. It makes the interaction asymmetric, with respect to the switch polarity. If the cathode is at the outer conductor (positive polarity) the direction of propagation of the wave is reversed. It now propagates into the plasma from the load side. If the plasma is unmagnetized, it will remain so, and all the current would flow in a narrow current channel on the generator side. Moreover, if the plasma is initially unmagnetized, then the wave propagating from the load side is expected to expel the magnetic field from the plasma.[4,12]

Since no nr^2 contours start at the electrodes, the effect of the boundary conditions there on the field evolution inside the plasma is small. The effect of the electrodes is restricted to their neighborhood. However, very different evolution is expected if a more realistic density profile is allowed. In Fig. 5 the nr^2 contours are shown for a density profile that decreases monotonically radially and also decreases gradually in both axial directions. This is the density profile that is expected for a plasma that is injected radially outward from the axis. In this case the nr^2 contours intersect the electrodes, and it is the evolution of the magnetic field at the electrodes that determines the evolution in the bulk of the plasma.

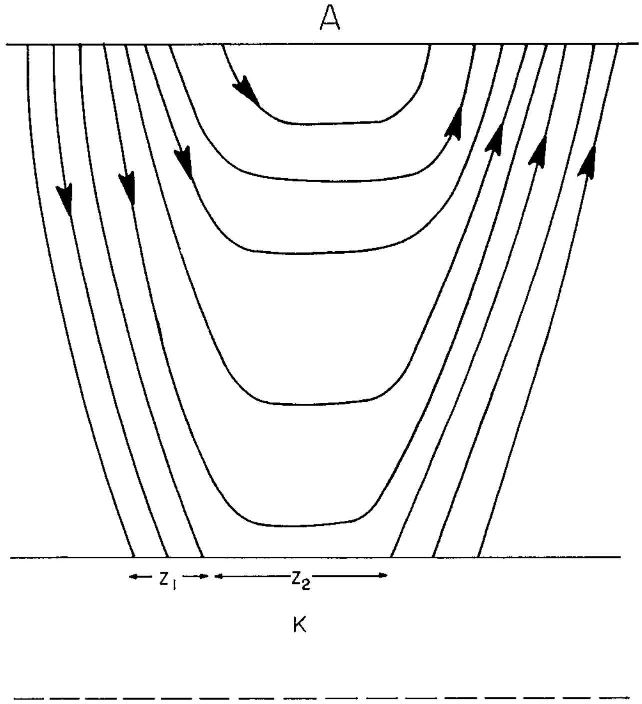

FIG. 5. The nr^2 contour levels when the density decreases radially and towards the axial boundaries. The arrows show the direction along which the magnetic field propagates.

Let us first assume that the magnetic field does not penetrate along the electrodes. The fast Hall penetration will occur only once the magnetic field has diffused through the layer of thickness z_1. The penetration time will then become

$$\tau\cong\frac{z_1^2}{4\pi c^2\eta}+\frac{4\pi enrz_2}{cB}. \qquad (21)$$

This time is the sum of the resistive time (the first term) and the Hall time (the second term). This time could be much longer than the time of the pure Hall penetration, if the resistivity is small enough

$$\frac{\eta}{B/nec}\ll\frac{z_1^2}{rz_2}. \qquad (22)$$

The penetration, even in this case, is shorter than the usual resistive time by $[z_1/(z_2+z_1)]^2$. If there is no field penetration along the electrodes and if the nr^2 contour lines always connect the electrodes, the penetration will occur on the slow resistive time, and no enhancement of the penetration will occur.

A correct treatment of the evolution near the electrodes involves an appropriate treatment of non-neutral sheaths. Elaborate models were developed for that purpose. Common to these models is the prediction of fast penetration of the magnetic field along the cathode[8,25,26] or along the anode.[27] Thus let us now assume that the magnetic field does penetrate into the plasma along the electrodes by some mechanism. Since the nr^2 contour lines intersect the plasma boundaries with the electrodes and not only the plasma boundaries with the vacuum, the magnetic

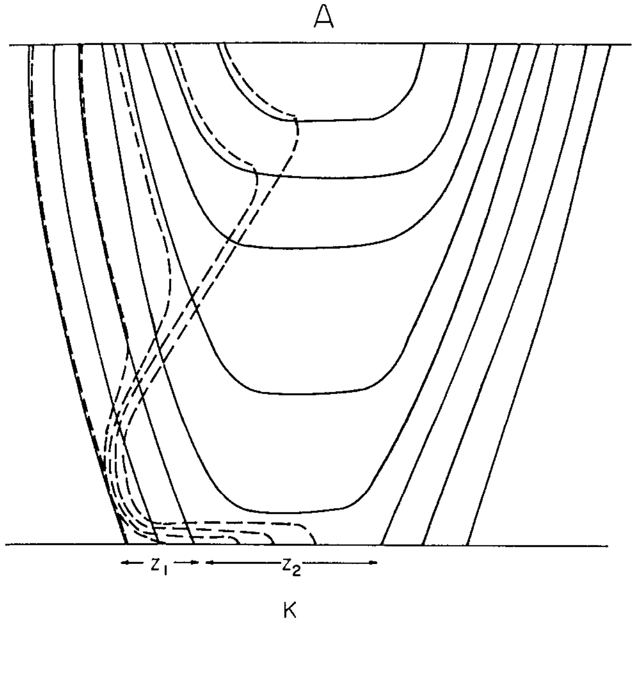

FIG. 6. Schematic of the field penetration when the density profile is realistic. The solid lines are the nr^2 contour levels and the dashed lines are the rB_θ contour levels. The magnetic field is assumed to penetrate along the electrodes.

field in this case will propagate along the nr^2 contour lines from the electrodes into the plasma. The arrows in Fig. 5 show the direction of propagation of the wave from the electrodes into the plasma for a negative polarity. Figure 6 shows schematically the magnetic field distribution at some time during the penetration. As a result of the more realistic density profile the distribution of the current is broad. An additional important result of the more realistic density profile, is that if the magnetic field penetrates along the electrodes, the field evolution is not as asymmetrical, with respect to the switch polarity. Some field penetration (though smaller) is expected also for positive polarity. In a uniform density plasma the magnetic field does not penetrate when the polarity is positive, since the propagation is along the nr^2 contour lines from the vacuum on the load side, where the field is zero. In a plasma of a realistic density profile, the magnetic field propagates along the nr^2 contour lines from the electrodes in both polarities (when the polarity is positive the direction is opposite to the arrows in Fig. 5). The result of field penetration along the electrodes will be certain extent of field penetration into the plasma in positive polarity as well. Detailed calculations of the magnetic field evolution in a plasma of a realistic density profile are given elsewhere.[28]

VI. DISCUSSION

We have described the recent theoretical progress in the study of fast magnetic evolution in nonuniform plasmas of scale length smaller than the ion skin depth. Despite the increasing understanding of these processes, important issues are not fully resolved. The central remaining issue concerns collisionality.

The collisionality of electrons that do not leave the plasma through the electrode boundary during the shock penetration could be small, but not too small. The electrons should stay in the shock layer for at least one collision time. For that to occur, the collision frequency should be at least the electron–ion hybrid cyclotron frequency. In fact, it should be at least $(\omega_{ce}\omega_{ci})^{1/2}[(c/\omega_{pi})/L]$, where L is the characteristic length of nonuniformity. The condition on the magnitude of the collision frequency is required for the neglect of the inertia in the electron momentum equation. It is also a necessary condition for the neglect of the kinetic energy associated with the electron drift velocity relative to that associated with the electron thermal velocity in the electron heat balance equation (19). The collisionality is also required in order for the stress tensor elements to be smaller than the isotropic part of the pressure, and for the thickness of the layer to be larger than c/ω_{pe}, the electron skin depth.

The collisional solution is thus valid for a collision frequency that might be higher than the collisionality that actually exists in many plasmas. The case of lower collisionality is therefore of much importance. Kalda and Kingsep have shown that when the electron inertia is included in the 1-D shock solution, the shock structure in the low collisionality case becomes oscillatory.[14] The oscillatory domain is large enough so that the electron remains there one collision time, a time sufficient for equilibration. However, one might question the existence of an oscillating current between electrodes. A different approach was taken by Rudakov *et al.*,[15,16] who showed a nonoscillatory 2-D fast penetration where the structure is determined by the inertia. One of us (AF) and Rudakov[18] showed that a 2-D low collisionality fast penetration can occur even into an initially homogeneous plasma. These recent studies advanced our understanding of the low collisionality case and showed the possibility of fast penetration. The full problem, with a treatment of the whole system, and an identification of where the dissipation occurs, has not yet been solved. It is believed that if the electrodes were sufficiently close, the shock could be totally collisionless with the exception of the electrode boundary.[29] The shock propagates entirely by replacing the original electrons with new electrons. It is easy to verify that in this case the distance between the electrodes should be smaller than L, the characteristic length (the radius or the length of density gradient). In this case, the electron transit time between the cathode and the anode (which is the effective collision time) would be shorter than the time the electron would have stayed inside the shock layer in the absence of the electrodes. The collisionless shock propagation in the presence of electrodes that are close should be studied further.

A second issue concerns the simultaneous evolution of the magnetic field and of the electron thermal energy. We described this evolution for the special case of field penetration into a plasma between two electrodes. This evolution of both the magnetic field and the electron thermal energy in a plasma surrounded by vacuum and for a realistic density profile requires further studies.

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