The effect of displacement current on whistler propagation of a fast-rising magnetic field

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The effect of the displacement current on the magnetic field propagation into plasmas in the form of a whistler wave is studied. For times between the electron and the ion cyclotron periods and if the plasma is tenuous so that the electron plasma frequency is smaller than the cyclotron frequency, the magnetic field is shown to be governed by the Telegraph equation with complex coefficients. The propagation of a fast-rising magnetic field is examined and at early times the magnetic field is shown to propagate as a left-polarized wave with the light velocity. At a later time the magnetic field propagates as the dispersive whistler wave and is governed by the diffusion equation with a complex coefficient [Fruchtman and Maron, Phys. Fluids B **3**, 1546 (1991)]. As expected, the front of the wave keeps propagating with the finite velocity of light. The effect of collisional resistivity is also considered.

I. INTRODUCTION

The magnetic field propagation in magnetized plasmas has been studied theoretically and experimentally in the context of space plasmas as in laboratory plasmas. The propagation of the magnetic field in processes for which the time scales are fast is also of wide interest and has been studied in laboratory plasmas, as in an afterglow plasma,[1] and in some pulsed power devices such as the magnetically insulated ion diode[2,3] and the plasma opening switch.[4,5] The time scale in such cases is between the electron cyclotron period and the ion cyclotron period. If the plasma collisionality is small, then for these time scales, the magnetic field evolution is highly governed by the Hall field.

In such short time scales, which are characterized by frequencies such that $\omega \gg \omega_{ci}$, where ω_{ci} is the ion cyclotron frequency, the electron motion becomes more important than the ion motion, and the ion motion can be neglected. In another paper where the ion motion is included,[6] we study the transition from field propagation to plasma pushing by the magnetic field, and we show that the neglect of the ion motion for $\omega \gg \omega_{ci}$, is a consistent assumption.

The electron inertia can be neglected as long as $\omega \ll \omega_{ce}$, where ω_{ce} is the electron cyclotron frequency. In such cases, the Hall term accounts for the electron motion. For $\omega \approx \omega_{ce}$ more complicated models should be considered. The Hall term has been already vastly introduced in the context of different plasma behaviors[7–14] and in particular also in the context of propagation of magnetic field.[6,15,16]

In a recent paper[16] it is shown that when the Hall term is introduced into the equations, the magnetic field propagation is enhanced in a dramatic way provided that there exists a strong magnetic field in the direction of propagation. In this case the propagation of a fast rising field is characterized by the whistler velocity rather than by the Alfvén velocity. Also, for large enough Hall term in comparison with the collisional term, the process is much faster than what a purely diffusive type process would be. The one-dimensional magnetic field evolution, when governed simultaneously by both whistler wave propagation and collisional diffusion, was shown to be described by a diffusion equation with a complex diffusion coefficient, where its real part results from the collisional resistivity and the imaginary part results from the Hall term. The displacement current has been neglected in this analysis. The purpose of the present paper is to examine how the whistler-dominated magnetic field evolution is modified by the inclusion of the displacement current.

Our model, as in Refs. 6 and 16, consists of a one-dimensional plasma slab with perpendicular magnetic field of intensity B_x. A parallel magnetic field is switched on at time $t=0$ on one of the slab faces, and it propagates into the plasma as a whistler wave, governed by the Hall term. We focus on the effect of the displacement current which was previously neglected.

In order to be able to neglect the displacement current in the Hall-field-dominated evolution, we previously assumed that the time scales should be such that $\omega \ll \omega_p^2/\omega_{ce}$, where ω_p is the plasma frequency. In the present paper we relax this assumption but still restrict ourselves to time scales such that $\omega_{ce} \gg \omega$. For such time scales, the electron inertia can still be neglected, but the displacement current has to be included. Such a regime exists for tenuous plasmas in which $\omega_{ce} \gg \omega_{pe}$.

We make the further assumption that the parallel fields are smaller than the perpendicular uniform field B_x. The problem is thus reduced to the linear one-dimensional evolution of the magnetic field along a background magnetic field in a cold uniform plasma. Under the combined effects of the Hall field, the collisional resistivity and the displacement current, the magnetic field evolution is shown to be governed by the Telegraph equation with complex coefficients. When the displacement current is neglected, the Telegraph equation is reduced to our previous diffusion equation with a complex coefficient. We study the propagation into the plasma of a magnetic field that is switched on with an infinitely fast rise time at the plasma boundary. We derive analytical solutions in this case for both semi-infinite and finite plasma slab.

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The propagation into the plasma of fast-rising currents, as well as that of the associated magnetic field, in the form of whistler wave propagation, has been recently observed experimentally in an afterglow plasma.[1] Although the geometry in that experiment was cylindrical rather than a slab geometry, as it is in our model, the currents were observed to propagate by dispersive oscillatory whistler waves governed by the electron motion, similar to the description in the present model. The typical velocity of the propagation was indeed observed to be the whistler wave velocity rather than the Alfvén velocity.

In Sec. II the model is presented and the governing equation for the parallel propagating field is derived. Also, typical time and space scales are calculated. In Sec. III an analytical solution is shown for the semi-infinite slab. The solution is compared to the solution that does not consider the displacement current, its development in time is shown and the role of the collisional resistivity is also shown. In Sec. IV a solution for finite boundaries at the perpendicular x direction is presented. Finally, a resumé is given in Sec. V.

II. THE MODEL

We assume that the process is so fast that the ions are immobile and the current as a result is determined by the electron motion only. On the other hand, we assume that the process is slow enough to allow the neglect of the electron inertia. The characteristic time scale is therefore between the electron and the ion cyclotron periods. We further assume that the pressure gradients are small. These assumptions are expressed in the following form of Ohm's law,

$$\mathbf{E} = \eta \mathbf{J} + \epsilon \mathbf{J} \times \mathbf{B}, \tag{1}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mathbf{J}$ is the current, η is the collisional resistivity, $\epsilon = 1/n_e ec$ and n_e is the electron density.

The other governing equations are Faraday's law and Ampère's law for the electric and magnetic fields, and the continuity equation for σ the charge density:

$$(1/c)\partial_t \mathbf{B} = -\nabla \times \mathbf{E}, \tag{2}$$

$$(1/c)(\partial_t \mathbf{E} + 4\pi \mathbf{J}) = \nabla \times \mathbf{B}, \tag{3}$$

$$\partial_t \sigma + \nabla \cdot \mathbf{J} = 0. \tag{4}$$

Equations (1)–(4) and the relation

$$n_e = n_0 - \sigma, \tag{5}$$

where n_0 is the constant-in-time ion density, comprise the governing equations for $\mathbf{E}$, $\mathbf{B}$, $\mathbf{J}$, and σ.

We consider an infinitely long plasma slab, with its normal at the x direction. The plasma is magnetized with a uniform magnetic field $B_x \hat{x}$. A constant parallel magnetic field B_0 is switched on the front face of the slab (a y-z plane), at a time $t = 0$. We assume that all quantities depend only on x.

When the displacement current is neglected, incompressibility is a property of the electron flow. When the displacement current is important, the electron flow is incompressible only if we consider the linearized one-dimensional case, B_z, $B_y \ll B_x$, on which we focus from now on. More exactly, B_z, $B_y \ll B_x \Rightarrow E_x \approx 0$ from this follows $J_x = 0$ and $n_i = n_e = n = n_0$ (constant and uniform, also charge neutrality is valid). Then Eq. (4) is satisfied trivially.

Defining $T = ct$, we obtain the equations for the y and z components:

$$-\partial_T B_z = (B_x c/4\pi)(\partial_x^2 - \partial_T^2)[\epsilon B_y - (\eta/B_x) B_z], \tag{6}$$

$$\partial_T B_y = (B_x c/4\pi)(\partial_x^2 - \partial_T^2)[\epsilon B_z - (\eta/B_x) B_y]. \tag{7}$$

By assuming B_y, B_z of the form $B_{y,z} = B_{0y,0z} e^{ikx + i(\omega/c)T}$ and $\eta = 0$, and by a substitution into Eqs. (6) and (7), we find the dispersion relation. Because the equations are of fourth order in T, we get two branches of positive frequencies:

$$\frac{\omega_{1,2}^2}{c^2} = \frac{1 + 2\epsilon^2 k^2 c^2 (4\pi)^2 B_x^2 \mp \sqrt{1 + 4\epsilon^2 k^2 c^2 (4\pi)^2 B_x^2}}{\epsilon^2 c^2 (4\pi)^2 B_x^2}. \tag{8}$$

This is the dispersion relation for the case $\omega_{ce} \gg \omega \gg \omega_p^2/\omega_{ce}$, for collisionless cold plasma electron waves. Generally there are three branches of electron waves with positive frequencies for a cold plasma of a low density where $\omega_{pe} \ll \omega_{ce}$. The whistler wave is the slower mode, and the two other modes are the left- and the right-polarized modes.[17] In our model, however, which corresponds to time scales for which the electron inertia is neglected ($\omega \ll \omega_{ce}$) the fastest mode, the right-polarized mode, does not appear. The presence of the two different frequencies here, in comparison to the previous work[16] that had only the whistler branch as a solution, is the reason for the basic differences between the solutions.

Defining $B \equiv B_y + iB_z$, Eqs. (6) and (7) can be rewritten as

$$\partial_T B = (B_x c/4\pi)[-\epsilon i + (\eta/B_x)](\partial_x^2 - \partial_T^2)B. \tag{9}$$

This is the Telegraph equation for complex B with complex coefficients.

The Telegraph equation describes magnetic field propagation that is governed by a superposition of whistler waves and left-polarized waves. When the time scale is long, $\omega \ll \omega_p^2/\omega_{ce}$, the dispersive whistler wave is dominant. When the time scale is shorter, $\omega_{ce} \gg \omega \gg \omega_p^2/\omega_{ce}$, the dominant wave is the left-polarized wave, which becomes nondispersive for $\omega \gg \omega_p^2/\omega_{ce}$.

The Telegraph equation is an hyperbolic equation, where c, the light velocity in vacuum, is the velocity of pulse propagation. As expected, this finite propagation velocity results from the inclusion of the displacement current, and the associated left-polarized wave, in our model. The neglect of the displacement current reduces the Telegraph equation to the parabolic diffusion equation (with a complex coefficient), which describes whistler wave propagation.[16] The Telegraph equation with real coefficients describes propagating waves in a lossy medium. The value of the parameter $\eta\omega$ determines whether the wave is diffusive ($\eta\omega \ll 1$) or a nondispersive light wave ($\eta\omega \gg 1$). In our case, the Telegraph equation with complex coefficients describes dispersive propagating waves where the analogous parameter $\omega_{ce}\omega/\omega_p^2$ determines whether the wave is dispersive ($\omega_{ce}\omega/\omega_p^2 \ll 1$) or a nondispersive light wave ($\omega_{ce}\omega/\omega_p^2 \gg 1$). The quantity $\omega_{ce}/\omega_p^2 = B_x/nec$ is called the Hall resistivity, because of the analogy to η in the Telegraph equation.

Scaling analysis. Equation (9) can be rewritten in a dimensionless form.

Defining $b\equiv B/B_x$, $T'\equiv 4\pi T/\epsilon B_x c$, $x'\equiv 4\pi x/c\epsilon B_x$, the equation becomes

$$\partial_{T'} b = [-i + (\eta/\epsilon B_x)](\partial^2_{x'} - \partial^2_{T'})b. \qquad (10)$$

From this equation it can be seen that we have left only one physical parameter, which is mainly the ratio between the resistivities, $\kappa\equiv\eta/\epsilon B_x$. If now we redefine $T''\equiv\beta T'$, and $x''\equiv\sqrt{\beta}x'$, we obtain the equation

$$\partial_{T''} b = (-i+\kappa)(\partial^2_{x''} - \beta\partial^2_{T''})b. \qquad (11)$$

We see that for $\beta\to 0$, the equation tends to a diffusion, or a Schrödinger equation, depending on the values of κ. This diffusion equation is exactly the equation given in the case when the displacement current is not considered.[16]

In terms of T' and x', we see that for $\beta\to 0$ and $T' = O(1/\beta)\to\infty$ and $x'^2/T'\approx O(\beta^0)$, the equation tends to the equation which does not consider the displacement current. Finally, we see that the conditions for tending to the previous solutions, which do not consider the displacement current, in terms of the unnormalized t and x and orders of β, are

(a) $t/\epsilon B_x \approx O(1/\beta)\to\infty$;
(b) $x^2/c^2\epsilon B_x t\approx O(\beta^0)$.

(Notice that this is the condition for the whistler waves to become dispersive, as mentioned before.)

From Eq. (10) we see that rescaling x', T' together with respect to a parameter β', where $\beta'\to 0$, in a way that both x' and T' have the same first-order dependence on β', then for small x' and T' both of order $\beta'\to 0$ the equation is a wave equation. In terms of x, t, the equation becomes a wave equation when

(c) $t/\epsilon B_x\approx O(\beta')$;
(d) $x/ct\approx O(\beta'^0)$.

We remark that the wave solution should not depend on κ.

Let us emphasize that this is true in general only when it is satisfied that none of the derivatives of the solution is singular and for both β and β' strictly tending to zero. The behavior near the front, therefore, cannot be predicted by these scalings, and it is studied separately in the following section.

In the next sections we find explicitly the solutions, and we will verify the conclusions of these scalings.

III. ANALYTICAL SOLUTION FOR A SEMI-INFINITE SLAB

We consider a plasma slab that is semi-infinite in the x direction. The magnetic field is applied at one of its faces, defined as $x = 0$. The magnetic field is applied at $T = 0$ and kept constant. The boundary conditions are then

$$B(0,x) = 0,\quad B(T,0) = B_0,\quad \partial_T B(0,x) = 0.$$

We define

$$a\equiv\{(B_x c/4\pi)[-i\epsilon + (\eta/B_x)]\}.$$

Then, applying a Laplace transform on Eq. (9), i.e., $f(x,s)\equiv L[B(x,t)]$, we obtain

$$as^2f - asB(0,x) - \partial_T B(0,x) + sf - B(0,x) = a\partial^2_x f. \qquad (12)$$

With the initial conditions Eq. (12) becomes

$$as^2f + sf = a\partial^2_x f. \qquad (13)$$

The solution of this equation, which is bounded at infinity, is of the form:

$$f = c(s)e^{-x\sqrt{s^2+(s/a)}}. \qquad (14)$$

To satisfy the initial condition at $x = 0$, we require that $L^{-1}f(0,s) = B_0$. Using the fact that $L^{-1}(1/s) = 1$, we obtain $c(s) = 1/s$.

In order to now find $B(T,x)$, we have to find the inverse Laplace transform of f. Using the relation

$$\frac{1}{s} = \frac{1}{as[s+(1/a)]} + \frac{as}{as[s+(1/a)]},$$

performing a change of a variable $s' = s + 1/2s$, and employing inverse Laplace transforms,[18] we obtain the solution for B:

$$x<T,\quad B = B_0 e^{T/2a}\Big\{J_0\Big(-\frac{i}{2a}\sqrt{T^2-x^2}\Big) - \frac{i}{2a}\int_x^T\Big[J_0\Big(-\frac{i}{2a}\sqrt{u^2-x^2}\Big)\Big] \times\Big[iJ_0\Big(-\frac{i}{2a}(T-u)\Big) - J_1\Big(-\frac{i}{2a}(T-u)\Big)\Big]du\Big\},\quad x>T,\quad B=0, \qquad (15)$$

where J_0 and J_1 are Bessel functions. This is the general solution with both resistivities, the collisional resistivity and the Hall resistivity.

Applying inverse Laplace transform to Eq. (14), it can be shown that using the conditions (a) and (b) of Sec. II, the solution tends to the known previous solutions as follows: We know that x' should scale as $\sqrt{T'}$ and $T'\to\infty$, that means $\gamma\equiv x'/\sqrt{T'}\approx[O(\beta^0)]$,

$$B(T,x) = L^{-1}(f) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{\exp\{-\gamma\sqrt{T[s^2+(s/a)]}\}}{s}e^{Ts}ds = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{\exp[-\gamma\sqrt{(u^2/T+u/a)}]}{u}e^{u}du.$$

Letting $T\to\infty$, we obtain[18]

$$B(T,x) = L^{-1}(\gamma,tt)(f) = L^{-1}(\gamma,tt=1)\left(\frac{e^{(-\gamma\sqrt{u/a})}}{u}\right) = \mathrm{erfc}\left(\frac{\gamma}{2\sqrt{a}}\right) = \mathrm{erfc}\left(\frac{x}{2\sqrt{Ta}}\right).$$

The last expression is exactly the one given in the previous paper.[16]

In the case of an infinitely fast-rising magnetic field at the plasma boundary, the magnetic field propagates into the plasma initially as a nondispersive wave moving with the light velocity and governed by the displacement current. At these early times, the Hall field and the collisional resistivity do not affect the magnetic field evolution much since the current is initially small. The finite velocity of the front prevails for all times and thus the magnetic field is identically zero for $x > ct$. Later, for times $t > B_x/nec$, the wave becomes dispersive due to the Hall field and the collisional resistivity. For $x \ll ct$, the magnetic field evolution resembles the solution of the diffusion equation, which validates the results of our previous paper.[16] Still, even for these later times, the magnetic field amplitude has small oscillations, rather than being monotonically decreasing in space. These small oscillations diminish for $t \to \infty$. Collisional resistivity damps the magnetic field amplitude but does not change the wavelength of the spatial oscillations. Near the front of the wave $x < ct$, the magnetic field is very different from the solution of the diffusion equation. The wavelengths of the oscillations of the values of the magnetic field components and of its amplitude decrease when approaching the front. These wavelengths near the front decrease also in time.

This behavior is demonstrated in Figs. 1(a)–1(d). The solution is plotted for various times T' as a function of x' in terms of the normalized units of Sec. II. We plot the amplitude of the magnetic field and one of its components (continuous lines). The collisional resistivity is zero ($\kappa = 0$). The solution of the previous paper,[16] where the displacement current is not taken into account, is also plotted (in dotted lines).

The rapid oscillatory behavior near the front, for

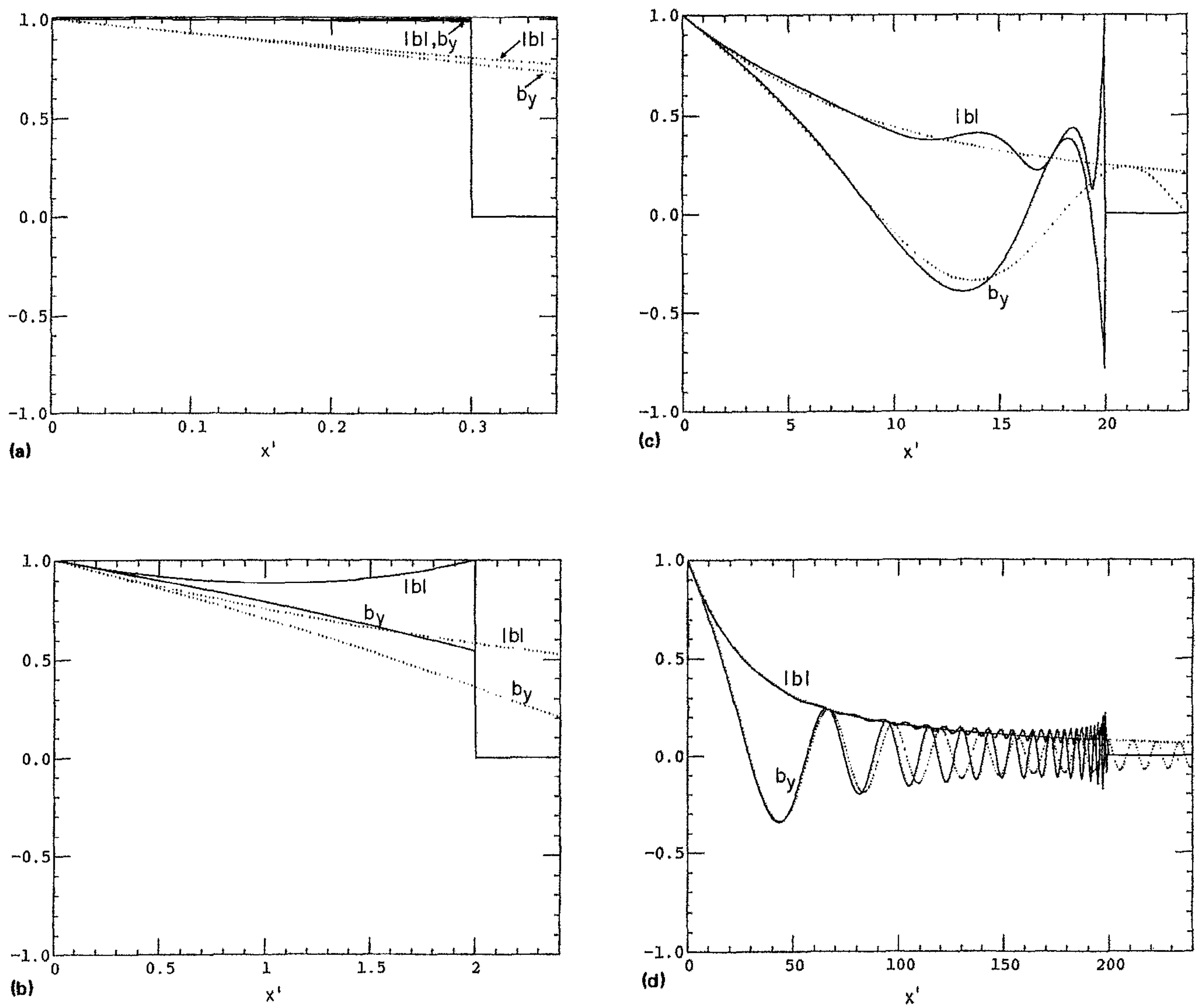

FIG. 1. For the semi-infinite slab, the y component of the parallel magnetic field and its amplitude $|b|$ are plotted versus normalized x' for different normalized T'. The solutions that do not consider displacement current are denoted by dotted lines, while those that do are denoted by continuous lines. Here $\kappa = 0$. The times for (a)–(d) are $T' = 0.3$, 2, 20, and 200, respectively.

$T' \rightarrow \infty$, cannot be predicted by the scaling analysis of Sec. II, because it is characterized by large values of the derivatives. It is rather obtained by an asymptotic estimate of the inverse Laplace transform of f. We perform a steepest descent analysis, which is correct for $T \rightarrow \infty$ and $x \approx T$ and we obtain for B the solution

$$B = C \frac{(1-\alpha)(2\alpha - \alpha^2)^{3/4}}{[1 - \sqrt{1/(2\alpha - \alpha^2)}]\sqrt{T}} \times \exp\left(T \frac{1}{2a} (-1 + \sqrt{2\alpha - \alpha^2}) \right), \quad (16)$$

where $\alpha = 1 - x/T$ and C is a numerical constant.

We can see that because a is imaginary when there is only Hall resistivity, the exponent is the source of the oscillations. The phase is varying as a function of α. Near the front, that is for $\alpha \rightarrow 0$, the slope of the phase with respect to α is proportional to $(1-\alpha)/\sqrt{(2\alpha - \alpha^2)}$ and results in a fast-rising phase as a function of x, of magnitude of order $T \rightarrow \infty$. The last oscillation near the front has a wavelength $\lambda = 4\pi^2(4a^2/T)$. This is the reason for the diminishing wavelength of the oscillations of the components of the magnetic field near the front, as observed in the figures, as well as its diminishing as T' grows. In the case of a purely collisional resistivity, the exponent is real and gives rise to a monotonically decaying $|B|$. For $\alpha \rightarrow 1$ or small x/T, but still large enough so that the method of steepest descent is applicable, the expression for B from the steepest descent calculation reduces to the asymptotic form of the previous expression without the displacement current, i.e., the asymptotic form of $B = B_0 \operatorname{erfc}(x/2\sqrt{ta})$.

In Figs. 2(a)–2(c) our solution is plotted, for a null Hall resistivity and for a normalized η to unity (dotted line), and for $\kappa = 0.085$ (continuous line). This is done for the same times as Figs. 1(a)–1(c). It is shown that the wave equation solution is not modified by the collisional resistivity. It is independent of whether we have Hall resistivity or collisional resistivity, and of their magnitudes. On the other hand, the solution for $T' > 1$ is modified and the propagation into the plasma is drastically diminished by the collisional resistivity. Oscillations are also diminished by the collisional resistivity, depending on the value of κ.

IV. FINITE BOUNDARIES

We suppose now that the slab is finite in the x direction, and we put a conductor at its end $(x = d)$. The boundary conditions are $B(T,0) = B_0$, $B(0,x) = 0$, $\partial_x B(T,d) = 0$, $\partial_T B(0,x) = 0$.

Then the solution for B, in terms of Fourier components, is

$$B(T,x) = B_0 \left[1 - \frac{4}{\pi(2n+1)} \sum_{n=0}^{\infty} \sin\left((2n+1) \frac{\pi}{2d} x \right) \times (A_k e^{i(\omega_2/c)T} + B_k e^{i(\omega_1/c)T}) \right], \quad (17)$$

where $k = (2n+1)\pi/2d$, $\omega_{1,2}$ are the positive frequencies given in Eq. (8) and

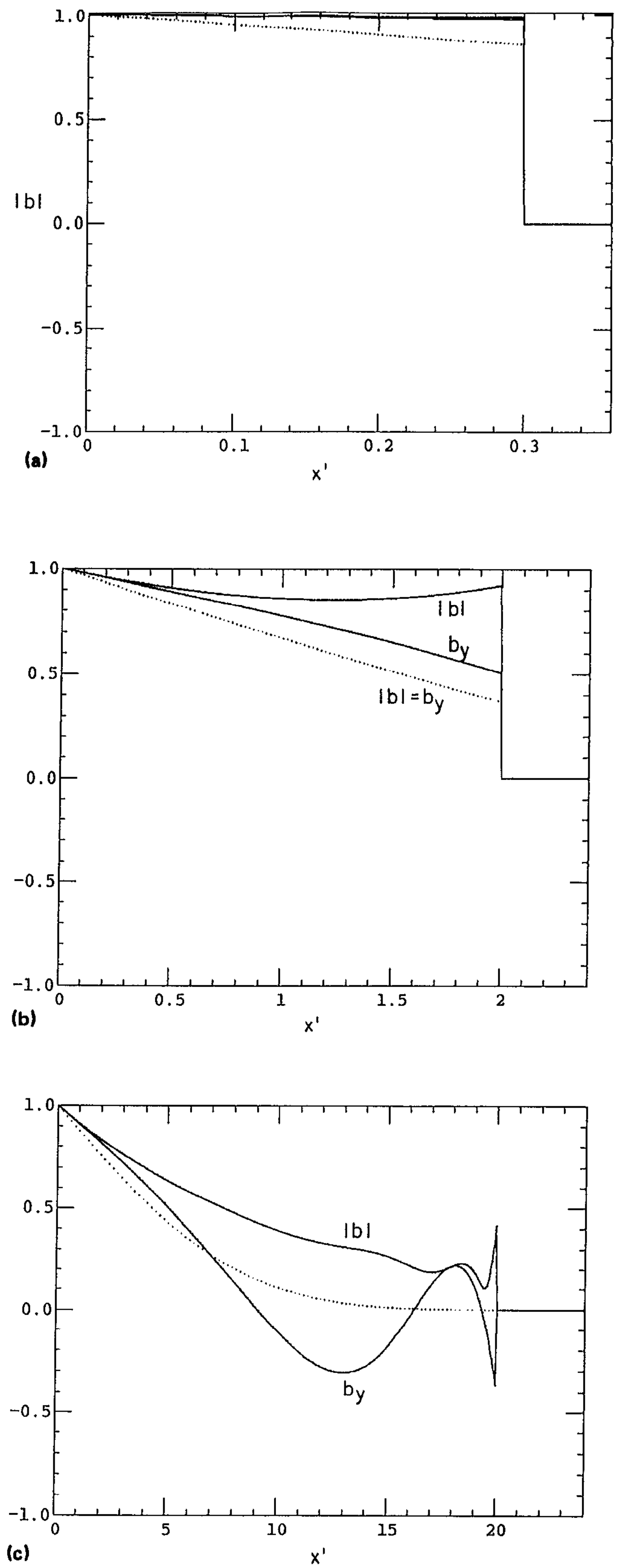

FIG. 2. For the semi-infinite slab the y component of the parallel magnetic field and its amplitude $|b|$ are plotted versus normalized x', for different normalized T' for two cases: $\kappa = 0.08$ (continuous lines), and for no Hall resistivity with collisional resistivity normalized to one (dotted lines). The times in (a)–(c), are $T' = 0.3$, 2, and 20, respectively. For no Hall term, there is only one component of the magnetic field, that is, $|b| = b_y$.

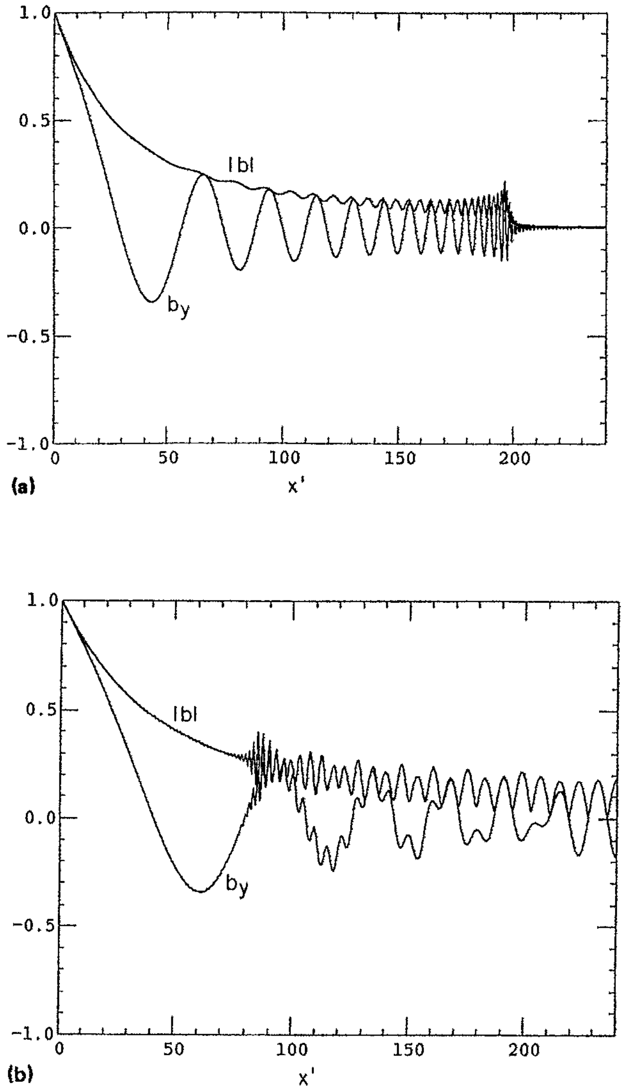

FIG. 3. For the finite slab with normalized width $d' = 240$, the y component of the parallel magnetic field and its amplitude $|b|$ are plotted versus normalized x' for different normalized T'; (a) 200, (b) 300. In (a), the wave has not reached the boundary, and therefore it is identical to the solution given in Fig. 1(d). In (b), the wave front has already reached the boundary, and therefore it is disturbed by the wall.

$$A_k = \omega_1(k)/[\omega_1(k) - \omega_2(k)],$$
$$B_k = \omega_2(k)/[\omega_2(k) - \omega_1(k)].$$

In order to get the limiting solution for the case that does not consider displacement current from Eq. (17), we can either use the conditions of Sec. II, or in this case to rescale a (a is defined in the previous section), so that $T \to \infty$ and $a \to 0$. In that case $\omega_1/\omega_2 \to \infty$ and $B_k = 0$, so that only one frequency, ω_2, appears. This is the solution given in Ref. 16 as we check numerically. This solution has no oscillations. Therefore the oscillations result from the presence of two distinct frequencies, following the equation being second order in T.

In Figs. 3(a) and 3(b) we present a solution for times that the wave has not reached the walls. We see that it is exactly the same solution as for the semi-infinite case (what also serves as a check). This is because our solution is exactly zero for $x > T$ and therefore exactly zero at the walls, otherwise it would be perturbed by the walls. In a later time, when the wave reaches the wall, it is disturbed and the solution differs from the semi-infinite case solution.

V. CONCLUSIONS

We added the displacement current to the model that describes the fast propagation of magnetic field into a one-dimensional plasma slab due to the Hall term. The governing equation was shown to be the Telegraph equation with complex coefficients rather than the diffusion equation with a complex coefficient, as it is when the displacement current is neglected. The propagation of the magnetic field was shown to be governed by two different waves, the left circularly polarized wave and the dispersive whistler wave. For a fast-rising magnetic pulse at the boundary of the plasma, the magnetic field evolution was studied analytically. As expected, we found out that the only prevailing effect for all later times due to displacement current is that the propagation is limited by the light velocity, being identically zero for $x > ct$. This is due to the nature of the governing hyperbolic equation. There are some effects which last for a long time but diminish as time grows, like oscillations of the absolute value of the magnetic field, around the solutions of the diffusion equation, and more importantly: short wavelength oscillations near the front of the pulse. This behavior results from the time varying dominance of the two different waves. At earlier times, the left circularly polarized wave is dominant, and the solution does not depend on the collisional resistivity or on the value of the Hall term. At a later time the dispersive whistler wave is dominant.

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