# Power dissipated during rapid magnetization or demagnetization of plasmas

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The power flow in rapid magnetization or demagnetization of a plasma that conducts current between two cylindrical conductors is analyzed through a consideration of the necessarily two-dimensional physics. This fast Hall-field-dominated evolution of the magnetic field must be accompanied by a large dissipation of power, partly in the bulk of the plasma and partly at the electrodes. Interestingly, precise statements can be made about this partitioning of the power.

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## I. INTRODUCTION

Fast-magnetic-field penetration is a crucial process in several pulsed-power devices, such as the magnetically insulated ion diode [1,2] and the plasma opening switch (POS) [3–13]. Often, the time scales are so short that the ions are essentially immobile and the so-called electron-magnetohydrodynamics (EMHD) [14] equations most aptly describe the plasma. There are a number of peculiar effects in this short-time-scale regime, even in the simplest case of a vacuum region adjacent to a plasma. If the plasma is unmagnetized but a magnetic field permeates the vacuum, and if the electrons that carry the current decelerate due to a density gradient parallel to the interface, a shocklike penetration of the magnetic field into the plasma ensues [15,5]. A similar shock penetration occurs if the deceleration is a result of a radial motion of the electrons in a cylindrical plasma and the magnetic field is azimuthal [6]. On the other hand, if the electrons accelerate due to a density gradient or cylindrical geometry, the magnetic field does not penetrate into the plasma. Moreover, in this case if the plasma initially is permeated by the field, then that field can be spontaneously expelled [6]. These physical phenomena can be found from one-dimensional (1D) penetration equations in the direction normal to the interface.

The puzzle in such 1D calculations is that the Poynting power flow is in the formally ignorable direction parallel to the interface. Actually, this is the direction in which the electrodes supporting the current must be. While much of the 1D treatment gives physical results, the supposition of power emanating from the electrodes either during magnetic-field penetration or during magnetic-field expulsion is a troubling picture. Moreover, the 1D treatment is unsatisfying in that it cannot answer the important question of exactly where between these electrodes the power is dissipated.

The resolution of the power flow is made possible here by a careful two-dimensional (2D) analysis of the magnetic-field evolution in a plasma that conducts current between two clindrical conductors. It is found that the magnetic-field energy penetrates into the plasma near the cathode and propagates radially into the plasma, and that the penetration is accompanied by a large energy dissipation both at the electrodes and in the bulk of the plasma. The dissipation is caused by the resistivity. However, the rate of energy dissipation and the partitioning of the power are determined by the nondissipative Hall field and by the geometry.

In Sec. II the model is described and the governing 2D equation is derived. Numerical solutions of the equation are then presented, followed in Sec. III by a detailed analysis of the sought-after 2D energy flow. Finally, the partitioning of the power is described.

## II. MODEL

The main assumption in the model used here, as in the EMHD model [14], is that the process is so fast that the ions are immobile and the current as a result is determined by the electron motion only. This assumption is justified if the characteristic scale length, here the radius, is smaller than the ion skin depth $c/\omega_{pi}$ ($\omega_{pi}$ is the ion plasma frequency). In this case the velocity of the magnetic-field evolution is larger than the characteristic velocity of mass flow, the Alfven velocity. On the other hand, it is assumed that the process is slow enough to allow the neglect of the electron inertia. It is further assumed that the pressure gradient and the stress tensor are small. The governing equation is not changed even if the pressure gradient is large, as long as the plasma is collisional enough so that the pressure is isotropic and the pressure and density gradients are parallel or the density is uniform. In the POS energetic electrons emitted from the cathode could provide significant anisotropic pressure gradient terms, if the plasma collisionality is low. Such a case is not treated here. All of the above assumptions are expressed in the following form of Ohm's law:

$$\mathbf{E}=\eta\mathbf{j}+\frac{\mathbf{j}\times\mathbf{B}}{enc} . \tag{1}$$

Here $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mathbf{j}$ is the current, $\eta$ is the collisional resistivity, $n$ is the plasma density (the ions are singly ionized), $-e$ is the electron charge, and $c$ is the velocity of light in vacuum. Quasineutrality is assumed to hold. The case of interest here is that the resistivity is small and the main contribution to the electric field is that of the Hall field, the second term on the right-hand side of (1). The other governing equations are Faraday's law and Ampere's law

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}\ ,\quad \nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{j}\ . \tag{2}$$

In Ampere's law the displacement current was neglected. The neglect of the displacement current implies that the flux of electric-field energy into the plasma in smaller than the flux of magnetic-field energy. The neglect of the electric-field energy relative to the magnetic-field energy is justified for not too high current densities. The validity of the assumptions mentioned above and the consistency of the results are discussed towards the end of the paper.

In a previous paper [6] it was explained how the magnetic field can increase or decrease in the plasma, when the resistivity is very small, while satisfying the frozen-in law. However, usually such an evolution results in the formation of narrow current channels, in which the magnetic flux changes despite the small resistivity. A detailed discussion of this important issue will be given elsewhere.

With the focus on the effects of cylindrical geometry, a system of azimuthal symmetry, $\partial/\partial\theta=0$, is analyzed, where $r$, $\theta$, and $z$ are the usual cylindrical coordinates. Equations (1) and (2) then combine to [6]

$$\frac{\partial B_\theta}{\partial t}=\frac{c^2\eta}{4\pi}\left[\frac{\partial^2 B_\theta}{\partial z^2}+\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)\right]\right]+\frac{c}{2\pi ne}\frac{B_\theta}{r}\frac{\partial B_\theta}{\partial z}\ , \tag{3}$$

where the density is assumed constant. It is now assumed that a plasma of length $a$ is located between an outer cylindrical conductor of radius $r_0$ and an inner conductor of radius $r_i$. The plasma conducts current so that at the generator side $B_\theta(r,z=0,t)=B_i(t)r_i/r$, while at the load side $B_\theta(r,z=a,t)=0$. By specifying $B_i(t)$, the total current in the plasma $I(t)=(c/2)r_iB_i(t)$ is specified. It is assumed that the plasma impedance is small relative to other impedances in the circuit and that the the total current is not affected by the magnetic-field evolution inside the plasma. For simplicity a fast rising current is chosen so that $B_i(t\geq 0)=B_0$, while $B_i(t<0)=0$. The electric field parallel to the conductors is zero,

$$E_z=\frac{c\eta}{4\pi r}\frac{\partial}{\partial r}(rB_\theta)-\frac{B_\theta}{4\pi ne}\frac{\partial B_\theta}{\partial z}=0,\quad r=r_i,r_0\ . \tag{4}$$

The assumption that Ohm's law holds at the plasma-conductor boundary excludes from the model the sheath between the plasma and the conductor. Instead of employing the boundary condition (4), the results of the model for the plasma could be matched to the results of a model that describes the sheath [12,13]. If the sheath physics is important in the POS, the present model could not be applied to the description of the plasma-electrode boundary in that device. However, the evolution of the magnetic field inside the plasma would not differ much. The boundary condition (4) does imply the important feature that the electromagnetic energy flux normal to the electrodes is zero. Therefore, by solving the 2D problem with the boundary condition (4) I am addressing the key question of how the magnetic field penetrates into the plasma when no energy is allowed to flow through the radial boundaries.

Two cases are studied. In the first case the cathode is at the inner conductor, i.e., $r_A>r_K$ and $B_0<0$. In the second case the polarity is reversed and the cathode is at the outer conductor, i.e., $r_A<r_K$ and $B_0>0$. In both cases $\eta nec/|B_0|$ is 0.01.

Figure 1 shows the shocklike penetration of the magnetic field into an initially unmagnetized plasma in the first case. The evolution of the magnetic field in the bulk of the plasma seems to be well described by the shock solution of the 1D Burgers equation [5,6]. The dependence of the shock velocity on the radius (as $1/r^2$) is demonstrated through the shape of the current channel.

Figure 2 shows the evolution of the magnetic field in the second case. Here it is assumed that initially the plasma is magnetized and the current is radial and axially uniform. Such a current distribution can be reached if the plasma resistivity is high. At a certain time, after the magnetic field has penetrated, the plasma resistivity abruptly decreases and the Hall field becomes dominant.

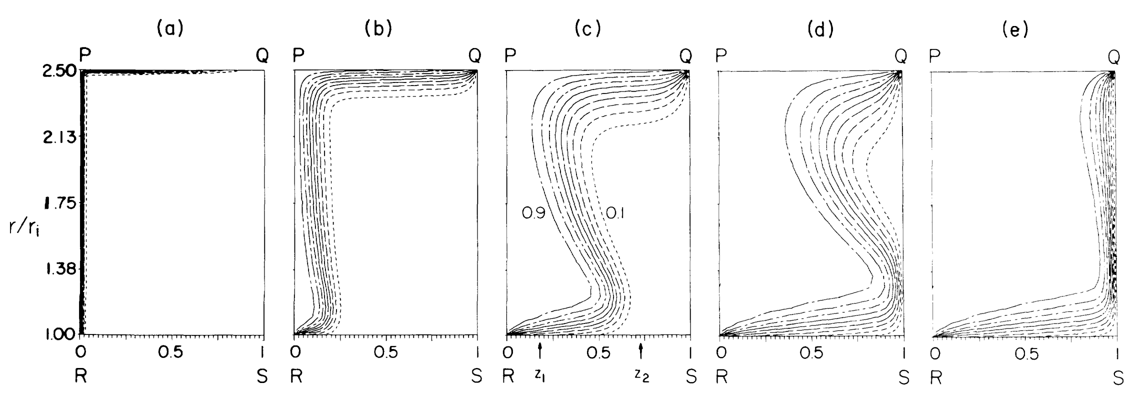


FIG. 1. The contour levels of the normalized magnetic field $-rB_\theta/(r_i|B_0|)$ for various normalized times $\tau\equiv c|B_0|t/(2\pi ner_i^2)$ in the case of fast penetration. The coordinates are normalized with respect to $r_i$. (a) $\tau=0.01$, (b) $\tau=0.5$, (c) $\tau=2$, (d) $\tau=4$, (e) $\tau=8$.

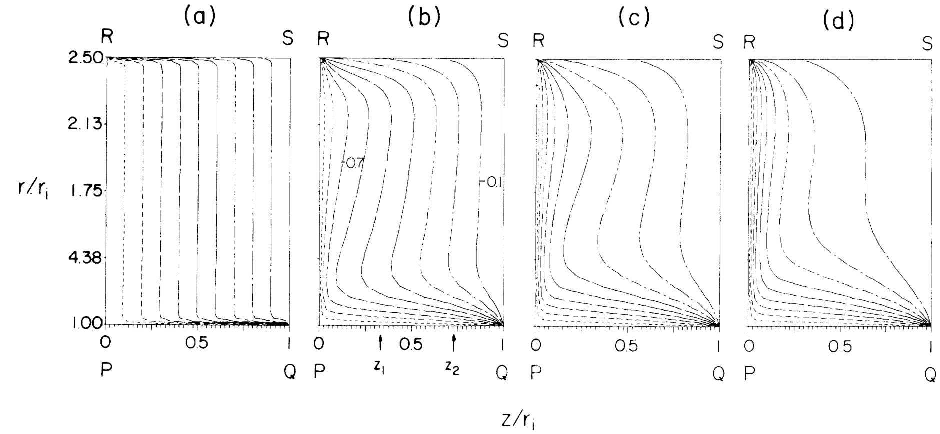


FIG. 2. The contour levels of the normalized magnetic field $-rB_\theta/(r_i|B_0|)$ for various normalized times in the case of expulsion. The time and coordinates are normalized as in Fig. 1. (a) $\tau=0.01$, (b) $\tau=1$, (c) $\tau=3$, (d) $\tau=9$.

Figure 2 shows the calculated magnetic-field evolution from that time. The magnetic field is seen to be expelled from the plasma.

In both cases the magnetic-field evolution is greatly modified near the electrodes. The magnetic-field energy flows into the plasma near the cathode, then axially along the cathode, radially into the plasma, and again axially along the anode.

## III. ENERGY FLOW

I now discuss the 2D energy flow in the plasma including the effect of the boundaries. The analysis of the magnetic field evolution is begun after the transient time in which the magnetic field penetrates the plasma along the anode. At the conclusion of this fast penetration the magnetic field all along the anode is approximately $B_0 r_i/r_A$, except near $Q$ (see Figs. 1 and 2) where its intensity drops to zero [16]. The magnetic field at the cathode, on the other hand, is approximately zero, except at the point $R$, where its intensity becomes $B_0 r_i/r_K$.

In order to analyze the energy flow, the potential drop $V(z,t)\equiv -\int_{r_i}^{r_0} dr\, E_r(r,z,t)$ and the axial flux of magnetic-field energy $P(z,t)\equiv(c/2)\int_{r_i}^{r_0} dr\, r\, E_r B_\theta$ are calculated at various axial locations. The potential drop and the energy flux depend on the magnetic-field distribution in the plasma. Different distributions of the magnetic field at different axial locations yield different values of $V$ and $P$ at these locations. Let us look at the difference between the axial energy flow in the plasma on the left of the radial current channel [$0<z\le z_1$ in Fig. 1(c)] and that on the right [$z_2\le z\le a$ in Fig. 1(c)]. The energy flows from the generator side towards the load in both axial positions. However, in the domain $z<z_1$ the current flows along the inner conductor, while for $z_2<z$ it flows along the outer conductor. The potential drop and the energy flux are approximately

$$V(0<z<z_1)=\frac{B_0^2}{8\pi ne}\,,\qquad V(z_2<z<a)=\left[\frac{B_0^2}{8\pi ne}\right]\left[\frac{r_i}{r_0}\right]^2; \tag{5a}$$

$$P(0<z<z_1)=\frac{cr_iB_0^3}{24\pi ne}\,,\qquad P(z_2<z<a)=\left[\frac{cr_iB_0^3}{24\pi ne}\right]\left[\frac{r_i}{r_0}\right]^2 . \tag{5b}$$

Since $V(z_1)$ is larger than $V(z_2)$ and $P(z_1)$ is larger than $P(z_2)$, the magnetic-field flux and energy between $z_1$ and $z_2$ grow in time. When the cathode is at the outer conductor, the energy flows from the vacuum into the plasma as well. However, the total axial energy flow at $z_1$ [see Fig. 2(b)], which is partially conducted near the outer conductor, is smaller than the total axial energy flow at $z_2$. Therefore the magnetic-field energy between $z_1$ and $z_2$ decreases in time. This is the reason for the remarkably different magnetic-field evolution in the cases of opposite polarities. This is also the reason why the evolution of the magnetic field in the cylindrical case occurs on a fast time scale and not on the slow resistive time scale. I emphasize, however, that in both penetration and expulsion cases the energy flows from the vacuum into the plasma.

In the calculation of the energy flux into the plasma $P(z=0)$, I cannot assume that the Hall field is dominant along the plasma boundary, since at $R$ its radial component is zero. I do exploit, however, the fact that $rB_\theta$ is constant at the plasma boundary, and that the potential drop $V$ in the first case is $z$ independent (since to the left of the radial current channel in Fig. 1 the magnetic field is approximately constant in time). The energy flux into the plasma is therefore found to be

$$P_T \equiv P(z=0) = \frac{cr_iB_0}{2}V = \frac{cr_i|B_0|^3}{16\pi ne} . \quad (6)$$

Comparing expressions (5b) and (6), it is found that the energy flux inside the plasma $P(0<z<a)$ equals $\frac{2}{3}$ of the energy flux into the plasma $P_T$. Therefore $\frac{1}{3}$ of the incoming energy is dissipated at $R$ on the cathode. The rest of the magnetic-field energy is convected by the electron current along the cathode and then radially towards the anode. The net energy flux into the plasma between radii $r_1$ and $r_2$ is

$$\frac{c|B_0|^3r_i^3}{24\pi ne}\left[\frac{1}{r_1^2}-\frac{1}{r_2^2}\right] .$$

The increase of the magnetic-field energy at a radius $r$ is $(B_0^2r_i^2/4r)v_p$, where $v_p \simeq cr_iB_0/(4\pi ner^2)$ is the shock-wave velocity [5,6]. Therefore $\frac{3}{4}$ of the energy convected along the radial current channel go to building the magnetic field in the plasma while the remaining $\frac{1}{4}$ is dissipated. The energy that flows at $z>z_2$ is dissipated near the anode. To summarize, during the magnetic-field penetration the dissipated energies at the cathode $Q_c$, in the plasma $Q_p$, and at the anode $Q_A$ are

$$Q_c=\tfrac{1}{3}P_T , \quad Q_P=\tfrac{1}{6}P_T(1-r_i^2/r_0^2) , \quad Q_A=\tfrac{2}{3}P_T(r_i/r_0)^2 , \quad (7)$$

while the energy that goes into building the magnetic field in the plasma $P_B$ is $P_B=(P_T/2)(1-r_i^2/r_0^2)$. In the slab limit $(r_i/r_0 \to 1)$ the energy is mostly dissipated at the two corners $R$ and $Q$, and almost none of the energy builds the magnetic field in the bulk of the plasma or is dissipated there. When the effect of cylindrical geometry is important $(r_i/r_0 \to 0)$) the ratio $P_B/P_T$ becomes $\frac{1}{2}$, i.e., half of the energy goes to building the magnetic-field energy in the plasma, and half is dissipated. A substantial amount $P_T/6$ is dissipated in the bulk of the plasma. When the radial current sheet finally reaches the plasma-vacuum boundary, a steady state is reached in which the energy dissipated in the plasma (at the vacuum boundary) is even larger,

$$Q_P=\tfrac{2}{3}P_T(1-r_i^2/r_0^2) . \quad (8)$$

Thus a large amount of energy is dissipated inside the plasma. It should be emphasized that this steady state is not an equilibrium and will be followed by plasma pushing by the magnetic field.

By similar analysis one can show that, when the cathode is in the outer conductor, the rate of energy dissipation at the steady state [Fig. 2(d)] is

$$Q_c=\tfrac{1}{3}P_T(r_i/r_0)^2 , \quad Q_p=\tfrac{1}{3}P_T(1-r_i^2/r_0^2) , \quad Q_A=\tfrac{2}{3}P_T . \quad (9)$$

The dissipation inside the plasma at the rate $Q_p$ occurs in the current channel at the vacuum-plasma boundary, on the generator side.

Finally, $R \equiv V/I$, the plasma resistance at steady state, is calculated to be

$$R=B_0/(4\pi necr_i) , \quad (10)$$

which is the accurate form of the approximate result deduced in Ref. [5] through an estimate of the radial energy flow. The resistance of a plasma of density $10^{13}$ cm$^{-3}$, inner radius 3 cm, and magnetic field 10 kG is 2 Ω. One could regard the hollow cylindrical plasma as composed of many thin hollow cylinders of various radii connected in series, the resistance of each being $B/(4\pi necr)$, where $B$ is the magnetic-field intensity at the thin plasma cylinder of radius $r$. The resistance of the whole plasma is found to be the resistance of the cylinder of largest resistance. This peculiar feature of the Hall resistance is analogous to the phenomenon explored in regard to the connection in series of two Hall conductors of different charge-carrier densities [17].

The energy dissipation is caused by the resistivity and the processes of magnetic-field penetration and energy dissipation require that the resistivity be nonzero. However, the rate of these processes is determined by the nondissipative Hall field and not by the resistivity. If the resistivity is smaller, the current density becomes higher so that the rate of energy dissipation in the shock layer and near the electrodes is unchanged. Even though the present model allows the collisionality to be arbitrarily small, the assumptions of the model cease to be valid for small collisionality and the electron inertia has to be included. The collision time $\tau_{col}$ ( $\equiv m/ne^2\eta$, $m$ is the electron mass) has to be smaller than the time an electron stays inside the shock $\tau_{tr}$. The time $\tau_{tr}$ is the ratio of the shock layer thickness and the shock velocity, which are approximately $2mcr/eB_\theta\tau_{col}$ and $cB_\theta/4\pi ner$. Therefore $\tau_{tr}$ is $8\pi mnr^2/B_\theta^2\tau_{col}$. The condition on the collisionality becomes

$$\tau_{col}^2 < \frac{8\pi mnr^2}{B_\theta^2} . \quad (11)$$

In this analysis the displacement current has been neglected, the electric field was assumed smaller than the magnetic field, and quasineutrality was assumed to hold $(\nabla \cdot \mathbf{E} \ll 4\pi ne)$. The condition that these assumptions are valid in the shock layer is

$$\tau_{col} < \frac{8\pi mnrc}{B_\theta^2} . \quad (12)$$

For a plasma of density $10^{13}$ cm$^{-3}$, a radius of 5 cm, and magnetic field of 10 kG, $\tau_{col}$ should be less than 1 ns in order to satisfy inequalities (11) and (12). It is clear, however, that both conditions (11) and (12) are not sufficient near the singular points at the electrodes where the current density and thus the electron velocity become infinite. A more detailed physical model is needed there.

The main prediction of the Hall field model, the penetration of the magnetic field into the plasma when the cathode is at the inner conductor, could provide the explanation to the Naval Research Laboratory (NRL) measurements [11] in the plasma opening switch experiment, which indicate that such a penetration does indeed occur. The further predictions of no penetration in the opposite polarity case and of large energy dissipation in the bulk of the plasma call for a clear experimental test. The fast penetration near the anode and the resulting large potential drop at the plasma-vacuum boundary on the load side could cause early power delivery to the load, which does not seem to be consistent with experiments. Possible processes which could limit the formation of this potential drop were discussed in Refs. [5,16]. A natural continuation of this research would be to modify the model at the electrodes in order to examine plasma pushing from the cathode and electron magnetization, processes which complete with the magnetic-field penetration and which could dominate the switch operation.

In conclusion, the distinct features of the Hall-field-dominated magnetic-field evolution in a hollow cylindrical plasma were demonstrated in the presence of perfect conductors at the radial boundaries. The energy flow during the shocklike fast penetration in one case and during the expulsion of the magnetic field in the second case was determined and a large dissipation was found in the bulk of the plasma as well as near the conductors.

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