Penetration and expulsion of magnetic fields in plasmas due to the Hall field

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The axial penetration of an azimuthal magnetic field into a short-duration hollow cylindrical plasma is studied. When the process is so fast that the ion motion is small and the plasma dissipative resistivity, electron inertia, and pressure are small, the evolution of the magnetic field is governed by the Hall field. When the radial current flows inward, the magnetic field penetrates in the form of a Hall-induced shock wave with a narrow current channel. When outward, the magnetic field does not penetrate the plasma. Moreover, in the latter case the magnetic field is expelled from an initially magnetized plasma. The increase and decrease of the magnetic field intensity in the cylindrical plasma are shown to result naturally from the frozen-in law.

I. INTRODUCTION

The effect of the dissipationless Hall field on the behavior of plasmas is often considered (see, for example, Refs. 1–9). We focus on the role of the Hall field in magnetic field penetration into short-duration plasmas. When a process is so fast that the ion motion is small and the plasma dissipative resistivity, electron inertia, and pressure are small, the evolution of the magnetic field is in fact governed by the Hall field. In a parallel paper[10] we examine the magnetic field penetration into an already magnetized short-duration plasma. In Ref.10 the Hall electric field enables the magnetic field to penetrate as a whistler wave along a background magnetic field. In the present paper we study the axial penetration of an azimuthal magnetic field into an initially unmagnetized short-duration hollow cylindrical plasma. Contrary to the mechanism of penetration described in Ref. 10, the mechanism described here does not rely upon the presence of a magnetic field component in the direction of penetration. We show that for an inward radial current the Hall field significantly enhances the magnetic field penetration. The magnetic field then penetrates in the form of a Hall-induced shock wave. However, in the case of an outward radial current, the magnetic field does not penetrate the plasma. Moreover, if in this case the plasma is initially magnetized, the magnetic field decreases the volume it occupies and is expelled from the plasma.

In Sec. II we explain the physical origin of the mechanism of magnetic field evolution and show that this evolution results naturally from the frozen-in law. We then derive the governing equation. In Sec. III we describe the magnetic field penetration in the form of a Hall-induced shock wave. The expulsion of the magnetic field from an initially magnetized plasma is described in Sec. IV. We conclude in Sec. V with a discussion of the limitations of our model.

II. THE MODEL

We consider plasmas of relatively low pressure and under strong magnetic fields for time scales longer than the electron cyclotron period. Ohm's law, which results from the electron momentum equation, becomes

$$\mathbf{E} = \eta \mathbf{j} - (\mathbf{v}_e \times B)/c. \tag{1}$$

Here $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mathbf{j}$ is the current, $\mathbf{v}_e$ is the electron flow velocity, c is the velocity of light in vacuum, and η is the collisional resistivity. In the limit of zero resistivity, the electrons move with their $\mathbf{E}\times\mathbf{B}$ velocity only. If we combine Ohm's law in this limit with Faraday's law we obtain

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_e \times \mathbf{B}). \tag{2}$$

Equation (2) expresses the familiar frozen-in law; the magnetic field is frozen into the electron fluid.

We first discuss the mechanism that governs the evolution of a magnetic field which satisfies the frozen-in law. Such an evolution was recently addressed by Kulsrud *et al.*,[11] who analyze an axially symmetric electron flow in cylindrical geometry, where the magnetic field has an azimuthal component only. Kulsrud *et al.* show that in such a flow nr/B is constant along an electron trajectory (n is the electron density); they then make the assumption [assumption (4) in Ref. 11] that the plasma is in a quasisteady state, so that the electron current lines are coincident with the electron trajectories. From this assumption follows the conclusion that along the electron current lines nr/B should be constant. Since this is usually not so, Kulsrud *et al.* conclude that a pure $\mathbf{E}\times\mathbf{B}$ motion of the electrons is not possible. This conclusion is in fact correct only with regard to steady-state current distributions. Pure $\mathbf{E}\times\mathbf{B}$ motion is possible and results in time-dependent current distribution where the current lines are not coincident with the electron trajectories. When nr/B is not constant along the current lines, the magnetic field and current lines evolve in time, so that nr/B remains constant along the electron trajectories. If the density is uniform and the electrons move radially from a small radius to a large radius the magnetic field grows in time, so that nr/B is constant along the electron trajectories. The magnetic field then penetrates into the plasma. If, however, the electrons move radially from a large radius to a small radius the magnetic field does not grow, but rather decreases in time. Thus the nonuniformity of nr/B is the source of the magnetic field evolution and results in either an increase or decrease of the magnetic field intensity in the cylindrical plasma. We emphasize that the magnetic field intensity may increase from a nonzero value to a larger value while satisfying the frozen-in law. However, in order for the magnetic field intensity to increase from a value of zero there must be some resistivity.

In rectangular geometry n/B has to be constant along electron trajectories. By an argument similar to that above, if electrons move from a low density region to a high density region, the magnetic field in the plasma grows in time, while if the electron motion is reversed, the magnetic field reduces in time.

We assumed above that the time scale is longer than the electron cyclotron period and therefore neglected the electron inertia. We now restrict ourselves to magnetic field evolution in short-duration plasmas, where the characteristic time is smaller than the ion cyclotron period and we assume, therefore, that the ions are immobile. These two assumptions correspond to the assumption that the ion mass is infinite, while the electron mass is zero. Equation (1) is then approximated as

$$\mathbf{E} = \eta \mathbf{j} + (\mathbf{j}\times\mathbf{B})/enc. \tag{3}$$

Here $\mathbf{j}$ is the current and e is the electron charge. At the limit of immobile ions the Hall field, which is the second term on the right-hand side of Eq. (3), results from the electron motion only. As often done for plasmas of high enough density, we neglect the displacement current in Ampère's law:

$$\nabla\times\mathbf{B} = (4\pi/c)\mathbf{j}. \tag{4}$$

Since the current is divergence-free the charge density is constant in time and since the ions are immobile the electron density is constant in time. Equations (3) and (4), combined with Faraday's law, become

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{c^2\eta}{4\pi}\nabla^2\mathbf{B} - \frac{c^2}{4\pi}\nabla\times\left[\left(\frac{1}{nec}\nabla\times\mathbf{B}\right)\times\mathbf{B}\right]. \tag{5}$$

Equation (4) governs the evolution of the magnetic field in short-duration plasmas in the presence of electron motion only. The first term on the right-hand side of Eq. (4) is the source of collisional diffusion; the second term results from the Hall field. In the present paper we examine the effect of the Hall field in cylindrical geometry. The case in which the Hall field enables the magnetic field to penetrate as a whistler wave is studied in Ref. 10. The magnetic field evolution, when governed by the electron dynamics, has been studied extensively in the Soviet literature.[9]

Assume a hollow cylindrical plasma that fills the gap between two concentric cylindrical conductors and closes the circuit for a current which flows in one conductor and returns in the other conductor. The system has cylindrical symmetry ($\partial/\partial\theta = 0$) and the magnetic field has only a θ component. The governing equation (5) becomes

$$\frac{\partial B_\theta}{\partial t} = \frac{c^2\eta}{4\pi}\left[\frac{\partial^2 B_\theta}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}\right)\right] + \frac{c}{2\pi ne}\frac{B_\theta}{r}\frac{\partial B_\theta}{\partial z}. \tag{6}$$

For simplicity the constant-in-time density is assumed uniform in space. Equation (6) is now written as

$$\frac{\partial \tilde{b}}{\partial t} + \frac{c\tilde{b}}{2\pi ner^2}\frac{\partial \tilde{b}}{\partial z} = \frac{c^2\eta}{4\pi}\frac{\partial^2\tilde{b}}{\partial z^2}, \quad \tilde{b} \equiv -rB_\theta. \tag{7}$$

Since we are interested in the case in which the collisionality is small, we have neglected the collisional terms in the radial direction. We have retained only the collisional term in the axial direction, which is the term that smooths the singularity at the possible shock front. The collisional terms that include radial derivatives are important near the radial boundaries. In a future analysis we will present a full 2-D solution of Eq. (6).

Assume now that a finite hollow cylindrical plasma is located at $-a \leqslant z \leqslant 0$ and $r_1 \leqslant r \leqslant r_2$. At $t \geqslant 0$ a constant-in-time current flows in the conductors and generates at $z = -a$ a constant-in-time magnetic field $\tilde{b} = r_1 B_0$, while at $z = 0$ the magnetic field is $\tilde{b} = 0$. Equation (7) becomes

$$\frac{\partial b}{\partial \tau} + b\frac{\partial b}{\partial \xi} = \nu\frac{\partial^2 b}{\partial \xi^2}, \tag{8}$$

where $\tau \equiv t/\tau_t$, $\xi \equiv z/a$, and $b \equiv \tilde{b}/(r_1|B_0|)$. The transit time τ_t and the normalized resistivity ν are

$$\tau_t \equiv 2\pi ner^2 a/c|B_0|r_1, \quad \nu \equiv \eta cner^2/2|B_0|ar_1. \tag{9}$$

Note that since in Eq. (8) there is no derivative with respect to r, we treat r as a parameter. The normalized resistivity ν measures the ratio of the collisional resistivity η to the effective Hall "resistivity" $(2|B_0|ar_1)/(cner^2)$. The characteristic velocity of penetration is $c|B_0|r_1/(4\pi ner^2)$. As long as this velocity is large compared to the Alfvén velocity $|B_0|r_1/(4\pi Mn)^{1/2}r$ (M is the ion mass), there is not much plasma pushing and the magnetic field penetration is the dominant process. In this case the plasma density remains constant. The criterion for the validity of our model is, therefore, $c/\omega_{pi} \gg r$ (ω_{pi} is the ion plasma frequency).

Equation (8) is the Burgers equation.[12] In a similar analysis of magnetic field penetration into plasmas[7] the Burgers equation is derived in relation to penetration perpendicular to a density gradient in a nonuniform plasma; the analogous penetration in cylindrical geometry is mentioned as well. Here we will explicitly solve the initial-value problem in finite hollow cylindrical plasma and discuss the expansion wave that describes magnetic field expulsion from the plasma.

III. FAST MAGNETIC FIELD PENETRATION

We solve Eq. (8) with the initial magnetic field $b(r,\xi,\tau=0) = b_0(r,\xi)$ and the boundary conditions $b(r,-1,\tau) = \pm 1$, $b(r,0,\tau) = 0$. The first set of conditions (the plus sign) corresponds to the case in which the cathode is in the inner conductor, while the second set (the minus sign) corresponds to the cathode in the outer conductor. We show that these two cases exhibit completely different behaviors of the magnetic field.

We solve Eq. (8) by transforming it into the linear heat equation $\partial\psi/\partial\tau = \nu(\partial^2\psi/\partial\xi^2)$ for $\psi(\xi,\tau)$ through[12]

$$b = -2\nu\frac{\partial\psi/\partial\xi}{\psi}. \tag{10}$$

The corresponding boundary conditions are

$$\mp \frac{1}{2\nu}\psi(-1,\tau) = \frac{\partial\psi}{\partial\xi}(-1,\tau), \quad \frac{\partial\psi}{\partial\xi}(0,\tau) = 0, \tag{11}$$

and the initial condition is

$$\psi(\xi,0) = \exp\left(-\int_0^\xi d\xi' \frac{b_0(\xi')}{2\nu}\right). \tag{12}$$

The magnetic field is therefore

$$b(\xi,\tau) = 2\nu \frac{\Sigma_{n=1}^\infty \alpha_n k_n \sin(k_n\xi)\exp(-\nu k_n^2\tau)}{\Sigma_{n=1}^\infty \alpha_n \cos(k_n\xi)\exp(-\nu k_n^2\tau)}, \tag{13}$$

where k_n are the roots of the dispersion relation

$$\pm 1/2\nu = k_n \tan k_n. \tag{14}$$

Let us first examine the case in which the cathode is in the inner conductor [the sign in Eq. (14) is minus]. The dispersion relation has an infinite number of real eigenvalues k_n, one in each interval $[\pi(n-\frac{1}{2}),\pi n]$. There is also a pair of complex conjugate purely imaginary eigenvalues. The solution of the heat equation is therefore unbounded, but the corresponding solution of the Burgers equation is perfectly physical. We assume that the plasma is initially unmagnetized $b_0(r,\xi) = 0$, $\psi(\xi,0) = 1$ for $-1 < \xi \leqslant 0$. The coefficients then are $\alpha_n = 4\sin k_n/(2k_n + \sin 2k_n)$. The asymptotic solution of the Burgers equation is determined by the growing mode of the heat equation and has the form of a steady shock

$$b(r,\xi,\tau = \infty) = -2\nu|k_0|\tanh(|k_0|\xi). \tag{15}$$

At the limit of low collisionality ($\nu \ll 1$) the magnetic field almost fills the plasma except for a narrow layer of a thickness proportional to the resistivity. The current is concentrated in this layer and the magnetic field drops to zero across it.

The real eigenvalues k_n ($n = 1,2,...$) are located in the intervals $[(n-\frac{1}{2})\pi, n\pi]$. The smaller k_n's, for which $2\nu k_n \ll 1$, are located near $(n-\frac{1}{2})\pi$ and are approximately $k_n = (n-\frac{1}{2})\pi(1+2\nu)$. The larger k_n's, for which $2\nu k_n \gg 1$, are located near $n\pi$ and are approximately $k_n = n\pi(1 - 1/2\nu n^2\pi^2)$. The imaginary eigenvalues are $k_0 = \pm(i/2\nu)(1 + 2e^{-1/\nu})$. The asymptotic solution becomes

$$\begin{aligned} b(\xi,\tau = \infty) = &-(1 + 2e^{-1/\nu}) \\ &\times\{[1 - \exp(\xi/\nu)]/[1 + \exp(\xi/\nu)]\} \\ &+ \theta[(e^{-1/\nu})^2]. \end{aligned} \tag{16}$$

The magnetic field deviates from -1 only in the narrow boundary layer, where $|\xi| = \theta(\nu)$.

In the second case the cathode is in the outer conductor and $b(r,-1,\tau) = -1$. The dispersion relation [Eq. (11) with the plus sign] has real eigenvalues only. In each interval $[(n-1)\pi,(n-\frac{1}{2})\pi]$ there is one eigenvalue k_n ($n = 1,2,...$). The asymptotic solution is determined by the slowest decaying mode of the heat equation

$$b(\xi,\tau = \infty) = 2\nu k_1 \tan k_1\xi. \tag{17}$$

When the collisionality is low the magnetic field does not penetrate the plasma except for a narrow boundary layer. The smaller k_n's, for which $2\nu k_n \ll 1$, are approximately $k_n = (n-\frac{1}{2})\pi(1-2\nu)$. The larger k_n's, for which $2\nu k_n \gg 1$, are approximately $k_n = (n-1)\pi[1 + 1/2\nu(n-1)^2\pi^2]$. The asymptotic solution is therefore

$$b(\xi,\tau = \infty) = \nu\pi \tan[(\pi/2)(1-2\nu)\xi] + \theta(\nu^2). \tag{18}$$

Thus the magnetic field is small except when $\xi + 1 = \theta(\nu)$.

Figure 1 shows the penetration of the magnetic field into the plasma as a function of time for both cases. When the inner conductor is the cathode the magnetic field penetrates the plasma in the form of a shock. The characteristic penetration time is the transit time τ_t. The geometry considered here is similar to that of the plasma opening switch (POS).[13,14] The rate of penetration of the magnetic field into the plasma in the POS is a central issue.[3,8,15–18] For the typical parameters of a POS ($n = 10^{13}$ cm^{-3}, $|B_0| = 10$ kG, $r_1 = 5$ cm, $r_2 = 8$ cm, $a = 10$ cm), the transit time is 30 nsec, which is much shorter than the resistive diffusion time. The mechanism described here could possibly be considered in conjunction with the fast magnetic field penetration measured in the POS.[15] However, further experimental and theoretical studies are necessary before any such relation between the mechanism described here and the POS is suggested. Measurements of the spatial distribution of the magnetic field in the plasma of the POS and the dependence of this distribution on the switch polarity are necessary. Previous measurements employing current loops[15] have been made only for the case in which the cathode is in the inner conductor and for this case seem to confirm the prediction of this model for fast penetration. However, we are not aware of any measurements that could support the prediction of strong dependence of the field penetration on the switch polarity. A theoretical study should include the physics of the sheaths near the electrodes[19] and in particular, the ion pushing from the sheaths by the magnetic pressure.[20] These sheaths are believed to play a dominant role in POS performance. We also note that there is no evidence of the effect described here in simulations of the POS.[18] The Soviet researchers[8] also suggested and discussed in detail the possibility that this shock wave, which they relate mainly to density nonuniformity, is associated with magnetic-field penetration

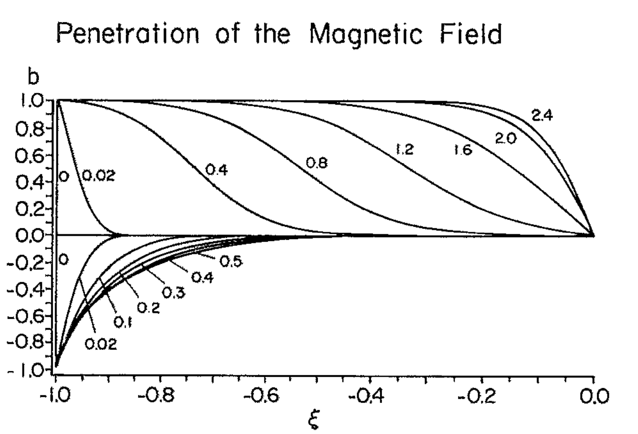

FIG. 1. Penetration of the magnetic field b into an initially unmagnetized plasma for a negative polarity (positive values) and for a positive polarity (negative values) for various times τ. Here $\nu = 0.05$.

in the POS. However, because of the simplicity of our model and the large number of assumptions involved, our purpose is not to describe or model any particular device, but rather to describe a basic physical effect in an idealized configuration.

IV. EXPULSION OF THE MAGNETIC FIELD

A somewhat unusual case arises when the plasma is initially magnetized when the outer conductor is the cathode. The initial condition is $b_0(r,\xi) = -1$, $\psi(\xi,0) = \exp(\xi/2\nu)$. The magnetic field behaves as an expansion wave and is expelled from the plasma on the transit-time time scale. The detailed forms of the boundary layer [Eq. (15)] and surface current are established on the slower resistive time scale. The coefficients are $\alpha_n = 8\nu/[1 + (2\nu k_n)^2](2 + \sin 2k_n)$. Figure 2 shows the expulsion of the magnetic field from the plasma. An initially magnetized plasma expels the magnetic field. Although of a completely different nature, the effect of the magnetic field relaxing to a steady state in which it is expelled from a large volume of the plasma without a change in the boundary conditions is somewhat reminiscent of the Meissner effect in superconductors.

Let us describe a scenario in which the magnetic field decreases the volume it occupies in the plasma and is even expelled from the plasma without a change in the magnetic field at the plasma axial boundaries. The nature of the expulsion of the magnetic field we will now describe is different from that of the expulsion shown in Fig. 2. The plasma resistivity will change and as a result the steady-state distribution of the magnetic field will be changed. We are not sure whether this scenario could be realized in practice. However, it exhibits how in principle the expulsion of the magnetic field could occur.

We assume a plasma in which the collisional resistivity is initially larger than the Hall resistivity. The magnetic field diffuses into the plasma and the current fills the plasma uniformly on the resistive time scale. As the plasma is heated by the currents, its collisional resistivity (if it is classical) decreases and the Hall resistivity becomes dominant. If the outer conductor is the cathode, the steady-state distribution of the magnetic field is modified and the magnetic field is expected to be expelled from the plasma.

Realization of such a scenario in an actual experiment is difficult. The rate of heating is not easy to predict, nor is the dependence of the plasma collisionality on its temperature. In addition, any such experiment must be short since the magnetic pressure begins to push the plasma.

For a collision-dominated magnetic field penetration the resistivity determines only the rate of penetration and the steady state does not depend on the resistivity. However, as mentioned above, for the Hall resistivity-dominated magnetic field penetration the rate of penetration and final steady state both depend on the resistivity. Figures 3 and 4 show the 2-D steady-state distributions of the magnetic field in the cases in which the cathode is in the inner and outer conductors, respectively:

$$b(\xi,\rho) = -[1 + 2\exp(-a/\epsilon r_1\rho^2)]\tanh(a\xi/\epsilon r_1\rho^2) \tag{19a}$$

and

$$b(\xi,\rho) = \frac{\pi\epsilon r_1\rho^2}{a}\tan\left[\frac{\pi}{2}\left(1 - \frac{2\epsilon r_1\rho^2}{a}\right)\xi\right], \tag{19b}$$

where $\rho \equiv r/r_1$ and $\epsilon \equiv \eta cne/|B_0|$ is a characteristic ratio of the collisional resistivity η to the Hall "resistivity" $|B_0|/(cne)$. In Figs. 3 and 4 $r_2/r_1 = 1.5$, $r_1/a = 0.5$, and $\epsilon = 0.1$. The current flows along the contour levels of b, which therefore also shows the current distribution.

A simple analytical form for the magnetic field penetration is obtained for the case that

$$b(-1,\tau) = -\frac{2\gamma\sinh(-\gamma/\nu)}{[\cosh(-\gamma/\nu) + \exp(-\gamma^2\tau/\nu)]},$$

$$b(0,\tau) = 0$$

and

$$b(\xi,0) = -2\gamma\sinh(\gamma\xi/\nu)/[\cosh(\gamma\xi/\nu) + 1].$$

Here $\gamma = ik_0\nu$ and k_0 is one of the two imaginary roots of the dispersion relation [Eq. (11) with the minus sign]. The solution is[21]

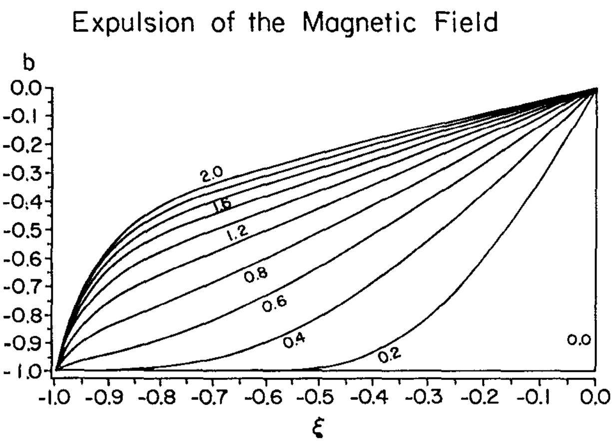

FIG. 2. Expulsion of a magnetic field from an initially magnetized plasma. Shown is the magnetic field b vs ξ for various τ. Here $\nu = 0.05$.

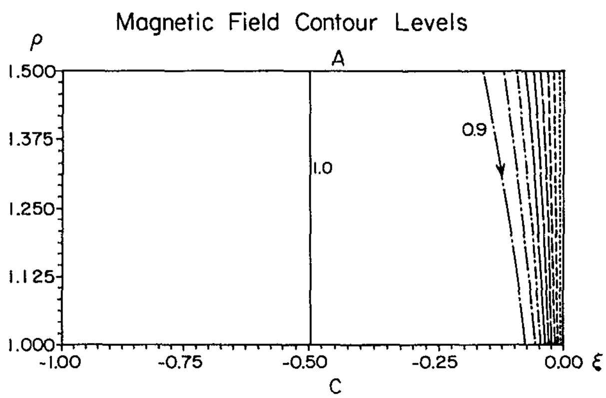

FIG. 3. The steady-state contour levels of the magnetic field in the (ξ,ρ) plane. The cathode is in the inner conductor [Eq. (19a)]: $r_1/a = 0.5$, $\epsilon = 0.1$.

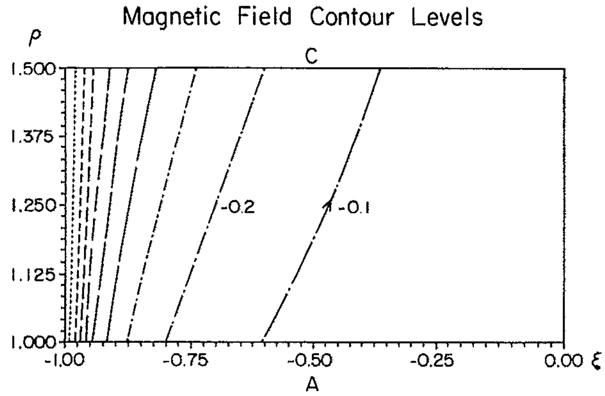

FIG. 4. The steady-state contour levels of the magnetic field in the (ξ,ρ) plane. The cathode is in the outer conductor [Eq. (19b)]: $r_1/a = 0.5$, $\epsilon = 0.05$.

$$b(\xi,\tau) = \frac{-2\gamma \sinh(\gamma\xi/\nu)}{[\cosh(\gamma\xi/\nu) + \exp(-\gamma^2\tau/\nu)]} \tag{20}$$

and becomes asymptotically the steady shock that is given by Eq. (15).

V. DISCUSSION

We have studied here the magnetic field evolution in a short-duration hollow cylindrical plasma. For times so short that the ion motion is small and if the dissipative resistivity, electron inertia, and pressure are small, the magnetic field evolution is governed by the Hall resistivity. We have shown that the magnetic field penetrates into the plasma in the form of a shock wave or is expelled from the plasma, depending on the direction of the accompanying radial current.

We have analyzed a simplified 1-D model. However, 2-D effects are very important. In a usual collisional diffusion the magnetic field energy flows in the direction of penetration. When the magnetic field penetration is driven by the Hall field, the magnetic field energy is carried by the electrons along current lines. In our problem this energy flows mainly in the radial direction, perpendicular to the axial direction of penetration. The magnetic field penetrates into or is expelled from the plasma in the axial direction not because energy flows axially into or out of the plasma, but because of the difference between the energy that flows radially into and out of the plasma. In the case of penetration the energy that flows radially into the plasma is greater than the energy that flows out of the plasma and vice versa in the case of expulsion. In the latter case, the magnetic field energy that flows radially out of the plasma is dissipated in a boundary layer near the anode. In a future study we will study the full 2-D problem with the radial boundary conditions.

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[1] J. D. Kilkenny, A. E. Dangor, and M. G. Haines, Plasma Phys. **15**, 1197 (1973).
[2] P. Rosenau, J. A. Tataronis, and G. Conn, J. Plasma Phys. **21**, 385 (1979).
[3] F. S. Felber, R. O. Hunter, Jr., N. R. Pereira, and T. Tajima, Appl. Phys. Lett. **41**, 705 (1982).
[4] U. Schaper, J. Plasma Phys. **29**, 1 (1983).
[5] M. Coppins, D. J. Bond, and M. G. Haines, Phys. Fluids **27**, 2886 (1984).
[6] E. A. Witalis, IEEE Trans. Plasma Sci. **PS-14**, 842 (1986); L. Turner, IEEE Trans. Plasma Sci. **PS-14**, 849 (1986).
[7] F. J. Wessel, R. Hong, H. J. Song, A. Fisher, N. Rostoker, A. Ron, R. Li, and R. Y. Fan, Phys. Fluids **31**, 3778 (1988).
[8] K. V. Chukbar and V. V. Yan'kov, Sov. Phys. Tech. Phys. **33**, 1293 (1988).
[9] A. S. Kingsep, K. V. Chukbar, and V. V. Yan'kov, in *Reviews of Plasma Physics*, edited by B. Kadomtsev (Consultants Bureau, New York, 1990), Vol. 16, p. 243.
[10] A. Fruchtman and Y. Maron, Phys. Fluids B **3**, 1546 (1991).
[11] R. Kulsrud, P. F. Ottinger, and J. M. Grossmann, Phys. Fluids **31**, 1741 (1988).
[12] G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974), Chap. 4.
[13] C. W. Mendel, Jr. and S. A. Goldstein, J. Appl. Phys. **48**, 1004 (1977).
[14] P. F. Ottinger, S. A. Goldstein, and R. A. Meger, J. Appl. Phys. **56**, 774 (1984).
[15] B. V. Weber, R. J. Commisso, R. A. Meger, J. M. Neri, W. F. Oliphant, and P. F. Ottinger, Appl. Phys. Lett. **45**, 1043 (1984).
[16] IEEE Trans. Plasma Sci., Special Issue on Fast Opening Vacuum Switches **PS-15** (1987).
[17] R. N. Sudan and P. L. Similon, in *Proceedings of the 7th International Conference on High-power Particle Beams*, Karlsruhe, Germany, 1988, edited by W. Bauer and W. Schmidt (Kernforschungszentrum Karlsruhe GmbH, Karlsruhe, 1988), Paper No. GC1.
[18] R. J. Mason, M. E. Jones, J. M. Grossmann, and P. E. Ottinger, Phys. Rev. Lett. **61**, 1835 (1988).
[19] J. M. Grossmann, P. F. Ottinger, and R. J. Mason, Bull. Am. Phys. Soc. **34**, 2076 (1989).
[20] C. W. Mendel, Jr. (private communication).
[21] E. R. Benton and G. W. Platzman, Q. Appl. Math. **30**, 195 (1972).