Fast magnetic-field penetration into plasmas due to the Hall field

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The enhancement of magnetic-field penetration into short-duration plasmas by the dissipationless Hall field is examined. Magnetic-field penetration along a background magnetic field is focused on, where the inductive Hall electric field enables the magnetic field to penetrate as a whistler wave. It is shown that the magnetic-field evolution, when governed simultaneously by both whistler wave propagation and collisional diffusion, is described by a diffusion equation with a complex diffusion coefficient. The imaginary part of this coefficient is proportional to the Hall resistivity associated with the background magnetic field. In the collisionless limit the governing equation is equivalent to the Schrödinger equation for a free particle, and the magnetic field propagates the way a free-particle wave packet expands by dispersion rather than by diffusion. This study was motivated by the enhanced magnetic-field penetration recently observed in the anode plasma of a magnetically insulated ion diode.

I. INTRODUCTION

The magnetic-field penetration into plasmas is usually viewed as a relaxation process due to dissipation. Hence it is described as a diffusion process where the diffusion coefficient is proportional to the dissipational (collisional) resistivity. We have recently measured the rate of magnetic-field penetration into the anode plasma in the magnetically insulated ion diode and have found it to be too high to be explained by classical resistivity (due to binary collisions).[1] We have suggested that a dissipative mechanism different from binary collisions, such as the lower-hybrid drift instability, could cause the anomalous resistivity.[2,3] In this paper we examine the possibility that a dissipationless (collisionless) mechanism, the whistler wave that results from the Hall field, enhances the magnetic-field penetration into plasmas in addition to instabilities.

The effect of the Hall field on the behavior of plasmas has been considered often (see, for example, Refs. 4–8). We are interested in the effect of the Hall field on magnetic-field penetration into short-duration plasmas. Motivated by the diode measurements, we focus on the role of the Hall field in magnetic-field penetration into an already magnetized short-duration plasma. We study a simple one-dimensional problem where the penetrated plasma is immersed in a background magnetic field. If the background magnetic field has a component in the direction of penetration, then, for times so short that the ions are immobile the inductive Hall field enables the magnetic field to penetrate as a whistler wave. We show that the magnetic-field evolution, when governed simultaneously by both whistler wave propagation and collisional diffusion, is described by a diffusion equation with a complex diffusion coefficient. The real part of this coefficient is proportional to the usual collisional resistivity and determines the rate of the dissipative collisional diffusion. The imaginary part is proportional to what we call the Hall resistivity and determines the velocity of the dissipationless whistler wave. When this Hall resistivity is much larger than the collisional resistivity, the rate of magnetic-field penetration scales with the Hall resistivity rather than with the collisional resistivity. In the collisionless limit the governing equation is equivalent to the Schrödinger equation for a free particle, and the magnetic field propagates the way a free-particle wave packet expands, by dispersion rather than by diffusion. The carrier of the magnetic field in our model problem, the whistler wave,[9,10] is dispersive and is characterized by electron flow perpendicular to the direction of penetration.

The presence of the whistler wave mechanism described here relies on the background magnetic field having a component in the direction of field penetration. In Sec. III we estimate the applied field component required to explain the fast field penetration in our diode to be 10% of the applied field. Since such a component is much too large to occur in our diode geometry we do not think that the fast magnetic-field penetration observed can be explained by this model. In general, the importance of the mechanism described here depends on the diode geometry, the magnetic-field intensity, and the diode plasma density and collisionality.

In Sec. II we present the simplifying assumptions and derive the governing equation. In Sec. III we solve the equation for several examples. In the collisionless limit of our model the magnetic field is frozen into the electron fluid. The propagation of a whistler wave that satisfies the frozen-in law is discussed in Sec. IV. The influence of finite system dimensions, two-dimensional effects, and ion motion on the solutions of the idealized model problem are also discussed.

II. THE MODEL

For times much shorter than the ion cyclotron period, the process might be too fast for the ions to move. We thus assume an infinite mass for the ions and consider the electron dynamics only. The velocity scaling we adopt, therefore, is $|\mathbf{v}_e| \cong |\mathbf{v}_e - \mathbf{v}_i| \gg |\mathbf{v}_i|$ where $\mathbf{v}_e$ and $\mathbf{v}_i$ are the

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electron and ion velocities. On the other hand, we assume that the time scale is much longer than the electron-cyclotron period, i.e., the process is slow enough so that we can neglect the electron inertia in the electron momentum equation. If we neglect also pressure gradients, Ohm's law, which follows the electron momentum equation, becomes

$$\mathbf{E}=\eta_c \mathbf{j}+[(\mathbf{j}\times\mathbf{B})/nec], \tag{1}$$

where η_c is the collisional resistivity, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mathbf{j}$ is the current, n the density, $-e$ the electron charge, and c the velocity of light in vacuum. We employed the relation $\mathbf{j}=-ne\mathbf{v}_e$ which follows the above neglect of the ion velocity. The second term on the right-hand side is the Hall field, which is perpendicular to the current and therefore dissipationless.

For plasmas dense enough, we neglect the displacement current and obtain from Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t}=\frac{c^2\eta_c}{4\pi}\nabla^2\mathbf{B}-\frac{c^2}{4\pi}\nabla\times\left[\left(\frac{1}{nec}\nabla\times\mathbf{B}\right)\times\mathbf{B}\right]. \tag{2}$$

Equation (2) governs the evolution of the magnetic field in short-duration plasmas, in the presence of electron motion only. The first term on the right-hand side of the equation is the source of collisional diffusion. The second term results from the Hall field. In the present paper we examine the case in which the Hall field enables the magnetic field to penetrate as a whistler wave along a background magnetic field. Other mechanisms for the magnetic-field evolution that result from the Hall field in Eq. (2) are described in a parallel paper.[11] The magnetic-field evolution, when governed by the electron dynamics, has been studied extensively in the Soviet literature.[12]

In the present case a magnetic field due to external currents is already present in the plasma, and the finite Hall electric field due to this magnetic field affects the penetration of an additional magnetic field. Such is the case in certain megnetically insulated ion diodes,[7] in which the magnetic field, generated by the electron sheath in the diode gap, penetrates the anode plasma, that is already penetrated by the magnetic field externally applied in the diode prior to the high-voltage pulse. This penetration and the accompanying loss of magnetic flux in the acceleration gap have a significant effect on the diode operation.[13–15] The insulating magnetic field is supposed to be parallel to the anode plasma surface and perpendicular to the direction of penetration. However, in the present model we treat a situation in which this field has a component in the direction of penetration. The presence of such a component is an essential ingredient in the mechanism of penetration, the whistler wave mechanism, we describe here.

For simplicity, we analyze the simple 1-D problem of a magnetic field $\mathbf{B}_1(x,t)$ which penetrates a plasma immersed in a uniform magnetic field $\mathbf{B}_0$. We assume that all the quantities vary along x only. If the density is uniform, Eq. (2) becomes

$$\frac{\partial B}{\partial t}=\frac{c^2\eta}{4\pi}\frac{\partial^2 B}{\partial x^2}, \quad \eta\equiv\eta_c+i\eta_{\rm H}, \tag{3}$$

where $B=B_{1z}+iB_{1y}$, and $\eta_{\rm H}\equiv B_{0x}/nec$ is the Hall resistivity. Equation (3) is the diffusion equation with a complex diffusion coefficient. The term "Hall resistivity" does not imply dissipation. We use the term "resistivity" since the rate of magnetic-field evolution scales with this Hall resistivity similarly to the way collisional diffusion scales with collisional resistivity. When the Hall resistivity is zero, the equations for B_{1y} and B_{1z} are decoupled, and for each component the usual diffusion equation with a real diffusion coefficient is recovered. In the other limit, when the collisional resistivity is zero, the diffusion coefficient is purely imaginary, and the resulting equation

$$\frac{\partial B}{\partial t}=\frac{ic^2\eta_{\rm H}}{4\pi}\frac{\partial^2 B}{\partial x^2} \tag{4}$$

is equivalent to the 1-D Schrödinger equation for a free particle. In this limiting case that $\eta_c=0$, Eq. (3) can be written also as

$$\frac{\partial^2 B_z}{\partial t^2}=-\left(\frac{c^2\eta_{\rm H}}{4\pi}\right)^2\frac{\partial^1 B_z}{\partial x^4}, \tag{5}$$

and B_y satisfies $\partial B_y/\partial t=(c^2\eta_{\rm H}/4\pi)(\partial^2 B_z/\partial x^2)$. Equation (5), known as the beam equation, governs the vibrations of a musical fork.[16] Equations (4) and (5) describe whistler waves in the limits of low frequency (relative to the electron-cyclotron frequency) and cold plasma. Whistler waves or helicons were observed in the ionosphere[17] and in laboratory plasmas[18,19] as well as in solids.[20] They have been shown by Haines *et al.*[21] to enhance current penetration in a toroidal Hall accelerator. The magnetic-field amplitude of those waves is much smaller than the background magnetic-field amplitude. In our idealized 1-D case these equations turn out to be linear without assuming that B_{0x} is much larger than $|B|$. Therefore, these equations may describe magnetic-field penetration into a plasma immersed in even a smaller magnetic field on a time scale of less than one oscillation period. However, if B_{0x} is too small one might have to consider also small two-dimensional effects which could make the problem nonlinear.

The amount of heat dissipated is calculated by using the Poynting theorem, which, following Eq. (3), is

$$\frac{c}{4\pi}{\rm Im}(B^*E)\Big|_{x=x_1}^{x=x_2}=\frac{\partial}{\partial t}\int_{x_1}^{x_2}dx\,\frac{|B|^2}{8\pi}+\int_{x_1}^{x_2}dx\,\eta_c|j|^2, \tag{6}$$

where $j\equiv j_z+ij_y$, $E\equiv E_z+iE_y$ and $j=-(ic/4\pi)(\partial B/\partial x)$, $E=\eta j$. Note that j_x is zero.

III. EXAMPLES

In the rest of this paper we solve Eq. (3) for few standard sets of boundary conditions of the diffusion equation. These solutions will help us in illuminating the difference between the diffusion of the magnetic field due to collisional resistivity and the whistler wave penetration of the magnetic field due to the Hall resistivity. In the first two cases we assume that at $t=0$, the magnetic field $B(x,0)$ is zero, and that at $t\geqslant 0$ a fixed magnetic field B_{10} is imposed at $x=0$, namely $B(0,t\geqslant 0)=B_{10}$. The first of the

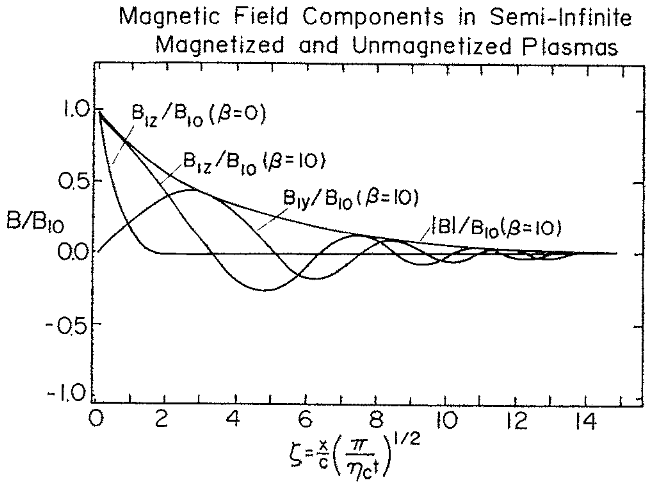

FIG. 1. The magnetic-field components B_{1z}/B_{10}, B_{1y}/B_{10}, and $|B|/B_{10}$ vs $(x/c)(\pi/\eta_c t)^{1/2}$ for a semi-infinite magnetized plasma ($\beta = 10$). Also shown is B_{1z}/B_{10} for an unmagnetized plasma ($\beta = 0$).

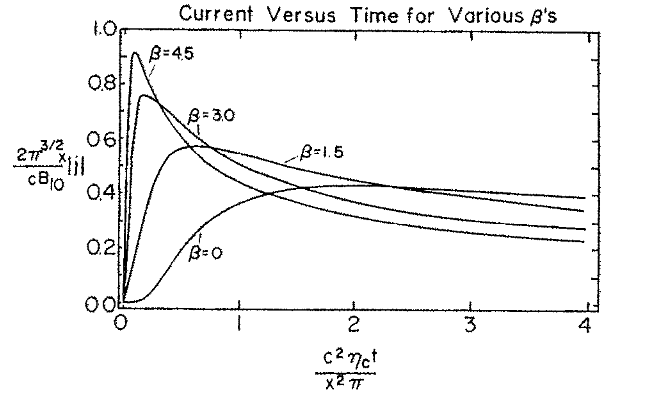

FIG. 2. Normalized current versus normalized time for various β's in a semi-infinite plasma.

two cases is of a semi-infinite plasma slab, and the second is of a finite plasma slab.

A. Semi-infinite plasma

The first case of a semi-infinite plasma is an approximation for the case that the depth of penetration is much smaller than the plasma thickness. The magnetic field is described by a standard solution of the diffusion equation

$$B(x\geqslant 0,t\geqslant 0) = B_{10}\,\mathrm{erfc}[(x/c)(\pi/\eta t)^{1/2}], \qquad (7)$$

where the argument of the complementary error function is complex. Figure 1 shows B_{1z}/B_{10}, B_{1y}/B_{10}, and $|B|/B_{10}$ as a function of $\xi \equiv (x/c)(\pi/\eta_c t)^{1/2}$. In three of the curves the value of $\beta(\equiv \eta_H/\eta_c)$ is 10. For such a large value of β, the penetration is dominated by the Hall resistivity. One curve shows B_{1z}/B_{10} for $\beta = 0$ (no Hall resistivity). In this case of usual diffusion B_{1y} is zero. The special features of the Hall-induced magnetic-field penetration are clear from the figure. When $\beta \gg 1$ the penetration is much faster, is accompanied by oscillations of the magnetic field, and the polarization is circular.

In the Weizmann ion diode,[1] for example, the classical resistivity is estimated to be 4×10^{-15} sec ($T_e = 8$ eV). In order to explain the fast magnetic-field penetration in our diode, the component of the magnetic field in the direction of penetration B_{0x} should be about 700 G, which is nearly 10% of the magnitude of the insulating field. Such a component of the applied magnetic field perpendicular to the electrodes is much larger than expected for our diode geometry. The mechanism we discuss here does not seem, therefore, to play a dominant role in our diode.

The current $j = -[iB_{10}/(2\pi)(\eta t)^{1/2}]e^{-x^2\pi/c^2\eta t}$ is also oscillatory and circularly polarized. The maximal current at a certain position $|j|_{\max}$ and the time $t_{\max}$ at which this current is reached are

$$|j|_{\max} = [cB_{10}/(2\pi)^{3/2}x](1+\beta^2)^{1/4}e^{-1/2},$$

$$t_{\max} = 2x^2\pi/c^2\eta_c(1+\beta^2). \qquad (8)$$

In an unmagnetized plasma ($\beta \ll 1$) the maximal current at each x is the same (because the pulse in our example is infinite in time) while in a magnetized plasma ($\beta \gg 1$) the maximal current is proportional to $\beta^{1/2}$. The dependence of $t_{\max}$ on η_c is not monotonic. When $\eta_c \gg \eta_H$ and the plasma is collisional, $t_{\max}$ grows as η_c decreases. However, when η_c becomes so small that $\eta_c \ll \eta_H$, $t_{\max}$ decreases with the decrease in η_c. In the collisionless limit ($\eta_c = 0$), the current is infinite at $t = 0$ everywhere in the plasma, and its amplitude is uniform in the plasma at $t > 0$, decreasing as $|j| = (B_{10}/2\pi)(\eta_H t)^{-1/2}$. A nonzero dissipation ($\eta_c \neq 0$) makes the current causal. The magnetic field nevertheless decreases for $x\to\infty$ even when $\eta_c = 0$ and is

$$B = B_{10} - B_{10}\exp\left(\frac{-i\pi}{4}\right)\left\{C\left[\frac{x}{c}\left(\frac{2}{\eta_H t}\right)^{1/2}\right] - iS\left[\frac{x}{c}\left(\frac{2}{\eta_H t}\right)^{1/2}\right]\right\}, \qquad (9)$$

where $C(z)$ and $S(z)$ are Fresnel integrals. Figure 2 demonstrates the time dependence of $|j|$ by presenting $(2\pi^{3/2}x/cB_{10})|j|$ as a function of $c^2\eta_c t/x^2\pi$ for various values of β. In the case of a magnetized plasma ($\beta \gg 1$), the current becomes peaked when β is increased.

The instantaneous energy flux into the plasma at $x = 0$ is $(c/8\pi^2)(B_{10}^2/t^{1/2})\mathrm{Re}(\eta^{1/2})$ while the rate of total heating is $(c/8\pi^2)(B_{10}^2/t^{1/2})\eta_c^{1/2}(1/\sqrt{2})$. The heating here does not depend on the Hall resistivity, but the magnetic energy entered into the plasma is enhanced. The Hall resistivity increases the coupling between the plasma and the source.

B. Finite plasma slab

The second case is of a finite plasma slab, where the surface at $x = a$ is of an infinite conductivity. With this boundary condition we model, for example, the metal anode in the magnetically insulated ion diode, whose conductivity is much higher than the plasma conductivity. The assumption of a perfect conductor at $x = a$ implies that

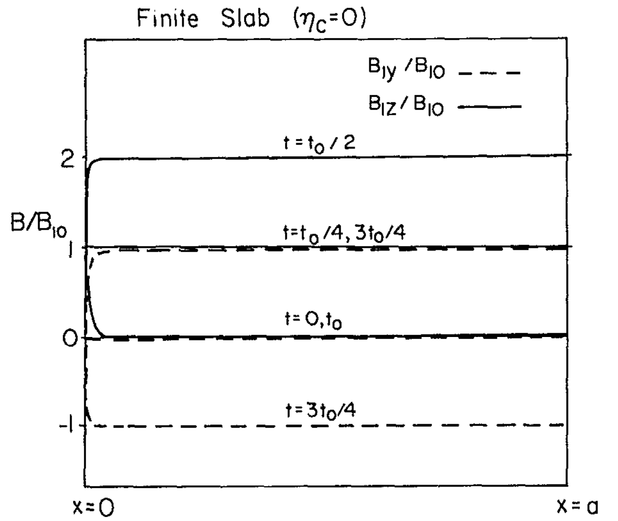

FIG. 3. Magnetic-field components versus x for various times in a collisionless finite plasma slab.

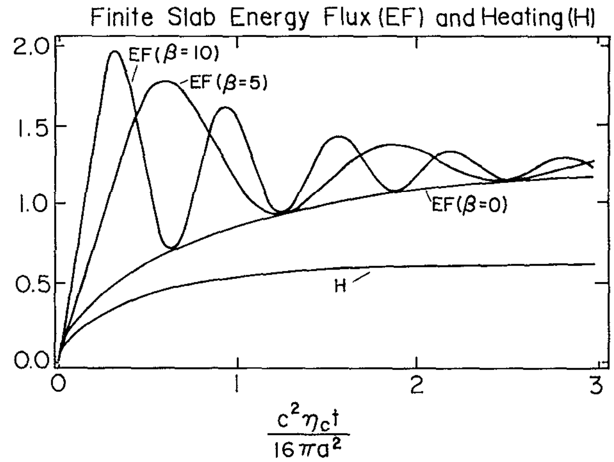

FIG. 4. Normalized energy flux $(2c/B_{10}^2a)\int_0^t dt'\,\mathrm{Im}(B^*E)$ and heating $(8\pi/B_{10}^2a)\int_0^t dt'\int_0^a dx\,\eta_c|j|^2$ versus normalized time for various β's in a finite plasma slab.

$E(\equiv E_z + iE_y) = 0$ there. The current, therefore, in the plasma is zero at $x=a$ and thus $(\partial B/\partial x)(a,t)=0$. If the magnetic-field flux between the cathode and the anode is constant and the diode gap is much wider than the plasma thickness, the magnetic field on the plasma boundary is approximately constant. Therefore, the rest of the boundary conditions are as before, $B(x,0)=0$ for $0\leqslant x\leqslant a$, and $B(0,t)=B_{10}$ for $t\geqslant 0$. The magnetic field in the plasma is

$$B(x,t)=B_{10}\left[1-\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\sin\left((2n+1)\frac{\pi x}{2a}\right)\times e^{-[(2n+1)^2c^2/16\pi a^2]\eta t}\right]. \quad (10)$$

The usual process of a relaxation to a steady-state equilibrium is accompanied by oscillations, again reflecting the dissipationless wavelike effect of the Hall field. For a collisionless plasma, when $\eta_c=0$, the solution is periodic with period $t_0=32a^2\pi^2/c^2\eta_{\mathrm{H}}$. Figure 3 shows B_{1z}/B_{10}, and B_{1y}/B_{10} for $t=t_0/4$, $t_0/2$, $3t_0/4$, and t_0. The dissipation (when $\eta_c\neq 0$) makes these whistler oscillations damped. The plasma is insulated at the conductor and the energy flux there is zero. The oscillatory energy flux into the plasma at $x=0$, until the time t, is

$$\int_0^t dt'\,\frac{c}{4\pi}\,\mathrm{Im}(B^*E)(x=0)$$

$$=\frac{2B_{10}^2a}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}[1-\exp(-s_n\eta_c t)\cos(s_n\eta_{\mathrm{H}}t)], \quad (11)$$

where $s_n\equiv\pi c^2(2n+1)^2/16a^2$. The heating of the plasma until t is

$$\int_0^t dt'\int_0^a dx\,\eta_c|j|^2=\frac{B_{10}^2a}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\times[1-\exp(-2s_n\eta_c t)]. \quad (12)$$

The rate of heating is independent of the Hall resistivity. The rate of change of the magnetic-field energy stored in the plasma is the difference of the energies in Eqs. (10) and (11) (as can be easily verified). The magnetic field is alternately pumped into and out of the plasma. Figure 4 shows the normalized energy flux into the plasma $(2c/B_{10}^2a)\int_0^t dt'\,\mathrm{Im}(B^*E)$ at $x=0$ and the normalized heating $(8\pi/B_{10}^2a)\int_0^t dt'\int_0^a dx\,\eta_c|j|^2$ as a function of the normalized time $(c^2/16\pi a^2)\eta_c t$ for various values of β. We see that for larger β the energy flux is more oscillatory. The total heating is

$$\int_0^\infty dt\int_0^a dx\,\eta_c|j|^2=\frac{B_{10}^2a}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}=\frac{B_{10}^2a}{8\pi}, \quad (13)$$

is independent of the resistivity, and is equal to the total magnetic-field energy entered into the plasma slab.

C. Oscillating field

A case different from the transient problem of magnetic-field penetration is the steady-state oscillating magnetic field in a plasma due to an oscillating source of the form $B(x=0,t)=B_{10}e^{-i\omega t}$. The steady-state solution in the plasma is

$$B_1(x\geqslant 0,t)=B_{10}e^{-(4\pi i\omega/c\eta)^{1/2}x-i\omega t}.$$

For $\beta\ll 1$, we obtain the usual collisional skin depth

$$\delta=\frac{c}{2\sqrt{\pi}}\left(\frac{\eta_c}{\omega}\right)^{1/2}=\frac{c}{\omega_p}\left(\frac{v_c}{\omega}\right)^{1/2},$$

where $v_c = ne^2\eta_c/m$ is the collision frequency, and m is the electron mass. However, for large β the skin depth becomes

$$\delta = \frac{c}{2\sqrt{\pi}} \left(\frac{\eta_H}{\omega}\right)^{1/2} = \frac{c}{\omega_p} \left(\frac{\omega_{cx}}{\omega}\right)^{1/2}, \tag{14}$$

where $\omega_{cx} = eB_{0x}/mc$. The skin depth thus may be broadened substantially by the whistler waves if the wave is propagating along a magnetic field. The energy conservation equation (6) averaged over $2\pi/\omega$ and integrated on x from zero to infinity shows that all the energy flux is transferred into heat. The total energy flux (or the total heating rate) is

$$\frac{c}{4\pi} \operatorname{Im}(B^*E)(x=0) = \int_0^\infty dx\, \eta_c |j|^2 = \frac{cB_{10}^2 \omega^{1/2}}{(4\pi)^{3/2}} \operatorname{Im}[(i\eta)^{1/2}]. \tag{15}$$

When $\beta \ll 1$, the heating is $[cB_{10}^2\omega^{1/2}/\sqrt{2}(4\pi)^{3/2}]\eta_c^{1/2}$ while for $\beta \gg 1$ the heating is $[cB_{10}^2\omega^{1/2}/(4\pi)^{3/2}]\eta_H^{1/2}$ and is independent of the collision frequency. Such an increase in the heating due to Hall field is expected when the magnetic energy is delivered over a finite pulse length, as it is indeed in practice (in diodes, for example), or when it is periodic (as in this example).

D. Expansion of a magnetic-field wave packet

In equivalence to the free-wave packet governed by the Schrödinger equation,[22] we write the expansion in time of a magnetic-field minimum packet,

$$B(x,t) = \frac{B_{10}}{(2\pi)^{1/4}} \left(\Delta x + \frac{c^2\eta t}{4\pi\Delta x}\right)^{-1/2} \times \exp\left(-\frac{x^2}{4[(\Delta x)^2 + (c^2\eta t/4\pi)]}\right). \tag{16}$$

The magnetic field flux $\int_{-\infty}^{\infty} dx\, B(x,t)$ is conserved for any η. The time-dependent magnetic-field energy in the wave packet is

$$\int_{-\infty}^{\infty} dx \frac{|B|^2}{8\pi} = \frac{B_{10}^2}{8\pi} \frac{\Delta x}{[\Delta x^2 + (c^2\eta_c/4\pi)t]^{1/2}}. \tag{17}$$

The total magnetic-field energy is conserved only if $\eta_c = 0$ (as does the total position probability in the solution of the Schrödinger equation) and decreases if there is dissipation. Thus, the Hall resistivity couples B_y and B_z and enables a magnetic-field propagation that conserves both the magnetic-field flux and the magnetic-field energy.

We note that for $t<0$ the wave packet does not expand but rather it shrinks. Figure 5 shows the evolution of the normalized energy $w \equiv (2\pi)^{1/2}\Delta x |B(x,t)|^2/B_{10}^2$ as a function of $x/\Delta x$ for normalized times $\tau = [4\pi(\Delta x)^2/c^2\eta_H]t = -6, -4, -2$, and 0. Note that the evolution of the energy is symmetrical around $t=0$.

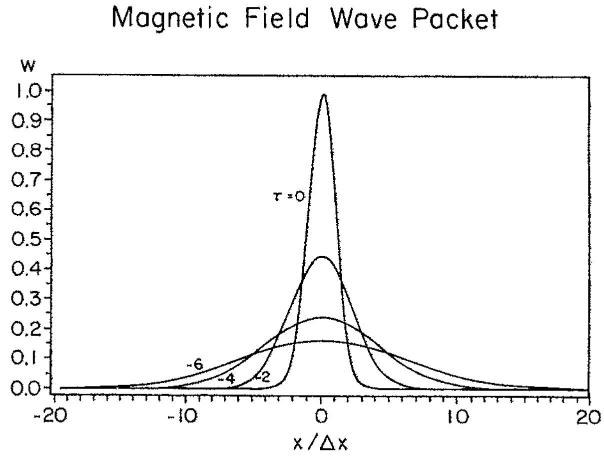

FIG. 5. The normalized magnetic-field energy w vs $x/\Delta x$ for various times τ.

IV. DISCUSSION

First we discuss the evolution of a whistler wave that satisfies the frozen-in law. Without dissipation (when $\eta_c = 0$) the governing equations can be written as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_e \times \mathbf{B}), \tag{18}$$

and

$$(4\pi/c)cn\mathbf{v}_e = -\nabla \times \mathbf{B}. \tag{19}$$

Equation (18) describes the freezing of the magnetic-field lines into the electron fluid. Nevertheless, in the 1-D problem we solved, the magnetic field propagates in the x direction even though there is no electron flow in that direction. It is easy to understand how that happens. Assume that at $t=0$ the magnetic field is only the uniform field $\mathbf{B}_0$ in the x direction. Consider a rectangular loop attached to the electron flow through which the initial magnetic-field flux ϕ_0 is zero. Denote by z the direction perpendicular to the loop plane and by Δx the (infinitesimal) side length in the x direction. At times larger than zero the loop sides move in the y and z directions. The displacement of a loop element in the z direction is $\xi_z(x)$. The change of the magnetic field flux per unit length in the y direction at $t>0$ is

$$\Delta\phi(x,t) = B_{1z}\Delta x + B_0\Delta\xi_z, \tag{20}$$

where

$$\Delta\xi_z(x,t) = \xi_z(x+\Delta x,t) - \xi_z(x,t).$$

This displacement is

$$\xi_z(x,t) = \int_0^t dt' v_{ez}(x,t') = \frac{c}{4\pi ne} \int_0^t dt' \frac{\partial B_y}{\partial x}(x,t'). \tag{21}$$

Therefore

$$\Delta\xi_z = \frac{\partial \xi_z}{\partial x}\Delta x = -\frac{B_{1z}}{B_0}\Delta x, \tag{22}$$

where we used the real part of Eq. (4). The increasing flux of the penetrating magnetic field is thus balanced by an increasing opposite flux of the background magnetic field resulting from the loop rotation. The flux change $\Delta\phi(x,t)$ is zero. The total magnetic-field flux through the loop thus remains constant.

The above description is not necessarily valid at the plasma boundaries in the y and z directions. For example, perfect conductors at the perpendicular boundaries that emit or absorb electrons would force those electrons to cross magnetic-field flux surfaces. This would be impossible in the absence of collisional resistivity. In our model we assumed that the plasma boundaries allow our solution to hold in the bulk of the plasma and we postpone the discussion on this issue to a later study.

The purpose of the simple 1-D model was to demonstrate the basic physical effect. It also approximates the behavior of systems where the variation with one space coordinate is much stronger than with the other space coordinates. Such is the case in the anode plasma whose thickness is much smaller than its dimensions parallel to the anode. However, even for the anode plasma some questions remain open: What is the path for the return current for the diamagnetic current which has both y and z components, what is the relative influence of the effect described here, which results from a small perpendicular component, with respect to the effects which result from small deviations from 1-D slab geometry, etc. Also, as said above, the applied field component perpendicular to the anode has to be sufficiently large in order to significantly affect the field penetration.

The behavior of plasmas when some of our assumptions are not valid will be different from the behavior we described. The magnetic-field penetration in the 1-D geometry turned out to be linear. Two-dimensional effects will make the problem nonlinear. The magnetic-field behavior governed by Eq. (2) probably will be modified even for propagation along a background magnetic field. We focused here on the short time before the ions respond and are pushed by the magnetic pressure. It is important to understand the transition from field penetration to ion motion, a motion that is a central phenomenon in plasma devices. Some of the issues mentioned above will be addressed in future studies.

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