Fundamental ion-cyclotron frequency heating in tokamaks

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The interaction of ions with waves at the fundamental ion-cyclotron frequency in tokamaks is studied without the assumption of geometrical optics. Instead, two small parameters, $\epsilon_1 = \rho/\lambda$ and $\epsilon_2 = B_P/B_T$, are introduced, where ρ is the Larmor radius, λ is the wavelength, and B_P and B_T are the poloidal and toroidal magnetic fields. The heating at the resonance surface is studied for a given incoming wave without considering the problem of accessibility. The case $\epsilon_2 \gg \epsilon_1$ is studied in detail and a boundary layer analysis is performed at the resonance surface. The cold plasma theory is not valid at the resonance surface and the currents and fields are found by solving the Vlasov–Maxwell equations. The current is of nonlocal form, so that an integrodifferential equation is derived and solved numerically. Unlike the case of mirror geometry, in the tokamak the electric field is not constant across the boundary layer. The profiles of the electric field and energy flux are presented. It is shown that all the incoming flux is absorbed, and for these particular values of parameters ($\epsilon_2 \gg \epsilon_1$), there is strong heating at the fundamental resonance. In addition, the case when ϵ_2 is not much larger than ϵ_1 is discussed and it is shown that there is not much heating.

I. INTRODUCTION

Absorption by ions of electromagnetic waves at the ion-cyclotron frequency range is a potentially useful way to heat a plasma. The understanding of the heating mechanism as well as the knowledge of the wave and the energy deposition profiles may lead to a more efficient heating scheme. Since the Maxwell equations for a collisionless plasma generally compose a set of hard-to-solve integrodifferential equations, one seeks an approximation, usually by expanding the equations in some small parameter. Three main scale lengths characterize the ion–wave interaction; those are the equilibrium scale length L, typically of the order of the system dimensions, the mean wavelength λ, and the ion Larmor radius ρ. Except at resonances the geometrical optics approximation may be used when λ is much less than L. However, for ion-cyclotron waves, λ is approximately c/ω_p, ω_p being the ion-plasma frequency, and when ω_p is not very large, the wavelength λ and the system dimension L are comparable. Thus, throughout this article we do not employ a geometrical optics approximation. Instead, the Larmor radius ρ is assumed to be much smaller than λ, and we expand the equations in the small parameter $\epsilon_1 = \rho/\lambda$. To lowest order in this parameter, the plasma is described by the cold plasma theory. The integrodifferential Maxwell equations are approximated by differential equations, which entails a considerable simplification of the problem. However, the cold plasma approximation fails near the ion-cyclotron resonances, where the wave frequency ω is close to an integer multiple of the ion-cyclotron frequency Ω. There the cold ion current is unbounded and the equations become singular.

In a previous article,[1] Weitzner treated the ion-cyclotron resonance in an axisymmetric mirror geometry by employing a boundary layer analysis near the resonance surface. A nonsingular expression for the ion current was found in the boundary layer by first solving an approximated Vlasov equation and then substituting into the Maxwell equations. Here we apply the same method to an axisymmetric tokamak. In both geometries the current is of nonlocal form, whose value at each point in the boundary layer depends on the wave fields across the whole layer. This nonlocal conductivity may create regions where the energy flows from the particles to the wave.

Upon analyzing the case of the fundamental ion-cyclotron resonance in a tokamak, we find that in addition to ϵ_1, there is a second small characteristic parameter ϵ_2, which is the ratio of the poloidal to toroidal magnetic fields. In the axisymmetric tokamak, ϵ_2 also measures the angle between the magnetic field and the gradient of the magnetic field intensity, and represents the degree of parallel stratification. The relation between ϵ_1 and ϵ_2 detemines the form of the equations in the boundary layer. When ϵ_2 is larger than ϵ_1, the presence of parallel stratification makes the form of the Vlasov equation in the boundary layer similar to that in the mirror case. The current obtained is nonlocal, as mentioned earlier.

We study in detail the fundamental ion-cyclotron resonance when ϵ_1 equals ϵ_2^3. The Maxwell equations become in this case integrodifferential equations. Though not very realistic for present day tokamaks, this ordering is interesting for two reasons. First, it results in strong heating, where to lowest order all the energy flux of the wave is absorbed in the resonance layer. Secondly the solution of the equations in this case can guide us in future problems where a similar analysis of integrodifferential equations in the boundary layer will be required. We discuss also other cases where the requirement on the relation between ϵ_1 and ϵ_2 is relaxed. The rate of heating is shown to be smaller in these cases.

Problems involving ion-cyclotron resonances have been treated in the past. Swanson[2] solved the Vlasov equation to higher order in ϵ_1, a process that yielded a higher-order system of equations including mode conversion. Colestock and Kashuba[3] used a variational principle analysis and expanded the conductivity tensor in **k** space to higher order in ϵ_1, reco-

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vering again higher-order differential equations. These two analyses were based on the assumption that the expansion in the small parameter ϵ_1 is still valid near the resonance and that what is needed is to include higher-order corrections. However, this assumption is not always correct and the cases studied in this article require, in fact, a solution of the full integrodifferential equations.

Let us make two comments before we proceed. First, the form of the equations depends crucially on the relation between various parameters of the system. We assumed here that λ and L are comparable. When geometric optics may be applied outside the boundary layer, a third small parameter, λ/L will appear in the equations. The relation between this new small parameter and ϵ_1 and ϵ_2 may change the form of the equations near the resonance surface, and a different analysis may be required. Secondly, the problem we treat here is the absorption for given fields at the edge of the boundary layer and the accessibility problem is not treated here. However, recent numerical studies by Jaeger[4] employing cold plasma theory indicate that in some cases wave energy flux does reach the resonance surface.

In Sec. II we write the Vlasov–Maxwell equations in natural coordinates tailored to the axisymmetric tokamak geometry. In Sec. III a boundary layer analysis is employed in solving the Vlasov equation near the fundamental ion-cyclotron resonance surface and a causal nonlocal form is derived for the current. A detailed analysis of the case of a relatively large poloidal field is given in Sec. IV, and cases of smaller poloidal field are discussed in Sec. V.

II. THE VLASOV–MAXWELL EQUATIONS IN NATURAL COORDINATES

The ion dynamics is determined by solving the linearized Vlasov equation,

$$\frac{\partial f}{\partial t} + (\mathbf{v}\cdot\nabla)f + \frac{e}{mc}\mathbf{v}\times\mathbf{B}_0\cdot\frac{\partial f}{\partial \mathbf{v}} = -\frac{e}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial F_0}{\partial \mathbf{v}}. \tag{1}$$

The tokamak equilibrium is axisymmetric. The equilibrium magnetic field $\mathbf{B}_0(r,z)$ is given as

$$\mathbf{B}_0(r,z) = B_0[\hat{\theta}\cos\alpha + (\hat{r}\sin\beta + \hat{z}\cos\beta)\sin\alpha], \tag{2}$$

where B_0, α, and β are functions of r and z, and the equilibrium ion distribution function F_0 is

$$F_0 = F_0(v_\parallel^2, v_\perp^2, r, z). \tag{3}$$

The angle β varies between zero and 2π, while the angle α is assumed to be small.

We now make successive changes of variables similar to those used for the mirror geometry by Weitzner[1] with modification to the tokamak geometry. Since this part is parallel to that of the earlier work, it is presented here in somewhat condensed form. We introduce three orthogonal unit vectors:

$$\begin{aligned}\hat{V}_1 &= \hat{r}\cos\beta - \hat{z}\sin\beta,\\ \hat{V}_2 &= \hat{\theta}\sin\alpha - (\hat{r}\sin\beta + \hat{z}\cos\beta)\cos\alpha,\\ \hat{V}_3 &= \hat{\theta}\cos\alpha + (\hat{r}\sin\beta + \hat{z}\cos\beta)\sin\alpha.\end{aligned} \tag{4}$$

The velocity vector $\mathbf{v}$ is characterized by $v_\perp$, $v_\parallel$, and the gyrophase angle ϕ, where

$$\mathbf{v} = v_\parallel \hat{V}_3 + v_\perp(\cos\phi\hat{V}_1 + \sin\phi\hat{V}_2). \tag{5}$$

Using complex unit vectors $\hat{V}_\pm = (\hat{V}_1 \mp i\hat{V}_2)/\sqrt{2}$, we define any vector $\mathbf{A}$ as $\mathbf{A} = A_+\hat{V}_+ + A_-\hat{V}_- + A_\parallel\hat{V}_3$. We take a Laplace transform in time, corresponding to time dependence $\exp(i\omega t)$, $\mathrm{Im}\,\omega < 0$, and in the azimuthal angle with mode number M. The perturbed distribution function $f(\mathbf{r},\mathbf{v},t)$ is

$$\begin{aligned}f(\mathbf{r},\mathbf{v},t) &= f(r,z,\theta,v_\parallel,v_\perp,\phi,t)\\ &= \sum_{n=-\infty}^{\infty} f_n(r,z,v_\parallel,v_\perp)\exp(in\phi)\\ &\quad\times\exp(i\omega t + iM\theta),\end{aligned} \tag{6}$$

where n is the harmonic number in the gyrophase angle.

We make now a final transformation in real space. We introduce the coordinate $\xi(r,z)$ that parametrizes the surfaces in real space on which the cyclotron resonance may occur,

$$\xi(r,z) = eB_0(r,z)/(mc\omega). \tag{7}$$

The second spatial coordinate $\eta(r,z)$ is chosen to be the coordinate of the orthogonal trajectories to the surface $\xi(r,z) = \text{const}$, such that $\nabla\xi\cdot\nabla\eta = 0$. We take

$$\begin{aligned}\eta_{,r} &= -\mu(r,z)\xi_{,z},\\ \eta_{,z} &= \mu(r,z)\xi_{,r},\end{aligned} \tag{8}$$

and the function $\mu(r,z)$ is

$$\mu(r,z) = \exp\left(-\int_{\xi_0}^{\xi}\frac{d\xi'(B_{0,rr} + B_{0,zz})}{|\nabla B_0|^2}\right). \tag{9}$$

We wish to represent spatial derivatives in terms of the new coordinates. For that purpose we define

$$\begin{aligned}\kappa' &= (\hat{B}_p\cdot\nabla)\xi = \xi_{,r}\sin\beta + \xi_{,z}\cos\beta,\\ \nu' &= (\hat{B}_p\times\nabla\hat{\theta}\cdot\nabla)\xi = \xi_{,r}\cos\beta + \xi_{,z}\sin\beta,\end{aligned} \tag{10}$$

where $\hat{B}_p$ is a unit vector in the direction of the poloidal magnetic field, $\hat{B}_p = \hat{r}\sin\beta + \hat{z}\cos\beta$. The derivatives in the (r,z) plane in the direction of the poloidal magnetic field and perpendicular to it are

$$\begin{aligned}\hat{B}_p\cdot\nabla &= \sin\beta\frac{\partial}{\partial r} + \cos\beta\frac{\partial}{\partial z} = \kappa'\frac{\partial}{\partial\xi} - \mu\nu'\frac{\partial}{\partial\eta},\\ \hat{B}_p\times\hat{\theta}\cdot\nabla &= -\cos\beta\frac{\partial}{\partial r} + \sin\beta\frac{\partial}{\partial z} = \nu'\frac{\partial}{\partial\xi} + \mu\kappa'\frac{\partial}{\partial\eta}.\end{aligned} \tag{11}$$

Similarly to Ref. 1 we normalize the quantities $\mathbf{v} = v_{\text{th}}\mathbf{u}$, $F_0(\mathbf{v})d\mathbf{v} = n_0G_0(\mathbf{u})d\mathbf{u}$, and $ev_{\text{th}}f(\mathbf{r},\mathbf{v})d\mathbf{v} = g(\mathbf{r},\mathbf{u})d\mathbf{u}$. The current in these units is

$$\mathbf{J} = \int \mathbf{u}g(\mathbf{r},\mathbf{u})d\mathbf{u}. \tag{12}$$

The wavelength of waves in an homogeneous plasma at the ion-cyclotron frequency range is c/ω_p. In the present work, we assume that this wavelength is comparable to the dimensions of the system. Thus the effective scalelength of waves and equilibrium quantities is

$$\lambda = c/\omega_p$$

and the dimensional space variable $\mathbf{r}'$ is

$$\mathbf{r} = \lambda \mathbf{r}'. \tag{13}$$

We assume the quantity ϵ_1 to be our small parameter

$$\epsilon_1 \equiv v_{\text{th}} \omega_p / (c\omega). \tag{14}$$

The quantity ϵ_1 is the ratio of the effective ion Larmor radius v_{th}/ω to the mean wavelength c/ω_p. This small quantity will be used for the expansion instead of the geometrical optics small parameter. We rescale the space gradients κ' and ν',

$$\kappa' = \kappa/\lambda, \quad \nu' = \nu/\lambda, \tag{15}$$

so that κ and ν are of order 1. The parallel and perpendicular derivatives are

$$\nabla_\parallel = \hat{B}_p \cdot \nabla = \frac{1}{\lambda}\left(\kappa \frac{\partial}{\partial \xi} - \mu\nu \frac{\partial}{\partial \eta}\right),$$

$$\nabla_\perp = \hat{B}_p \times \hat{\theta} \cdot \nabla = \frac{1}{\lambda}\left(\nu \frac{\partial}{\partial \xi} + \mu\kappa \frac{\partial}{\partial \eta}\right).$$

In the new variables, the Vlasov equation for g_n, the nth Fourier component of the normalized distribution function g is

$$\begin{aligned} i(1 - n\xi) g_n &+ \epsilon_1 \sum_{j=-2}^{2} O^j_n g_{n+j} \\ &= -(\omega_p^2/\omega)\left[\delta_n^0 E_\parallel G_{0,u_\parallel} + (i/2)(E_- \delta_n^{-1} - E_+ \delta_n^1)\right. \\ &\quad \left. \times G_{0,u_\perp} - (v_{\text{th}}/2)(B_- \delta_n^{-1} + B_+ \delta_n^{+1}) R G_0\right], \end{aligned} \tag{16}$$

where we employ the notation $G_{0,u_\perp} = \partial G_0/\partial u_\perp$ and $G_{0,u_\parallel} = \partial G_0/\partial u_\parallel$, and $R = u_\perp(\partial/\partial u_\parallel) - u_\parallel(\partial/\partial u_\perp)$. The operators O^j_n are as follows:

$$\begin{aligned} O_n^0 &\equiv \frac{u_\perp}{2}\left[\sin\alpha\left(\nabla_\perp\beta + \frac{\sin\beta}{r}\right) + \cos\alpha\nabla_\parallel\alpha\right]R + u_\parallel\left[\sin\alpha\nabla_\parallel + \frac{in\cos\alpha}{r} - \frac{3in}{2}\sin\alpha\cos\alpha\left(\nabla_\parallel\beta + \frac{\cos\beta}{r}\right) + \frac{in}{2}\nabla_\perp\alpha\right], \\ O_n^{\pm 1} &\equiv \frac{u_\perp}{2}\left\{\nabla_\perp - \cos\alpha\nabla_\perp\beta\cdot i(n\pm 1) \pm i\left[\left(-\cos\alpha\nabla_\parallel + \frac{iM\sin\alpha}{r}\right) + i(n\pm 1)\nabla_\parallel\beta\cos^2\alpha\right]\right\} \pm \frac{u_\perp}{2r}\sin^2\alpha\cos\beta(n\pm 1) \\ &\quad + \frac{u_\parallel}{2}\left\{-\frac{\cos^2\alpha\cos\beta}{r}R - \frac{iu_\parallel\cos\alpha\sin\beta}{u_\perp r}(n\pm 1) + \sin^2\alpha\nabla_\parallel\beta R + \frac{iu_\parallel}{u_\perp}\sin\alpha\nabla_\parallel\alpha(n\pm 1)\right. \\ &\quad \left.\pm i\left[\left(\frac{\cos\alpha\sin\beta}{r} - \sin\alpha\nabla_\parallel\alpha\right)R + i(n\pm 1)\frac{u_\parallel}{u_\perp}\left(-\frac{\cos\beta\cos^2\alpha}{r} + \nabla_\parallel\beta\sin^2\alpha\right)\right]\right\}, \\ O_n^{\pm 2} &= \frac{u_\perp}{4}\left\{\left[\sin\alpha\nabla_\perp\beta - \left(\frac{\sin\alpha\sin\beta}{r} + \cos\alpha\nabla_\parallel\alpha\right)\right]R \pm i\left(-\frac{\sin\alpha\cos\alpha\cos\beta}{r} + \nabla_\perp\alpha - \sin\alpha\cos\alpha\nabla_\parallel\beta\right)R\right\} \\ &\quad + \frac{u_\parallel}{4}\left[\nabla_\perp\alpha + \sin\alpha\cos\alpha\left(\nabla_\parallel\beta + \frac{\cos\beta}{r}\right)i(n\pm 2) \mp \left(-\frac{\sin\alpha\sin\beta}{r}(n\pm 2) + (\sin\alpha\nabla_\perp\beta - \cos\alpha\nabla_\parallel\alpha)(n\pm 2)\right)\right]. \end{aligned} \tag{17}$$

The current J_+ is

$$J_+ = \int_0^\infty du_\perp \int_{-\infty}^{\infty} du_\parallel\, u_\perp^2 g_1. \tag{18}$$

We introduce here a second small parameter ϵ_2 that characterizes the system

$$\sin\alpha = \bar{\alpha}\epsilon_2, \tag{19}$$

where $\bar{\alpha}$ is of order 1. This small parameter is of the order of the ratio of the magnitudes of the poloidal and toroidal magnetic fields, and its value varies according to the location of the resonance surface relative to the tokamak axis. The relation between ϵ_1 and ϵ_2 is crucial in determining the forms of the Vlasov–Maxwell equations and thus is important to the character of the interaction.

As can be seen from the last form of the Vlasov equation, far from the ion-cyclotron resonances, as long as $1 - n\xi \neq 0$, the terms proportional to ϵ_1 can be neglected. Then the cold plasma theory is valid to lowest order. The perturbed currents thus obtained from the approximated solution of the Vlasov equation are the same as those obtained from the cold plasma theory. However, near the ion-cyclotron resonances, when for some integer n, $1 - n\xi$ is nearly zero, thermal effects must be included. The solution of the Vlasov equation is modified, and as a result the forms of the currents and the electric fields are changed.

We turn now to Maxwell's equations. In the normalized units they become

$$\nabla' \times (\nabla' \times \mathbf{E}) - (\omega^2/\omega_p^2)\mathbf{E} = (-\omega/\omega_p^2)\mathbf{J}_T, \tag{20}$$

$$\mathbf{B} = i(\omega_p/\omega)\nabla' \times \mathbf{E}, \tag{21}$$

where $\mathbf{J}_T$, the total current, is the sum of the ion current $\mathbf{J}$ and the electron current $\boldsymbol{\sigma}_e \cdot \mathbf{E}$:

$$\mathbf{J}_T = \mathbf{J} + \boldsymbol{\sigma}_e \mathbf{E}. \tag{22}$$

We employ the approximation commonly used for waves at the ion-cyclotron frequency range, that is

$$E_\parallel = 0. \tag{23}$$

This approximation is justified by the fact that the parallel electron conductivity $\sigma^e_{\parallel\parallel}$ is proportional to the ion to electron mass ratio and $E_\parallel$ must vanish in order to balance this term.[1]

For the high-density plasma in a tokamak and for waves at the ion-cyclotron frequency ω^2/ω_p^2 is small. Thus the perpendicular components of the first of the scaled Maxwell equations [Eq. (20)] are approximated to be

$$\nabla' \times (\nabla' \times \mathbf{E})_\perp = (-i\omega/\omega_p^2)\mathbf{J}_{T_\perp}. \tag{24}$$

Far from the ion-cyclotron resonances the various terms in the last equation are of order 1 in the wave electric field amplitude. Near a resonance surface the current may grow as well as derivatives of the fields relative to ξ. One may recall that $\sin\alpha$ is much smaller than 1, even though we do not yet specify its magnitude. We are interested in an approximate solution of the Vlasov–Maxwell equations near the resonance surfaces, which will give us the currents and fields to lowest order. For that purpose we write the components of $\nabla' \times \nabla' \times \mathbf{E}$, omit the terms that are proportional to $\sin\alpha$, and do not involve derivatives and thus are obviously of high order in the wave field amplitude,

$$\begin{aligned}
\nabla\times(\nabla\times\mathbf{E})\cdot\hat{V}_1 &= \left(\frac{\sin\beta}{r} + \kappa\frac{\partial}{\partial\xi} - \mu\nu\frac{\partial}{\partial\eta}\right)\left(\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}\right) + \frac{M^2}{r^2}E_1 - \frac{iM}{r}\left(\nu\frac{\partial}{\partial\xi} + \mu\kappa\frac{\partial}{\partial\eta}\right)(E_2\sin\alpha),\\
\nabla\times(\nabla\times\mathbf{E})\cdot\hat{V}_2 &= -\cos\alpha\left[\left(-\frac{\cos\beta}{r} + \nu\frac{\partial}{\partial\xi} + \mu\kappa\frac{\partial}{\partial\eta}\right)\left(\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}\right)\right] - \frac{M^2}{r^2}\cos\alpha E_2\\
&\quad + \frac{iM}{r}\left(\kappa\frac{\partial}{\partial\xi} - \mu\nu\frac{\partial}{\partial\eta}\right)(E_2\sin\alpha)\\
&\quad + \sin\alpha\left\{-\frac{iM}{r}\left(\kappa\cos\alpha\frac{\partial E_2}{\partial\xi} + \nu\frac{\partial E_1}{\partial\xi}\right) - \left[(\kappa^2+\nu^2)\frac{\partial^2}{\partial\xi^2} + \frac{1}{2}\left(\frac{\partial}{\partial\xi}(\kappa^2+\nu^2)\right)\frac{\partial}{\partial\xi}\right](E_2\sin\alpha)\right\}.
\end{aligned} \tag{25}$$

We now turn to study the Vlasov–Maxwell equations near the fundamental ion-cyclotron surface.

III. A NONLOCAL CURRENT

In order to solve the equations near the resonance surface we apply a boundary layer analysis. The equations are expanded inside the boundary layer and their solutions are matched to the outer cold plasma soutions. First we solve the Vlasov equation in the boundary layer, use this solution to determine the currents and then solve Maxwell's equations with those currents.

A basic assumption in the analysis is that near the resonance surface the perturbed quantities have fast variations in the direction perpendicular to the surface, while they retain the same space dependence parallel to it. The fundamental resonance occurs when $1-\xi$ is small. We thus rescale ξ and introduce a boundary layer variable $\bar{\xi}$ such that

$$\epsilon_3\bar{\xi} = \xi - 1 \tag{26}$$

and

$$\frac{\partial}{\partial\xi} = \frac{1}{\epsilon_3}\frac{\partial}{\partial\bar{\xi}}. \tag{27}$$

The variable $\bar{\xi}$ is of order 1 in the boundary layer and ϵ_3 is the small width of the boundary layer. Examining the Vlasov equation we notice that if $\epsilon_2 \gg \epsilon_1$ the equation for g_1 in the boundary layer is approximately

$$-i\epsilon_3\bar{\xi}g_1 + \epsilon_3\kappa\bar{\alpha}u_\parallel\frac{\partial g_1}{\partial\bar{\xi}} = -\frac{\omega_p^2}{2\omega}E_+G_{0,u_\perp}, \tag{28}$$

where the width of the boundary layer ϵ_3 is related to ϵ_1 and ϵ_2 through

$$\epsilon_3 = (\epsilon_1\epsilon_2)^{1/2}. \tag{29}$$

A similar form for the Vlasov equation was obtained for an axisymmetric mirror geometry.[1] This form originated from the presence of parallel stratification, which was comparable to the perpendicular stratification. In the tokamak there is only small parallel stratification. Nevertheless, as long as the parallel stratification is not too small, or explicitly as long as ϵ_2 is larger than ϵ_1, we obtain the above approximated form (28) for the Vlasov equation.

The unique causal solution of Eq. (28) is[1]

$$g_{+1} = -\left(\frac{\omega_p^2}{2\omega\epsilon_3}\right)\left(\frac{G_{0,u_\perp}}{u_\parallel\kappa\bar{\alpha}}\right)\int_{-\infty\,\mathrm{sgn}(u_\parallel\kappa\bar{\alpha})}^{\bar{\xi}} dx \times E_+(x,\eta)\exp[(\bar{\xi}^2 - x^2)/-2iu_\parallel\kappa\bar{\alpha}]. \tag{30}$$

By integration in the velocity space, we obtain the resonance current,

$$J_+ = \left(\frac{\omega_p^2}{\omega\epsilon_3}\right)\int_0^\infty du_\perp \int_{-\infty}^\infty du_\parallel\, 2\pi u_\perp\left(\frac{G_0}{u_\parallel\kappa\bar{\alpha}}\right) \times \int_{-\infty\,\mathrm{sgn}(u_\parallel\kappa\bar{\alpha})}^{\bar{\xi}} dx\, E_+(x,\eta)\exp\left(\frac{\bar{\xi}^2 - x^2}{-2iu_\parallel\kappa\bar{\alpha}}\right). \tag{31}$$

The current is of nonlocal form. The current at each point is obtained by an integration across the whole boundary layer. The nonlocal conductivity causes some peculiarities in the interaction of waves and particles and considerably complicates the analysis of the problem and its solution.

The resonant current is of order $1/\epsilon_3$, and we substitute it into the Maxwell equations. The terms of order $1/\epsilon_3^2$ must be zero so that

$$\frac{\partial^2}{\partial\bar{\xi}^2}(\nu E_2 - \kappa E_1) = 0. \tag{32}$$

Since we expect the fields at the edge of the boundary layer to be of order 1 in the fields, we look for a solution that does not have linear dependence in $\bar{\xi}$, since such dependence results in field amplitude of order $1/\epsilon_3$ at the edge. Thus, excluding linear dependence, we obtain to lowest order

$$\nu E_2^{(0)} - \kappa E_1^{(0)} = a(\eta), \tag{33}$$

where $a(\eta)$ is a constant that does not depend on $\bar{\xi}$.

We turn now to higher-order terms in $1/\epsilon_3$, which will balance the resonant current. In order to identify those terms we have to specify the relation between ϵ_1 and ϵ_2. We study two cases. First we let ϵ_1 be much smaller than ϵ_2, namely $\epsilon_1 = \epsilon_2^3$. This ordering may not fit present day experiments, but it is interesting since explicit solution of the equations in this case shows a strong absorption of the waves. Secondly, we examine the case where ϵ_1 is larger than ϵ_2^3 but smaller than ϵ_2, and show that the absorption is smaller and of higher order in the wave fields. Finally, the case when ϵ_2 is comparable to ϵ_1 or smaller than ϵ_1 is discussed.

IV. RELATIVELY LARGER POLOIDAL FIELD

The largest terms in the Maxwell equations that can balance the resonant current are

$$(\nu \hat{V}_1 + \kappa \hat{V}_2)\cdot\nabla\times(\nabla\times\mathbf{E}) \cong \frac{\kappa\bar{\alpha}^2}{2}(\kappa^2+\nu^2)\frac{\partial^2 E_2}{\partial\bar{\xi}^2}\frac{\epsilon_2^2}{\epsilon_3^2} = \frac{i\omega}{\omega_p^2}(\nu J_1 + \kappa J_2). \quad (34)$$

This balancing requires the relation $\epsilon_2^2 = \epsilon_3$ and with the previously used equality $\epsilon_3 = \epsilon_1\epsilon_2$ we have

$$\epsilon_1 = \epsilon_2^3. \quad (35)$$

Using (31) we obtain the following integrodifferential equation for E_+:

$$(\kappa\bar{\alpha})^3\frac{d^2E_+}{d\bar{\xi}^2} = \int\int 2\pi u_\perp\, du_\perp \frac{du_\parallel}{u_\parallel}G_0 \times\int_{-\infty\,\mathrm{sgn}(u_\parallel\kappa\bar{\alpha})}^{\bar{\xi}} dx\, E_+(x,\eta)\exp\left(\frac{\bar{\xi}^2 - x^2}{-2iu_\parallel\kappa\bar{\alpha}}\right). \quad (36)$$

For mirror geometry,[1] the electric field is constant to lowest order in the boundary layer. However the current has fast oscillations because of the strong $\bar{\xi}$ dependence of the nonlocal conductivity. In the tokamak the electric field itself has fast oscillations to lowest order and in order to find its form, we have to solve this far from the trivial integrodifferential equation. Since the resonant current is of order $1/\epsilon_3$, and the width of the boundary layer is of order ϵ_3, the rate of heating $\mathbf{J}\cdot\mathbf{E}$ is of order 1 in the wave field amplitude.

Assuming an equilibrium Maxwellian distribution function, and defining

$$G \equiv \int_0^\infty 2\pi u_\perp\, du_\perp\, G_0 = \left(\frac{1}{\sqrt{2\pi}}\right)e^{-u_\parallel^2/2},$$
$$k \equiv |\kappa\bar{\alpha}|, \quad \beta = 4\bar{\xi}/k^2, \quad p = 32\sqrt{2}/k^3, \quad (37)$$
$$E(\beta(\xi)) = E_+(\xi),$$

Eq. (36) becomes

$$\frac{d^2E}{d\beta^2} = \frac{i}{2\sqrt{\pi}p}\left[\int_{-\infty}^{\beta} d\beta'\, E(\beta')F\left(\frac{\beta^2-\beta'^2}{p}\right) + \int_\beta^\infty d\beta'\, E(\beta')F^*\left(\frac{\beta^2-\beta'^2}{p}\right)\right] \equiv j(\beta), \quad (38)$$

where

$$F(y) = \int_0^\infty \frac{dt\, e^{-t^2}}{t}e^{iy/t}. \quad (39)$$

We may obtain the asymptotic form of $j(\beta)$ for large $\beta > 0$ by expressing it as an integral on the whole real axis plus an integral from β to infinity. The integral on the whole real axis can be shown, by an application of the steepest descent method, to decay faster than any power of $1/\beta$. The second integral can be shown, through integration by parts, to be of the form $-E/4\beta$. This is also true for large $|\beta|$, $\beta < 0$. Thus asymptotically Eq. (38) is

$$\frac{d^2E}{d\beta^2} + \frac{E}{4\beta} = 0. \quad (40)$$

Two independent solutions of this equation are

$$W_j(\beta) = \beta^{1/2}H_1^{(j)}(\beta^{1/2}), \quad j = 1,2, \quad (41)$$

where $H_1^{(j)}$ are the Hankel functions. In the high-field side, when $\beta > 0$, the two solutions are oscillatory. We expect the general solution in this region to be asymptotically of the form

$$E = W_1(\beta) + RW_2(\beta). \quad (42)$$

In the low-field side, for $\beta < 0$, one of the solutions, $W_1(\beta)$, is exponentially decaying, while the second, $W_2(\beta)$, is exponentially growing. Not expecting an exponentially growing solution, we look in this region for an asymptotic solution of the form

$$E = TW_1(\beta). \quad (43)$$

The values of the quantities R and T are determined by matching these asymptotic solutions on both sides of the boundary layer to the solution of the integrodifferential equation. The value of R determines the rate of absorption as can be shown by use of the Poynting theorem. The Poynting theorem says that to lowest order the rate of absorption is

$$P = \epsilon_3\int_{-\infty}^{\infty} d\bar{\xi}\, \mathrm{Re}(J_+^* E_+) = \frac{1}{4\pi}\left(\mathrm{Re}(\mathbf{E}^*\times\mathbf{B})\cdot\frac{\nabla\xi}{|\nabla\xi|}\right). \quad (44)$$

Here [ ] denotes a jump of the quantity in the brackets across the boundary layer. In order to know the rate of absorption there is no need to perform the integration of $J_+^* E_+$ across the boundary layer. It is sufficient to find the asymptotic values of the Poynting vector on both sides of the layer and to evaluate the jump in this quantity. The relations between $\mathbf{E}$ and $\mathbf{B}$ [Eq. (21)] and between E_+ and E_- allow us to write the value of the Poynting vector flux in the normalized units

$$\mathbf{S}\cdot\frac{\nabla\xi}{|\nabla\xi|} = \frac{\omega_p^2}{\pi|\nabla\Omega|}\mathrm{Im}\left(E_+^*\frac{\partial E_+}{\partial\beta}\right). \quad (45)$$

Using the Wronskian relations between the Hankel functions, we obtain

$$\mathrm{Im}\left(E_+^*\frac{\partial E_+}{\partial\beta}\right) = \frac{1}{\pi}(1-|R|^2), \quad \beta\to\infty,$$
$$\mathrm{Im}\left(E_+^*\frac{\partial E_+}{\partial\beta}\right) = 0, \quad \beta\to-\infty. \quad (46)$$

We retained only that part of the flux that is not oscillatory. One can see that R is the reflection coefficient and $|R|^2$ expresses the fraction of the wave energy that is reflected from the resonance surface. No wave propagates in the low-field side. The calculation of the absorption rate is thus reduced to finding R since there is no transmission. The rate of absorption is

$$P=(\omega_p^2/\pi^2|\nabla\Omega|)(1-|R|^2). \quad (47)$$

In order to solve Eq. (38) we treat it as a differential equation with an inhomogeneous part $j(\beta)$. We write an equivalent differential equation, whose inhomogeneous part vanishes faster than $j(\beta)$ at infinity. The equivalent equation is

$$\frac{d^2E}{d\beta^2}+\frac{E}{4\beta}=j(\beta)+\frac{E}{4\beta}. \quad (48)$$

However, a Green's function built from the homogeneous solutions given above has singular behavior near the origin. Thus we write the equation in a new form again,

$$\frac{d^2E}{d\beta^2}+f(\beta)=j(\beta)+f(\beta)=h(\beta), \quad (49)$$

where

$$f(\beta)=(E/4\beta)[1+H(\beta-\beta_0)-H(\beta+\beta_0)] \quad (50)$$

and $H(x)$ is the Heaviside step function. The Green's function of this equation is not singular near the origin. We are looking for a solution that does not grow exponentially at negative infinity. The general solution of this form is

$$E=W_1(\beta)+\int_{-\infty}^{\infty}d\beta'\,G(\beta,\beta')h(\beta'), \quad (51)$$

where the Green's function G is

$$G(\beta,\beta')=\begin{cases}[g_1(\beta)g_2(\beta')]/[q(\beta')], & \beta<\beta',\\ [g_2(\beta)g_1(\beta')]/[q(\beta')], & \beta>\beta',\end{cases} \quad (52)$$

and g_1 and g_2 are solutions of the homogeneous equation $d^2E/d\beta^2+f(\beta)=0$. These solutions are

$$g_i(\beta)=\begin{cases}W_i(\beta), & \beta<-\beta_0,\\ a_i\beta+b_i, & |\beta|<\beta_0,\quad i=1,2,\\ c_iW_1(\beta)+d_iW_2(\beta), & \beta>\beta_0.\end{cases} \quad (53)$$

The constants a_i, b_i, c_i, and d_i are found by requiring continuity of the functions g_i and their first derivatives at $\beta=\pm\beta_0$. The Wronskian $q(\beta)$ is constant, $q(\beta)=b_1a_2-b_2a_1$. The asymptotic solutions are

$$\begin{aligned}E&=W_1(\beta)+[c_2W_1(\beta)+d_2W_2(\beta)]\\ &\quad\times\int_{-\infty}^{\infty}d\beta'\,\frac{g_1(\beta')}{q}h(\beta'),\quad \beta\to\infty,\\ E&=W_1(\beta)+W_1(\beta)\\ &\quad\times\int_{-\infty}^{\infty}d\beta'\,\frac{g_2(\beta')}{q}h(\beta'),\quad \beta\to-\infty,\end{aligned} \quad (54)$$

which correspond to an exponentially decaying wave in the low-field side, and to incoming and reflected waves in the high-field side. The relative reflection coefficient is

$$\begin{aligned}R&=\lim_{\beta\to\infty}\{d_2[E(\beta)-W_1(\beta)]/[g_2(\beta)+c_2E(\beta)\\ &\quad-c_2W_1(\beta)]\}.\end{aligned} \quad (55)$$

Equation (51) is an integral equation. As such, it is more convenient to write it in the following form,

$$\begin{aligned}E(\beta)&=W_1(\beta)+\int_{-\infty}^{\infty}d\beta''\,E(\beta'')\kappa_0(\beta,\beta'')\\ &\quad+\int_{-\infty}^{\infty}d\beta''\frac{d}{d\beta''}\left(\frac{E(\beta'')}{\beta''}\right)\kappa_1(\beta,\beta'').\end{aligned} \quad (56)$$

The first kernel is

$$\begin{aligned}k_0(\beta,\beta'')&=\left(\frac{i}{2\sqrt{\pi p}}\right)\int_{-\infty}^{\infty}d\beta'\,G(\beta,\beta')F_R(y)\\ &\quad+\left(\frac{1}{2\sqrt{\pi p}}\right)\int_{-\infty}^{\beta_1}d\beta'\,G(\beta,\beta')F_I(y)\\ &\quad-\left(\frac{1}{2\sqrt{\pi p}}\right)\int_{\beta_1}^{\infty}d\beta'\,G(\beta,\beta')F_I(y),\end{aligned} \quad (57)$$

where F_R and F_I are, respectively, the real and imaginary parts of the function F defined above (39), and

$$y=(\beta'^2-\beta''^2)/p, \quad (58)$$

$$\beta_1=\begin{cases}\mathrm{sgn}(\beta'')\beta_0, & |\beta''|>\beta_0,\\ \beta'', & |\beta''|<\beta_0.\end{cases} \quad (59)$$

The second kernel is

$$k_1(\beta,\beta'')=\begin{cases}\dfrac{1}{2\sqrt{\pi}}\displaystyle\int_{\beta''}^{-\beta_0}d\beta'\,G(\beta,\beta')F_1(y), & \beta''<-\beta_0,\\ 0, & |\beta''|<\beta_0,\\ \dfrac{1}{2\sqrt{\pi}}\displaystyle\int_{\beta_0}^{\beta''}d\beta''\,G(\beta,\beta')F_1(y), & \beta''>\beta_0,\end{cases} \quad (60)$$

where y denotes the same as above and the function F_1 is

$$F_1(y)=\int_0^{\infty}dt\,e^{-t^2}\cos\left(\frac{y}{t}\right). \quad (61)$$

Equation (56) is approximated as a set of algebraic equations. We divide the interval $[-L/2, L/2]$ into N equal subintervals, each of length $\Delta\beta=L/N$, and denote $\beta_{j+1}=\beta_j+\Delta\beta$, where $\beta_0=-L$. We solve the $N+1$ coupled algebraic equations

$$\begin{aligned}E(\beta_n)&=W_1(\beta_n)+\Delta\beta\sum_{m=1}^{N+1}E(\beta_m)k_0(\beta_n,\beta_m)\\ &\quad+\frac{1}{2}\sum_{m=2}^{N}\left(\frac{E(\beta_{m+1})}{\beta_{m+1}}-\frac{E(\beta_{m-1})}{\beta_{m-1}}\right)k_1(\beta_n,\beta_m),\\ n&=1,\ldots,N+1,\end{aligned} \quad (62)$$

for which the kernels are calculated at $(N+1)^2$ points. The kernels at each point are found by a double integration. Their calculation imposes a computing time problem when the number of subintervals N is relatively large. In order to save computing time we calculate the values of the functions F and F_1 for a large number of points and store them. Then during the calculation of the kernel we perform only one integration. The value of the function of F or F_1 is found by using the vector of stored numbers and a spline method, instead of performing an integration each time.

The values of the function $F(y)$ are found using the fact that this function is a solution of the third-order differential equation,

$$y\frac{d^3F}{dy^3}+2\frac{d^2F}{dy^2}-2F=0. \quad (63)$$

Using numerical integrations we find the values of F and its first and second derivatives at $y=i$. Then, with these initial values, we find the values of F on the real axis by employing a standard ODE solver. This method is not accurate for large values of the argument, since the ODE solver picks the exponentially growing solution of Eq. (63), while the function F is a decaying solution of this equation. Thus, for large values of the argument we integrate Eq. (63) from infinity, choosing appropriate initial values.

The function $F_1(y)$ can be easily calculated by using the equality

$$\frac{dF_1}{dy} = -\operatorname{Im}(F). \qquad (64)$$

The electric field profile of the wave was found for several values of the parameter p by solving Eq. (51) and the reflection coefficient R was calculated using Eq. (55). In all the cases, R was found to be zero, which means that to lowest order, all the incoming energy flux is absorbed. The full solution of the Maxwell–Vlasov equations for the warm plasma thus justifies the cold plasma calculation of the heating. The cold plasma equation (40) has a solution of total absorption for waves coming from the high-field side. An analytic proof that the reflection coefficient is zero indeed is given elsewhere.[5] It is interesting to note that for this specific case, where ϵ_1 equals ϵ_2^3, there is a strong heating mechanism in the fundamental resonance for a one-species plasma. The rate of heating is proportional to the plasma density, the wave energy density and the reciprocity of the magnetic field gradient magnitude at the resonance surface [Eq. (47)]. This heating has not been observed experimentally probably because in present day tokamaks there is a different relation between ϵ_1 and ϵ_2.

In addition to the total amount of absorption, the energy deposition profile is also of major importance. We present the numerical values of the electric field, the energy flux and the absorption rate across the boundary layer when the parameter p takes the values 50 and $2\times 10.^4$ The quantities are plotted versus the normalized coordinate β. The relation between β and ξ may be written in the form

$$\xi - 1 = \beta(c/\omega_p)^2(\hat{B}\cdot\nabla\xi/2)^2, \qquad (65)$$

where the gradient is in cgs units. Note that this relation does not depend on the ion temperature. Thus, increasing the value of p by increasing the temperature only, means keeping constant the ratio of $1-\xi$ and β. The figures in this case show that influence of the temperature on the interaction. In Figs. 1 and 2 the real and imaginary parts of the electric field are compared with the cold plasma electric field, the Hankel function. The deviations of the electric field profiles from the cold plasma fields grow as p increases. Figure 3 shows the energy flux magnitude. For the cold plasma case the Poynt-

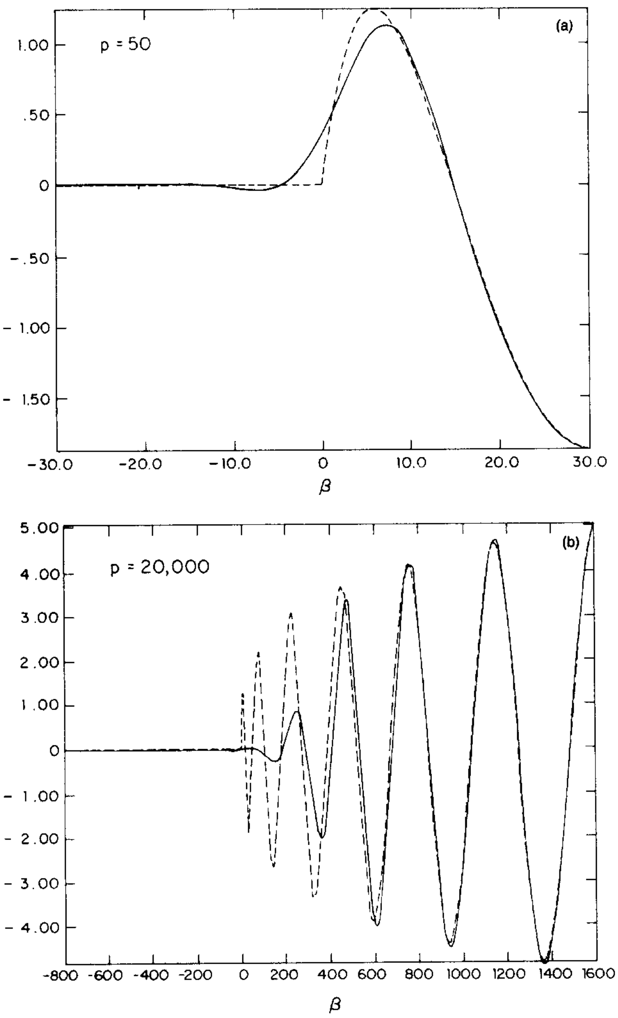

FIG. 1. The real part of the electric field (solid line) versus the real part of $H_1^{(1)}(\beta)$ (dotted line) near the resonance. (a) $p = 50$, (b) $p = 20\,000$.

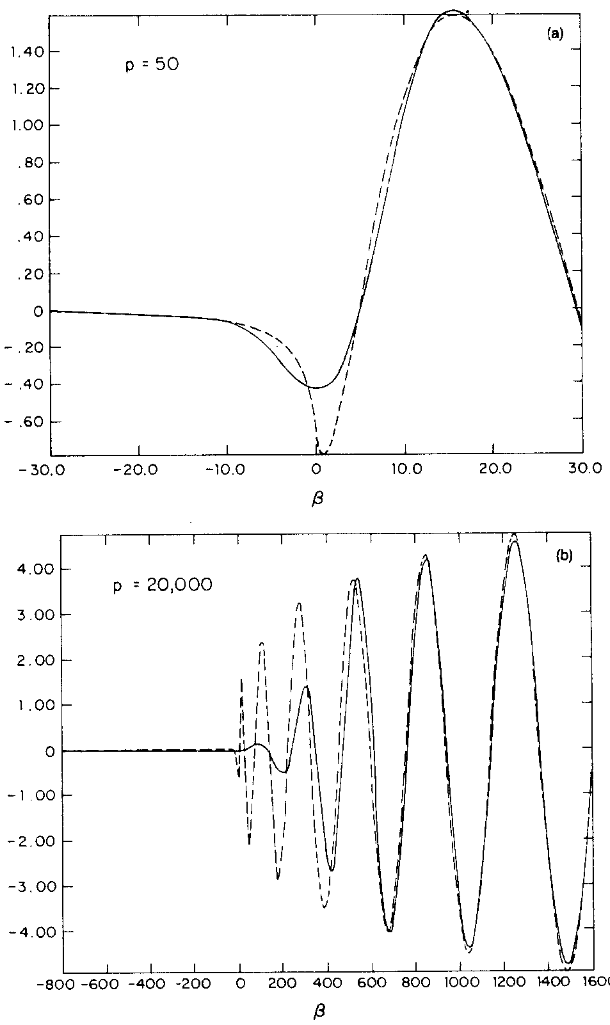

FIG. 2. The imaginary part of the electric field (solid line) versus the imaginary part of $H_1^{(1)}(\beta)$ (dotted line). (a) $p = 50$, (b) $p = 20\,000$.

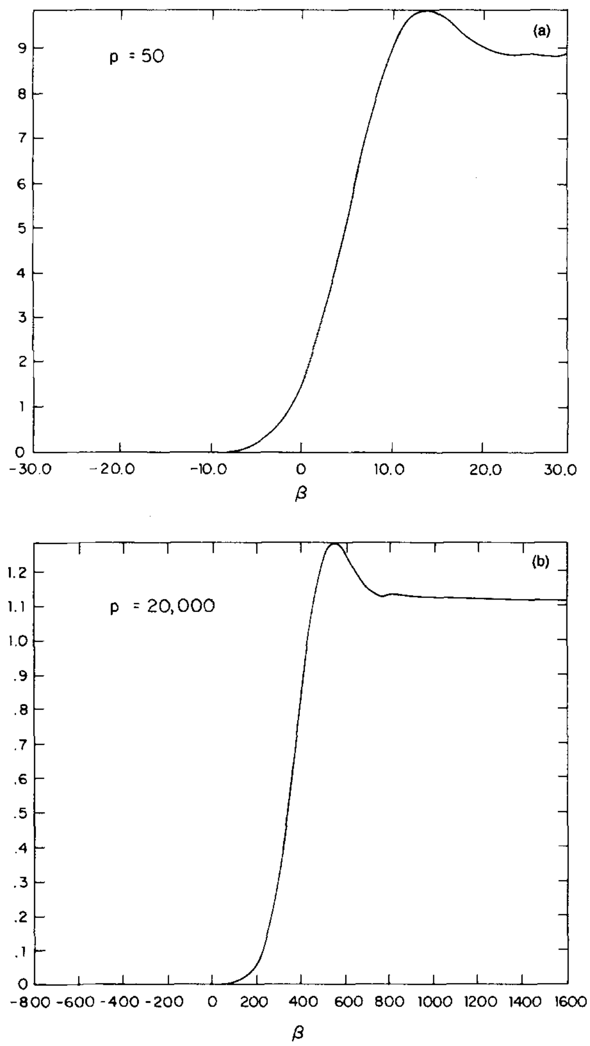

FIG. 3. The energy flux near the resonance. (a) $p=50$, (b) $p=20\,000$.

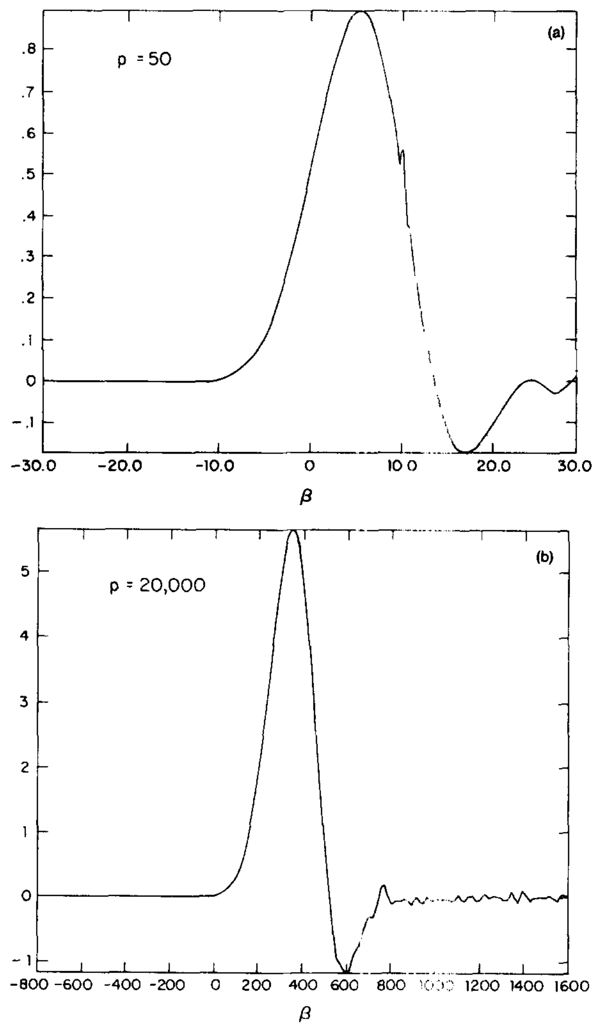

FIG. 4. The rate of absorption near the resonance. (a) $p=50$, (b) $p=20\,000$.

ing vector magnitude is a step function, a finite constant flux for positive β and zero flux for negative β. In the warm plasma case the flux is continuous and changes over a broader interval as p gets larger. The same feature is observed in Fig. 4, where the absorption rate, found by taking the derivative of the energy flux, is plotted. In both Figs. 3 and 4 we notice that while broadening with increasing p, the regime where absorption occurs also moves to the high-field side region. For large values of p most of the absorption takes place before the wave reaches the resonance layer. We also notice that there are regimes where the particles emit radiation rather than absorb it. This phenomena is related to the nonlocal character of the current. On the whole, the particles absorb energy from the wave and do not emit energy, as expected.

To sum up this case, there is a strong heating mechanism at the fundamental cyclotron frequency, whose origin is the finite parallel stratification.

V. SMALLER POLOIDAL FIELD

First we study the case when ϵ_2 is still larger than ϵ_1 but ϵ_1 is larger than ϵ_2^3. In this case the solution of the Vlasov equation in the boundary layer is the same as before [Eq. (30)]. The form of the Maxwell equations, however, is different. In order to balance J_+ in the Maxwell equations, the electric field E_+ has to be small in the boundary layer. To lowest order E_+ becomes

$$E_+(\bar{\xi},\eta)=\epsilon_3 E_+^{(1)}(\bar{\xi},\eta), \tag{66}$$

and the current J_+ is of order 1. Using (33) we find that E_- is constant to lowest order,

$$E_-(\bar{\xi},\eta)=E_-^{(0)}(\eta)+\epsilon_3 E_-^{(1)}(\bar{\xi},\eta). \tag{67}$$

Since the currents are at most of order 1, $E_1^{(1)}$ and $E_2^{(1)}$ obey the same relation as $E_1^{(0)}$ and $E_2^{(0)}$, so that

$$\nu E_2^{(1)}\cos\alpha - kE_1^{(1)} = a^{(1)}(\eta)+b^{(1)}(\eta)\bar{\xi}. \tag{68}$$

Here we allow linear dependence, since such dependence of the first-order field does not cause divergence of the fields at the edges of the boundary layer. To lowest order the term

$$\nabla\times(\nabla\times\mathbf{E})(\nu\cos\alpha\bar{V}_1+k\hat{V}_2) \tag{69}$$

does not contain derivatives with respect to $\bar{\xi}$ and is thus constant. The current J_- is also constant since $E_-^{(0)}$ is constant. The Maxwell equations thus yield

$$J_+ = \int_0^\infty du_\perp \int_{-\infty}^\infty du_\parallel\, 2\pi u_\perp \frac{G_0}{(u_\parallel k\bar{\alpha})} \times \int_{-\infty\,\mathrm{sgn}(u_\parallel k\bar{\alpha})}^{\bar{\xi}} dx\, E_+^{(1)}(x,\eta) \exp\left(\frac{\bar{\xi}^2 - x^2}{-2iu_\parallel k\bar{\alpha}}\right) = \text{const.} \tag{70}$$

An electric field that obeys this equation is

$$E_+^{(1)} = A(\eta)\bar{\xi}. \tag{71}$$

The total heating is

$$\int_{-\infty}^\infty d\bar{\xi}\, \mathrm{Re}(E_+^{(1)*} J_+) \propto \int_{-\infty}^\infty d\bar{\xi}\, \bar{\xi} = 0. \tag{72}$$

To the order of ϵ_3^2 ($= \epsilon_1\epsilon_2$) the heating is zero, so that nonzero heating may be found only at higher order. One may note that the net zero absorption here is composed of two equal parts of both absorption and emission. Again the source of emission is the nonlocal form of the current.

To the next order, the fields and distribution function are

$$g_1 = g_1^{(0)} + (\epsilon_1/\epsilon_3) g_1^{(1)},$$
$$E_+ = \epsilon_3 E_+^{(1)} + \epsilon_1 E_+^{(2)}, \tag{73}$$
$$E_- = E_-^{(0)}(\eta) + \epsilon_3 E_-^{(1)} + \epsilon_1 E_-^{(2)}.$$

Substituting g_1 and E_+ into the Vlasov equation, we have

$$g_1^{(1)} = -\left(\frac{\omega_p^2}{2\omega}\right)\left(\frac{G_{0,u_\perp}}{ku_\parallel}\right) \int_{-\infty\,\mathrm{sgn}(ku_\parallel)}^{\bar{\xi}} dx\, E_+^{(2)}(x) \times \exp\left(\frac{\bar{\xi}^2 - x^2}{-2iku_\parallel}\right) - \frac{(O_1^0 g_1^{(0)} + O_1^{-2} g_{-1}^{(0)})}{ku_\parallel} \times \int_{-\infty\,\mathrm{sgn}(ku_\parallel)}^{\bar{\xi}} dx\, \exp\left(\frac{\bar{\xi}^2 - x^2}{-2iku_\parallel}\right). \tag{74}$$

Collecting terms of order ϵ_1/ϵ_3 in Maxwell's equations, we have

$$\nabla\times\nabla\times\mathbf{E}\cdot(\nu\cos\alpha\hat{V}_1 + k\hat{V}_2) = \text{const} = \nu\cos\alpha J_1 + kJ_2. \tag{75}$$

Expressing J_+ via the integral of $g_1^{(1)}$ and substituting it into the last equation, we derive the following integral equation for E_+:

$$\int_0^\infty du_\perp \int_{-\infty}^\infty du_\parallel\, 2\pi u_\perp \frac{G_0}{ku_\parallel} \int_{-\infty\,\mathrm{sgn}(ku_\parallel)}^{\bar{\xi}} dx \times E_+^{(2)}(x,\eta) \exp\left(\frac{\bar{\xi} - x^2}{-2iku_\parallel}\right) = C + F(\bar{\xi}), \tag{76}$$

where

$$F(\bar{\xi}) = \frac{\omega}{2\omega_p^2} \int\int \frac{2\pi u_\perp du_\perp du_\parallel}{ku_\parallel} (O_1^0 g_1^{(0)} + O_1^{-2} g_{-1}^{(0)}) \times \int_{-\infty\,\mathrm{sgn}(ku_\parallel)}^{\bar{\xi}} dx\, \exp\left(\frac{\bar{\xi}^2 - x^2}{-2iku_\parallel}\right). \tag{77}$$

The heating resulting from products of $E_+^* J_+$ is of order ϵ_1 and the total heating is of order $\epsilon_1\epsilon_3$ after multiplying by the width of the boundary layer, namely of order $\epsilon_1^{3/2}\epsilon_2^{1/2}$.

We do not solve the equation for E_1 explicitly in this case, but estimate the order of the heating rate and find it to be small. The heating is smaller than in the case of higher poloidal field since the parallel stratification is small.

Next we discuss the case of even smaller poloidal magnetic field, when $\epsilon_2 \ll \epsilon_1$. In this case the parallel stratification is too small to provide the heating mechanism described in the previous sections. The equations for the various Fourier components g_n are coupled and a simple approximate solution is not available. However, the decrease in the absorption rate with the decrease of the poloidal field found when $\epsilon_2 > \epsilon_1$ indicates that the heating is small. An analysis of electron-cyclotron resonance heating for perpendicular stratified plasma[6] showed that for such geometry, $E_\parallel$ is the source of heating. This parallel component of the wave electric field is zero in the ICRH case. This may be an additional indication that there is no heating when the poloidal magnetic field is small.

Usually in tokamaks the poloidal field is small, which explains why heating is not observed experimentally at the fundamental resonance.

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