Wiggler-free free electron waveguide laser in a uniform axial magnetic field: Single particle treatment

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A wiggler-free free electron laser operating in a waveguide is analyzed by using a single particle treatment. The use of either a TE or a TM mode is shown to enhance the gain for a resonant frequency much higher than the cyclotron frequency. It is demonstrated that a source of a submillimeter radiation, based on this analysis, may have output power comparable to that of a wiggler-type free electron laser.

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I. INTRODUCTION

Considerable effort has been made in recent years to develop sources of coherent radiation, using relativistic electron beams moving along helical trajectories. The radiation wavelength in these so-called "free electron lasers" (FELS) is the Doppler-shifted pitch of the electron motion $\lambda \simeq \lambda_0/2\gamma^2$ where λ_0 is the electron pitch, $\gamma = [1 - (v/c)^2]^{-1/2}$ is the relativistic factor, and v is the velocity of the beam. One class of such devices is the wiggler-type free electron laser,[1] where a periodic magnetic structure forces the electrons into helical motion. Most of the experimental and theoretical research up to now has been aimed at this type of FEL.

Recently interest arose in a second class of FEL, the "wiggler-free free electron lasers." Here the electrons move on helical orbits in a simple uniform magnetic field (which is different from the longitudinally[2] or transversally[3] modulated axial magnetic field). In contrast to the gyrotron,[4] the frequency here is the Doppler up-shifted cyclotron frequency. Chu and Hirshfield[5] treated the collective interaction and showed the existence of an unstable growing mode. They also compared in detail the two bunching mechanisms. Later it was demonstrated that gain enhancement can be achieved by a careful choice of the electron momentum distribution function.[6]

The various gain mechanisms were clearly explained using a single-particle approach.[7,8] Ride and Colson[7] showed that two sources of bunching exist as a result of the electron-wave interaction. One source of bunching is the ponderomotive force due to the product of the perpendicular component of the electron equilibrium velocity and the magnetic vector of the electromagnetic wave. The second source of bunching is the modulation of the cyclotron frequency due to the relativistic change of the electron mass. Each one of the sources causes gain proportional to L^3 (L is the length of the amplifier), but acting simultaneously they nearly cancel each other. There remains a lower order gain proportional to L^2.

In all these previous papers the wave was assumed to propagate parallel to the direction of the uniform magnetic field. In a realizable device there must be a waveguide within which the radiation propagates. Ott and Manheimer published a collective theory for a thin slab beam in a parallel plate waveguide.[9] The difference between bunching mechanisms for TE and TM modes which we describe below using a single-particle model are not easily identified in their treatment. Moreover for practical devices the applicability of the thin beam model may be limited. The main role of this paper is to study the influence of the waveguide modes on the interaction within the framework of a single-particle approach. The two aforementioned sources of bunching which cancel that part of the gain proportional to L^3 will be shown here not to do so when, as in the case of waveguide modes, propagation is not exactly parallel to the magnetic field. There is a residual term proportional to L^3, similar to the case of a wiggler-type free electron laser. Thus the use of waveguide modes may enhance the gain. This enhanced gain mechanism can be exploited for the design of a practical device for submillimeter wave generation or amplification within the constraints of a single-particle interaction. A practical example, similar to that in Ref. 9, will be described based on the present analysis.

Electron beam sources of radiation for the submillimeter portion of the spectrum usually employ high current densities, where collective effects play an important role. The present single-particle calculation, by describing clearly the physical picture, may be used as an important first step for a self-consistent collective description in future work.

II. THE EQUATIONS OF MOTION

A relativistic electron beam is guided by a uniform magnetic field along a waveguide within which an electromagnetic wave propagates in the same direction. The gain is found by calculating the energy loss of the electrons as they pass through the structure. In doing it, two assumptions are used. The first is that the intensity of the radiation is big enough (or the electron density low enough), so that the wave amplitude remains constant. Secondly we assume that the intensity of the radiation relative to the magnetostatic field is small enough to allow the use of a perturbation method to solve the electron equations of motion. The uniform magnetic field is

$$\mathbf{B}_0 = B_0 \mathbf{e}_z. \tag{1}$$

For simplicity we choose a waveguide made of two infinite plane parallel plates with distance a between them. The wave is assumed to be coherent and is either a TE or a TM mode. Its components are[10]

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$$\left.\begin{aligned} E'_y &= -(A'\omega'/k_\perp c)\cos(k_\perp x)\cos(\beta z - \omega' t),\\ B'_x &= (A'\beta/k_\perp)\cos(k_\perp x)\cos(\beta z - \omega' t),\\ B'_z &= A'\sin(k_\perp x)\sin(\beta z - \omega' t),\end{aligned}\right\}\ \text{TE mode,}$$

$$\left.\begin{aligned} E'_z &= A'\sin(k_\perp x)\sin(\beta z - \omega' t),\\ E'_x &= (\beta/k_\perp)A'\cos(k_\perp x)\cos(\beta z - \omega' t),\\ B'_y &= (\omega'/k_\perp c)A'\cos(k_\perp x)\cos(\beta z - \omega' t).\end{aligned}\right\}\ \text{TM mode.} \tag{2}$$

ω' is the wave frequency, x is the coordinate perpendicular to the plates, and β and $k_\perp$ are the components of the wave vector related by

$$(\omega'/c)^2 = k_\perp^2 + \beta^2. \tag{3}$$

$k_\perp$ will have discrete values

$$k_\perp = (n\pi/a) \quad n = 1,2\ldots . \tag{4}$$

The equation of motion of the electron is

$$\frac{d}{dt}(\gamma \mathbf{v}) = -\frac{e}{mc}\mathbf{v}\times(\mathbf{B}_0 + \mathbf{B}') - \frac{e}{m}\mathbf{E}'. \tag{5}$$

e and m are the electron charge and mass, respectively. The equation of motion is easily solved by using a rotating system of coordinates which is better suited to this problem, because of the helical nature of the electron orbit. A similar system of coordinates was used previously in dealing with the wiggler-type FEL problem.[11,12] For an electron, whose perpendicular velocity in the entrance makes an angle ψ_0 with the negative x axis, we define

$$\begin{aligned} \mathbf{e}_1(z,\psi_0) &= -\mathbf{e}_x \sin(k_0 z + \psi_0) + \mathbf{e}_y \cos(k_0 z + \psi_0),\\ \mathbf{e}_2(z,\psi_0) &= -\mathbf{e}_x \cos(k_0 z + \psi_0) - \mathbf{e}_y \sin(k_0 z + \psi_0),\\ \mathbf{e}_3(z,\psi_0) &= \mathbf{e}_z .\end{aligned} \tag{6}$$

k_0 is chosen later. Let us use the following notations:

$$\begin{aligned} &\mathbf{u} = \mathbf{v}/c, \quad \tau' = tc, \quad \mathbf{E} = e\mathbf{E}'/mc^2,\\ &\mathbf{B} = e\mathbf{B}'/mc^2, \quad \omega = \omega'/c, \quad A = eA'/mc^2,\\ &\Omega = eB_0/mc^2.\end{aligned} \tag{7}$$

With these notations Eq. (5) becomes

$$\begin{aligned} \dot u_1 &= u_2(k_0 u_3 - \Omega/\gamma) - \frac{u_1}{\gamma}\dot\gamma + \frac{1}{\gamma}(u_3 B_2 - u_2 B_3 - E_1),\\ \dot u_2 &= -u_1(k_0 u_3 - \Omega/\gamma) - \frac{u_2}{\gamma}\dot\gamma - \frac{1}{\gamma}(u_3 B_1 - u_1 B_3 + E_2),\\ \dot u_3 &= -\frac{u_3}{\gamma}\dot\gamma - \frac{1}{\gamma}(u_1 B_2 - u_2 B_1 + E_3),\end{aligned} \tag{8}$$

where

$$\begin{aligned} &E_3 = 0, \quad \text{TE mode},\\ &B_3 = 0, \quad \text{TM mode},\end{aligned} \tag{9}$$

and the dot represents differentiation with respect to τ'. Conservation of energy dictates that the energy change of the electrons equals the work done by the wave fields:

$$\dot\gamma = -\mathbf{u}\cdot\mathbf{E}. \tag{10}$$

The components of the wave in the rotating system of coordinates are

$$\left.\begin{aligned} E_1 &= -(A\omega/2k_\perp)\cos(k_\perp x)\cos[(\beta + k_0)z - \omega(\tau + \tau_0) + \psi_0],\\ E_2 &= (A\omega/2k_\perp)\cos(k_\perp x)\sin[(\beta + k_0)z - \omega(\tau + \tau_0) + \psi_0],\\ B_1 &= -(\beta/\omega)E_2, \quad B_2 = (\beta/\omega)E_1,\\ B_3 &= A\sin(k_\perp x)\sin[\beta z - \omega(\tau + \tau_0)],\end{aligned}\right\}\ \text{TE mode,}$$

$$\left.\begin{aligned} E_1 &= -(A\beta/2k_\perp)\cos(k_\perp x)\sin[(\beta + k_0)z - \omega(\tau + \tau_0) + \psi_0],\\ E_2 &= -(A\beta/2k_\perp)\cos(k_\perp x)\cos[(\beta + k_0)z - \omega(\tau + \tau_0) + \psi_0],\\ B_1 &= -(\omega/\beta)E_2, \quad B_2 = (\omega/\beta)E_1,\\ E_3 &= A\sin(k_\perp x)\sin[\beta z - \omega(\tau + \tau_0)].\end{aligned}\right\}\ \text{TM mode.} \tag{11}$$

Terms which oscillate with high frequency were omitted keeping only terms which might be resonant. τ_0 is the time the electron is at $z = 0$, and τ is the time which has passed since then.

The equations of motion (8) are solved perturbatively. First we find the steady-state electron orbit in the absence of the wave. Then its perturbed velocity and position are calculated when the EM fields are taken along the steady-state orbits. The energy transfer is found only in the second order.

To zero order there are no wave fields:

$$\mathbf{E} = \mathbf{B} = 0, \quad \gamma = \gamma_0. \tag{12}$$

The equations of motion are

$$\begin{aligned} \dot u_{10} &= u_{20}(k_0 u_{30} - \Omega/\gamma_0),\\ \dot u_{20} &= u_{10}(k_0 u_{30} - \Omega/\gamma_0),\\ \dot u_{30} &= 0.\end{aligned} \tag{13}$$

The third of these equations yields $u_3 = u_{30} = \text{const}$. The definition of our rotating system of coordinates is completed by setting $k_0 = \Omega/\gamma_0 u_{30}$, in which case u_{10} and u_{20} are constant too. We are still free to choose u_{10} and u_{20}, with ψ_0 determining the initial velocity of each electron. For our convenience, u_{10} is set equal to 0, in which case for each electron $u_2 = u_{20}$ and $\mathbf{e}_2$ is always in the direction of the perpendicular velocity. This means that $\mathbf{e}_i$ can be different for different electrons. The electrons are assumed to enter the waveguide with the same velocity components parallel and perpendicular to the magnetic field. The solution of Eq. (13) is therefore

$$u_{10} = 0, \quad u_{20} = \text{const}, \quad u_{30} = \text{const}, \qquad (14)$$
$$\gamma_0 = (1 - u_{20}^2 - u_{30}^2)^{-1/2}.$$

The x and z coordinates of the electron position are to zero order

$$z_0(\tau) = u_{30}\tau,$$
$$x_0 = (\tau) = x_e - r_0 \sin(k_0 u_{30}\tau + \psi_0), \qquad (15)$$
$$r_0 = u_{20}/k_0 u_{30}.$$

r_0 is the Larmor radius. From now on assume

$$k_\perp r_0 \ll 1. \qquad (16)$$

Due to Eq. (16) we approximate the amplitude of the wave in the first-order equations of motion

$$\cos(k_\perp x_0) = \cos \rho_e,$$
$$\sin(k_\perp x_0) = \sin \rho_e, \qquad (17)$$
$$\rho_e = k_\perp x_e,$$

and exclude the excitation of higher cyclotron harmonics. The fields along the steady-state trajectories are

$$E_{10} = -(E_0/2)\sin(\nu\tau + \xi),$$
$$E_{20} = -(E_0/2)\cos(\nu\tau + \xi), \qquad (18)$$

where

$$\left.\begin{aligned} E_0 &= A(\omega/k_\perp)\cos\psi_e, \\ \xi &= -\omega\tau_0 + \psi_0 + \pi/2, \end{aligned}\right\} \text{TE mode},$$
$$\left.\begin{aligned} E_0 &= A(\omega/k_\perp)\cos\psi_e, \\ \xi &= -\omega\tau_0 + \psi_0. \end{aligned}\right\} \text{TM mode}. \qquad (19)$$

The "resonance parameter" ν is

$$\nu = (\beta + k_0)u_{30} - \omega. \qquad (20)$$

The interaction between the electrons and the wave fields is strongest when the resonance condition is fulfilled, namely when $\nu \simeq 0$. In order that the resonant frequency will be high we require that $\beta \gg k_\perp$ and that $u_{30} \gg u_{20}$.
Then

$$\omega \simeq \frac{k_0}{1 - u_{30}} \simeq 2k_0\gamma_0^2. \qquad (21)$$

B_{30} or E_{30} were omitted because they oscillate with high frequency. We linearize the electron velocity and energy.

$$u_1 = w_1(\tau,\tau_0),$$
$$u_2 = u_{20} + w_2(\tau,\tau_0), \qquad (22)$$
$$u_3 = u_{30} + w_3(\tau,\tau_0),$$
$$\gamma = \gamma_0 + \Gamma(\tau,\tau_0).$$

Next we write the equations of motion for these perturbed quantities

$$\dot{w}_1 = k_0 u_{20} w_3 + k_0 u_{20} u_{30}\frac{\Gamma}{\gamma_0} + \frac{E_{10}}{\gamma_0}(g u_{30} - 1),$$
$$\dot{w}_2 = -\frac{u_{20}}{\gamma_0}\dot{\Gamma} + \frac{E_{20}}{\gamma_0}(g u_{30} - 1), \qquad (23)$$
$$\dot{w}_3 = -\frac{u_{30}}{\gamma_0}\dot{\Gamma} + \frac{B_{10}}{\gamma_{10}}u_{20},$$

where

$$g = \beta/\omega, \quad \text{TE mode},$$
$$g = \omega/\beta, \quad \text{TM mode}. \qquad (24)$$

The third of Eqs. (23) shows that the longitudinal velocity is perturbed by two forces. The first term on the right-hand side of this equation gives rise to the cyclotron maser instability. Its origin lies in the relativistic change of mass of the electrons. The second term represents the ponderomotive force of the magnetic component of the wave on the perpendicular velocity of the electron. This force drives the Weibel-type instability. A detailed comparison of these two bunching mechanisms was given by Chu and Hirshfield[5] and also by Ride and Colson.[7] To first order the energy Eq. (10) is

$$\dot{\Gamma}_1 = -u_{20}E_{20}. \qquad (25)$$

The solutions of Eq. (23) using Eq. (25) are

$$\dot{w}_1 = \left(\frac{E_0}{2\gamma_0}\right)\left(\frac{S_1}{\nu^2}\right)[\cos\xi - \cos(\nu\tau + \xi) - \nu\tau\sin\xi] + \left(\frac{E_0}{2\gamma_0}\right)(1 - g u_{30})\tau\sin\xi,$$
$$\dot{w}_2 = \left(\frac{E_0}{2\gamma_0}\right)\left(\frac{S_2}{\nu}\right)[\sin(\nu\tau + \xi) - \sin\xi],$$
$$\dot{w}_3 = \left(\frac{E_0}{2\gamma_0}\right)\left(\frac{S_3}{\nu}\right)[\sin(\nu\tau + \xi) - \sin\xi], \qquad (26)$$
$$S_1 = \nu(1 - g u_{30}) + k_0 u_{20}^2 g,$$
$$S_2 = 1 - g u_{30} - u_{20}^2,$$
$$S_3 = u_{20}(g - u_{30}).$$

III. THE ENERGY GAIN

We solved the equations of motion to first order. This enables us to calculate the net energy loss of the electrons to second order, which is the lowest order where it does not vanish. To second order Eq. (10) is

$$\dot{\Gamma} = -w_1 E_{10} - w_2 E_{20} - u_{20}E_{21} - w_3 E_{30} - u_{30}E_{31}. \qquad (27)$$

For the TM mode $E_{30} = E_{31} = 0$. The net energy transfer is found by averaging on τ_0, the time of entrance ($\langle \ldots \rangle$ denotes this averaging). It is in fact averaging on ξ, which means that the distribution of ψ_0 is irrelevant. This distribution has an influence on higher cyclotron harmonics; it also can be important when collective effects become dominant.[6]

Thus

$$\langle -w_1E_{10}\rangle = \left(\frac{E_0^2}{8\gamma_0}\right)\left[\left(\frac{S_1}{\nu^2}\right)(\sin\nu\tau - \nu\tau\cos\nu\tau) + (1-gu_{30})\tau\cos\nu\tau\right],$$

$$\langle -w_2E_{20}\rangle = \left(\frac{E_0^2}{8\gamma_0}\right)\left(\frac{S_2}{\nu}\right)\sin\nu\tau. \quad (28)$$

The resonant term in E_{21} is due to modulations in axial position Δz.

$$\Delta z = \int_0^\tau w_3 d\tau = \left(\frac{E_0}{2\gamma_0}\right)\left(\frac{S_3}{\nu^2}\right)[-\nu\tau\sin\xi - \cos(\nu\tau+\xi) + \cos\xi]. \quad (29)$$

Using again the fact that the wave is only a perturbation on the steady-state orbit, E_{21} is

$$E_{21} = \left(\frac{E_0}{2}\right)(\beta + k_0)\Delta z\sin(\nu\tau+\xi), \quad (30)$$

and the energy transfer is

$$\langle -u_{20}E_{21}\rangle = \left(\frac{E_0^2}{8\gamma_0}\right)\left(\frac{u_{20}S_3}{\nu^2}\right)\times(k_0+\beta)(\nu\tau\cos\nu\tau - \sin\nu\tau). \quad (31)$$

Adding the terms in Eqs. (28) and (31) we obtain the contribution to total energy transfer from the perpendicular part of the radiation

$$\langle\dot{\Gamma}_{2_1}\rangle = \frac{1}{\nu}\left(\frac{E_0^2}{8\gamma_0}\right)[2(1-gu_{30})\sin\nu\tau - u_{20}^2\nu\tau\cos\nu\tau] + \frac{1}{\nu^2}\left(\frac{E_0^2}{8\gamma_0}\right)u_{20}^2(\omega-\beta g)(\sin\nu\tau - \nu\tau\cos\nu\tau). \quad (32)$$

The last expression is different for the two modes

$$\omega - \beta g = k_\perp^2/\omega, \quad \text{TE mode,}$$
$$\omega - \beta g = 0, \quad \text{TM mode.} \quad (33)$$

For the TE mode there remains the term proportional to $1/\nu^2$, whilst it vanishes for the TM mode. This residual term proportional to $1/\nu^2$ is the major contribution to the gain. Thus the gain for the TE mode is

$$\langle\dot{\Gamma}_2\rangle_{\text{TE}} = \left(\frac{E_0^2}{8\gamma_0}\right)u_{20}^2\frac{k_\perp^2}{\omega}\left(\frac{\sin\nu\tau - \nu\tau\cos\nu\tau}{2}\right). \quad (34)$$

When the wave propagates parallel to the magnetic field $k_\perp = 0$ and the gain is the first term in Eq. (32) only, and is proportional to $1/\nu$ instead of to $1/\nu^2$. This result, when $k_\perp$ is 0, agrees with Ride and Colson's result.[7] Thus the use of a TE mode may indeed enhance the gain.

It is interesting to note that in the opposite case, namely when $\beta = 0$ and $k_\perp = \omega$, our result for the TE mode gain agrees with the gain in the gyrotron.[7] In fact, being near cutoff the magnetic component of the wave is in the z direction only, and the ponderomotive force, which is one of the two bunching sources, vanishes. The second bunching source exists alone; this is the cyclotron maser bunching mechanism. This case does interest us since $\beta = 0$ gives no Doppler up-shift.

Let us now complete our study of the gain of the TM mode. Until now the gain for the TM case due to the work done on the electrons by the perpendicular fields is low and proportional to $1/\nu$ only. But for this mode there is still the work done by the axial electric field of the wave.

The work done on the electrons by the axial field is composed of two terms. The first term is $-w_3E_{30}$ and its average vanishes since E_{30} oscillates with high frequency. The second term is $u_{30}E_{31}$. E_{31} contains resonant terms. The perturbation on the axial field due to the perturbed trajectory is, after linearization

$$E_{31} = A(\cos\rho_e)(k_\perp\Delta x)\sin[(\beta u_{30}-\omega)\tau - \omega\tau_0]. \quad (35)$$

High frequency terms were omitted. Only terms linear in Δx or Δz were kept.

Using the rotating coordinate system Δx is

$$\Delta x = -\Delta x_1\sin(k_0u_{30}\tau + \psi_0) - \left(\frac{u_{20}}{u_{30}}\Delta x_3 + \Delta x_2\right)\cos(k_0u_{30}\tau + \psi_0). \quad (36)$$

Equations (35) and (36) yield for E_{31}

$$E_{31} = \frac{A}{2}(\cos\rho_e)k_\perp\left[\Delta x_1\cos(\nu\tau+\xi) - \left(\frac{u_{20}}{u_{30}}\Delta x_3 + \Delta x_2\right)\sin(\nu\tau+\xi)\right]. \quad (37)$$

The next step is to calculate the Δx_1 and Δx_2. Using the identities

$$\dot{\mathbf{e}}_1 = k_0u_3\mathbf{e}_2, \quad \dot{\mathbf{e}}_2 = -k_0u_3\mathbf{e}_1, \quad (38)$$

we obtain the equations

$$\Delta\dot{x}_1 = w_1 + k_0u_{30}\Delta x_2,$$
$$\Delta\dot{x}_2 = w_2 - k_0u_{30}\Delta x_1 - \frac{u_{20}}{u_{30}}w_3. \quad (39)$$

The solutions of these equations (keeping resonant terms only) are

$$\Delta x_1 = \left(\frac{E_0}{2\gamma_0}\right)\frac{(1-gu_{30})}{k_0u_{30}}\left[\frac{\sin(\nu\tau+\xi) - \sin\xi}{\nu}\right],$$

$$\Delta x_2 = \left(\frac{E_0}{2\gamma_0}\right)\left(\frac{1}{k_0u_{30}}\right)\times\left\{S_1\left[\frac{\cos(\nu\tau+\xi) - \cos\xi + \nu\tau\sin\xi}{\nu^2}\right] - (1-gu_{30})\tau\sin\xi\right\}. \quad (40)$$

Therefore the work done by the axial field is

$$\langle -u_{30}E_{31}\rangle = \left(\frac{E_0^2}{8\gamma_0}\right)\left(\frac{k_\perp^2}{\beta}\right)\times\left[u_{20}^2\left(\frac{\nu\tau\cos\nu\tau - \sin\nu\tau}{\nu^2}\right) - \frac{(1-gu_{30})}{k_0u_{30}}\frac{\sin\nu\tau}{\nu}\right]. \quad (41)$$

The main contribution to the gain comes from the term proportional to $1/\nu^2$. Thus the gain for the TM mode is

$$\langle\dot{\Gamma}_2\rangle_{\text{TM}} = \left(\frac{E_0^2}{8\gamma_0}\right)u_{20}^2\frac{k_\perp^2}{\beta}\left(\frac{\nu\tau\cos\nu\tau - \sin\nu\tau}{\nu^2}\right). \quad (42)$$

The gain for both the TE and the TM mode may be written in a similar form

$$\langle \dot{\Gamma} \rangle = \left(\frac{E_0^2}{8\gamma_0} \right) u_{20}^2 \frac{k_\perp^2}{P} \left(\frac{\nu\tau \cos \nu\tau - \sin \nu\tau}{\nu^2} \right), \qquad (43)$$

$P = -\omega$, TE mode,

$P = \beta$, TM mode.

The total energy loss of an electron along the amplifier is obtained by integrating $\langle \dot{\Gamma} \rangle$

$$\langle \Delta\gamma \rangle = \int_0^\tau \langle \dot{\Gamma} \rangle d\tau. \qquad (44)$$

The energy gain of the wave is the energy loss of all the electrons divided by the energy of the fields at $z = 0$ across the plates

$$G(\tau) = -\frac{\langle \Delta\gamma \rangle_T mc^2 N_0}{W_{n_0}}, \qquad (45)$$

where n_0 and W_{n_0} are the electron and the initial wave energy densities, respectively. $\langle \Delta\gamma \rangle_T$ is the sum of energy changes across the plates. The electron beam is assumed to fill uniformly the gap between the plates.

Thus

$$\langle \Delta\gamma \rangle_T = \int_{x'}^{x'+a} \langle \Delta\gamma \rangle dx_e,$$
$$W_{n_0} = \frac{(mc^2)^2 A^2}{e^2 16\pi} a \left(\frac{\omega}{k_\perp} \right)^2. \qquad (46)$$

Writing $L = u_{30}\tau$ where L is the length of the amplifier, the gain along the amplifier is

$$G(L) = \frac{1}{8} \frac{\omega_p^2}{c^2} \frac{k_\perp^2}{\gamma_0 \omega} \frac{u_{20}^2}{u_{30}^3} L^3 F'(\theta), \quad \text{TE mode},$$
$$G(L) = -\frac{1}{8} \frac{\omega_p^2}{c^2} \frac{\beta k_\perp^2}{\gamma_0 \omega^2} \frac{u_{20}^2}{u_{30}^3} L^3 F'(\theta), \quad \text{TM mode}, \qquad (47)$$

where $F'(\theta)$ is the line-shape function

$$F(\theta) = \left(\frac{\sin \theta}{\theta} \right)^2,$$
$$\theta = \frac{\nu\tau}{2}. \qquad (48)$$

The gain for the TE mode is higher by the factor (ω/β) than for the TM mode. In our case, far from cutoff $\beta \simeq \omega$, the gain in both cases is about the same. The form of the gain (47) is very similar to the form of gain obtained for the wiggler-type FEL.[13] As a matter of fact we can write a general expression for the gain in these devices.

$$G = \left(\frac{\omega_p^2}{8c^2\gamma_0} \right) \eta u_{20}^2 L^3 F'(\theta),$$
$$\eta_{\text{WFEL}} = k_0,$$
$$\eta_{TE} = k_\perp^2/\omega u_{30}^3,$$
$$\eta_{TM} = k_\perp^2/\omega^2 u_{30}^3. \qquad (49)$$

WFEL denotes wiggler-type FEL.

The gain in the proposed wiggler-free FEL is decreased relative to the wiggler-type by the factor

$$\frac{G_{\text{TE}}}{G_{\text{WFEL}}} = \frac{k_\perp^2}{\omega k_0 u_{30}^3} = \frac{k_0}{\omega u_{20}^2 u_{30}} (k_\perp r_0)^2. \qquad (50)$$

Since $k_0/\omega u_{20}^2 \gtrsim 1$ and $(k_\perp r_0)^2 \ll 1$ this last ratio is smaller than 1. The gain here, even though enhanced relative to the case without waveguide, is still small relative to the case where one uses the wiggler. Yet the advantages gained by the simplicity of the magnetic configuration and the possible use of large interaction volume could outweigh the somewhat smaller gain in many situations.

IV. DISCUSSION

Here we sketch a possible practical device based on the ideas described hitherto.

A magnetron injection gun emits an electron beam into a hollow coaxial cylindrical waveguide. The inner and outer radii are 10 and 13 cm, respectively. Since the gap 3 cm is small relative to each radius our analysis of the two infinite plane parallel plates may be applied here. The electron beam fills the waveguide uniformly [in contrast to the case in Eq. (9)]. A radiation of wavelength 785 μm is launched into the waveguide. The fourth mode has $k_\perp = 4.2\ \text{cm}^{-1}$. The electrons are injected with $\gamma_0 = 5$ (energy of 2 MeV). They enter with perpendicular velocity $u_{20} = 0.1$. We apply a uniform magnetic field of 16.5 kG which yields $k_0 = 2\ \text{cm}^{-1}$. Following Eq. (21) the above wavelength is resonant. $k_\perp r_0$ is 0.2 and obeys the condition (16). For a gain of 10% the required current density is 6 A cm^2 or a total current of about 1.2 kA. Other modes are not excited for $L \simeq 100$ cm since $\Delta\nu L = \Delta\beta L, \beta\Delta\beta = k_\perp \Delta k_\perp$, and $\Delta k_\perp = \pi/a$ yield $\Delta\nu L = (k_\perp/\beta)(\pi/a)L > \pi$. In order to satisfy the resonance condition for gain, $\Delta\nu L$ should be less than 2π, where $\Delta\nu$ is due to the spread in energy and angle in the initial electron beam. From the definition of ν [Eq. (20)] it follows that $\Delta\alpha/\alpha (tg\alpha \equiv u_{20}/u_{30})$ should be less than $\lambda / L u_{20}^2$, and $\Delta\gamma/\gamma$ less than $1/N (= 2\pi/k_0 L)$.

We now compare the proposed device to a wiggler-type FEL. Imagine that the electrons in the wiggler-type FEL move on similar helical orbits. By Eq. (49) the current density needed is 0.6 A/cm^2 only. On the other hand when $k_0 = 2\ \text{cm}^{-1}$ the pitch of the wiggler is 3.1 cm. Considering that the desired wiggler-field is only at a radius of less than 0.3 cm,[14] the volume of interaction has a cross section of 0.3 cm^2. In our device it is much bigger, about 200 cm^2 so that its power output would undoubtedly be larger. In addition, the current required to create the wiggler-field (320 G) is about 15 kA. In view of these facts the advantage of the wiggler-type FEL on the proposed device is not clear.

In summary, we have demonstrated the possibility of operation of a novel source of submillimeter radiation. It is built simply from a waveguide immersed in an intense uniform magnetic field in which a relativistic electron beam interacts with one of its modes. By amplifying the Doppler-shifted electron cyclotron frequency, it becomes, in terms of its gain and its simplicity, a viable source of submillimeter radiation.

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