*International Journal of Infrared and Millimeter Waves, Vol. 2, No. 5, 1981*

# ORBIT STABILITY IN FREE ELECTRON LASERS*

**P. Avivi, F. Dothan, A. Fruchtman, A. Ljudmirsky, and J. L. Hirshfield†**

*Center for Plasma Physics*
*Hebrew University*
*Jerusalem, Israel*

Received June 2, 1981

Helical magnetic wigglers for free electron lasers can produce non-helical electron trajectories if a uniform axial guide magnetic field is imposed. Friedland's necessary criterion for the existence of helical orbits is reviewed and shown to apply for non-relativistic electron energies. An experiment designed to test this criterion is described and results are compared with theory.

Key words: free electron laser, magnetic wiggler, electron orbits.

## Introduction

Considerable effort is currently underway in the analysis (1), design (2), and construction (3) of free electron lasers for amplification of infrared and far infrared radiation. A typical device comprises a good quality electron beam with energy of 10's of MeV which moves through a periodic static pump magnetic field, termed a magnetic wiggler. Radiation propagating along the electron beam has been shown experimentally (4) to be amplified, but the single-pass small-signal gain may be quite small (7% was reported for a 520 cm length at $\lambda = 10.6\mu$ in Ref. 4).

Suggestions for enhancing the small-signal gain by superposing a uniform axial magnetic upon the wiggler field have appeared, based upon both single-particle (5,6) and collective (7) models. The gain enhancement can result from either increased equilibrium undulatory momentum (5), or from dynamical resonance between induced electromagnetic perturbations and the natural oscillations of electrons on helical orbits (6,7). The increased undulatory momentum results in a decreased axial momentum, and thus a decreased Doppler up-shift, i.e. the laser output frequency is shifted to longer wavelength. Gain enhancement may still be achieved without this wavelength increase by operating the device with a reduced wiggler field.

A necessary condition for achievement of the gain enhancement is that the equilibrium electron orbits in the wiggler be nearly helical. Without the axial guide field a helical magnetic wiggler produces a helical orbit; this result follows from the constancy of canonical angular momentum. But when the axial guide field is present, the orbits are generally not helical (8). They can be arranged to be nearly helical if the entry conditions into the wiggler are suitably tailored, and if the wiggler and guide field parameters are in a regime of stability, determined from the orbit parameters (9).

In this paper, we shall review the basis underlying the criterion for orbit stability, and shall present results of an experiment designed to test this criterion quantitatively.

## Orbit Stability

Here we summarize (8) some aspects of the dynamics of charged particles moving in a static magnetic field given by

$$\begin{aligned} \underline{B}(z) &= \hat{e}_z B_o + (\hat{e}_x \cos k_o z + \hat{e}_y \sin k_o z) B_\perp \\ &= \hat{e}_3 B_o - \hat{e}_2 B_\perp \quad . \end{aligned} \tag{1}$$

Here $B_o$ is the magnitude of the uniform axial guide field, and $B_\perp$ is the magnitude of the transverse helical field with pitch $\ell_o = 2\pi/k_o$. It has been shown (1) that the charged particle dynamics in this field are described compactly if a coordinate system with basis vectors ($\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$) is used, rather than the Cartesian system ($\hat{e}_x$, $\hat{e}_y$, $\hat{e}_z$). The coordinate transformations follow from the definitions of $\hat{e}_2$ and $\hat{e}_3$ given in Eq. (1), and by $\hat{e}_1 = \hat{e}_2 \times \hat{e}_3$.

Of course, the field given by Eq. (1) does not satisfy $\nabla \times \underline{B} = 0$; it is however a good approximation to the actual field near the axis of two identical interspersed helical conductors carrying currents in opposite directions. The exact field, and the precise nature of the approximations leading to Eq. (1) will be discussed in a forthcoming paper (10).

For a particle of charge e, rest mass m, and relativistic energy factor $\gamma$, the steady-state solutions of the equation of motion $m d(\gamma \underline{v})/dt = -e\underline{v} \times \underline{B}$ [with $\underline{B}$ given by Eq. (1)] are

$$u_1 = 0$$

$$u_2 = \frac{k_o \xi u_3}{k_o u_3 \gamma - \Omega/c} \qquad (2)$$

$$u_3 = (1 - u_2^2 - \gamma^{-2})^{1/2}$$

where $\underline{u} = \underline{v}/c$, $\Omega = eB_0/m$, and $\xi = eB_\perp/k_o mc$. These components correspond to ideal helical trajectories, since $u_2$ and $u_3$ are constants. However, these steady-state values can only be approached asymptotically, for an actual wiggler, because of coupling between the components in the transition region at the entrance to the wiggler (8), and because the form given by Eq. (1) is only an approximation.

The solutions given by Eq. (2) are depicted (for $\gamma$ = 10.0, $k_o$ = 6.0 $cm^{-1}$, and $\xi$ = 1.0) in Fig. 1. For $\Omega > \Omega_{cr}$ the equations are single-valued, whilst for $\Omega < \Omega_{cr}$ they are triple-valued. The critical axial guide field cyclotron frequency $\Omega_{cr}$ is given by

$$\Omega_{cr} = k_o c[(\gamma^2 - 1)^{1/3} - \xi^{2/3}]^{3/2} \quad . \qquad (3)$$

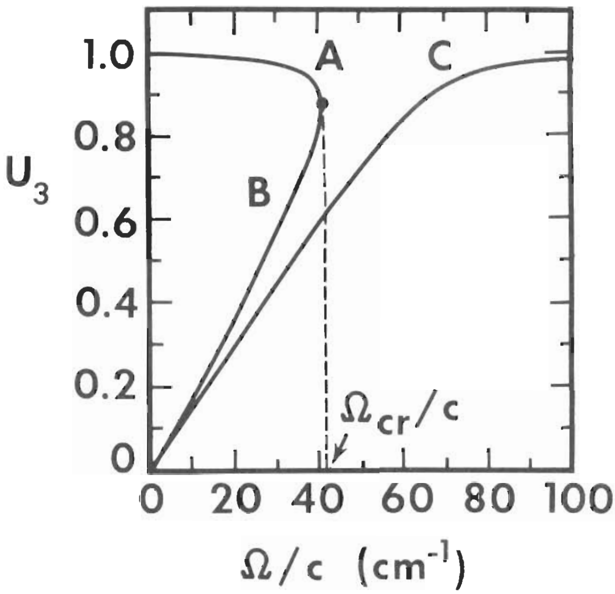


Figure 1. Solutions for steady-state axial momentum $u_3$, as a function of axial magnetic field, for $\gamma = 10$, $k_0 = 6\ \mathrm{cm}^{-1}$, and $\xi = 1$.

For smaller values of $\xi$ than that chosen for Fig. 1, the curves hug more closely the asymptotes $u_3 = (1 - \gamma^{-2})^{1/2}$ and $u_3 = \Omega/k_0 c\gamma$. Perturbation theory shows (8) that branches A and C in Fig. 1 are stable, whilst branch B is unstable. Thus, if a particle enters a wiggler along a gradually increasing guide field, it would move on a stable helical orbit along branch A, but at $\Omega = \Omega_{cr}$ the orbit would become unstable and thus severely non-helical. Examples of non-helical orbits are shown in Ref. 8. If $\Omega$ = const and the wiggler field increases gradually, a similar phenomenon occurs at $\xi_{cr}$, where

$$\xi_{cr} = [(\gamma^2 - 1)^{1/3} - (\Omega/k_0 c)^{2/3}]^{3/2} , \qquad (4)$$

or $$(B_\perp/B_0)_{cr} = [(\gamma^2 - 1)^{1/3}(k_0 c/\Omega)^{2/3} - 1]^{3/2} . \qquad (5)$$

Thus for a charged particle moving through a wiggler in a uniform axial guide field, the orbit can be nearly helical if $\xi < \xi_{cr}$ all along the wiggler but would depart significantly from helicity if $\xi > \xi_{cr}$.

## Experiment

Although electron beams of interest for practical free electron lasers have relativistic energies, the phenomenon of helical orbit stability discussed above is not fundamentally a relativistic effect. Thus if the electron energy V is much less than 511 keV, so that we approximate $\gamma^2 - 1 \approx (2eV/mc^2)$, we can write Eq. (5) as

$$(B_\perp/B_o)_{cr} = [(8\pi^2 mV/eB_o^2 \ell_o^2)^{1/3} - 1]^{3/2}$$
$$= [(21.2V^{1/2}/B_o\ell_o)^{2/3} - 1]^{3/2} \tag{6}$$

where, in the final expression, V is in volts, $B_o$ is in gauss, and $\ell_o$ is in cm.

In the experiments to be described, electron beams in the energy range 4-14 keV were employed; a simple dc low-current ($\sim$ 10's of $\mu$A) crt electron gun could then be used to provide the electron beam with a diameter of about 1 mm and energy resolution of better than 1%. The helical wiggler, to be described more fully below, had a period $\ell_o$ = 3.6 cm. Thus, from Eq. (6), one sees that the transition from stable to unstable orbits would occur for very small wiggler fields indeed if the axial magnetic field were adjusted to be slightly above $5.89V^{1/2}$ gauss, i.e. in the range between 350 and 700 gauss. The axial magnetic field was in fact adjusted to dc values between about 300 to 3000 gauss. For a given electron energy V and axial field $B_o$ the wiggler field amplitude $B_\perp$ was varied continuously in time by triggering a spark gap to discharge a capacitor in series with the wiggler coil. The ensuing RC-decay could be calibrated to give $B_\perp$ values as a function of time during each discharge pulse.

The wiggler coil itself was a bifilar periodic winding of 3 mm diam conductor would on a 53 mm diam cylinder with a uniform pitch of 36 mm. The uniform portion was 666 mm long, i.e. 18.5 periods. At each end the wiggler diameter tapered outward to 100 mm over a 175 mm length. It was found that, in addition to provision of these tapered end portions, careful symmetrizing of the conductors at the end turns was essential for obtaining stable beam transmission through the wiggler. Furthermore, flux shunts at the ends were required to produce a smooth uniformly tapered transition into the wiggler. A plot of one component of the transverse field produced by this wiggler is shown in Fig. 2. (The uniform portion is not shown, as this portion is

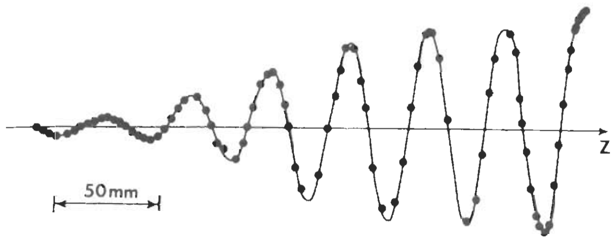


Figure 2. Measured transverse magnetic field at the entrance end of the wiggler.

relatively easy to produce.) This wiggler produced a field of about 20 gauss/kA, and fields up to 250 gauss have been routinely produced.

Several beam analyzers were constructed to examine the properties of the beam within the uniform portion of the wiggler. For the data to be presented in this paper, a movable analyzer was used consisting of two parallel plates spaced by 9 mm and positioned normal to the axial magnetic field. The first plate had a 3 mm hole in its center through which the beam would pass either in the absence of any wiggler field, or for wiggler field values below the critical value. In this case, paraxial helical orbits with diameter less than 3 mm were ascertained to be produced, so that the beam current was collected by the back plate. If the orbit were to involve excursions of more than 3 mm away from the axis, current would be collected by the front plate. When the beam was seen to migrate back and forth between the two analyzer plates as the wiggler field decayed with time, this was taken as direct evidence for a strongly non-helical orbit. Two examples of this migration are shown in Fig. 3, which is traced from oscillograms of the

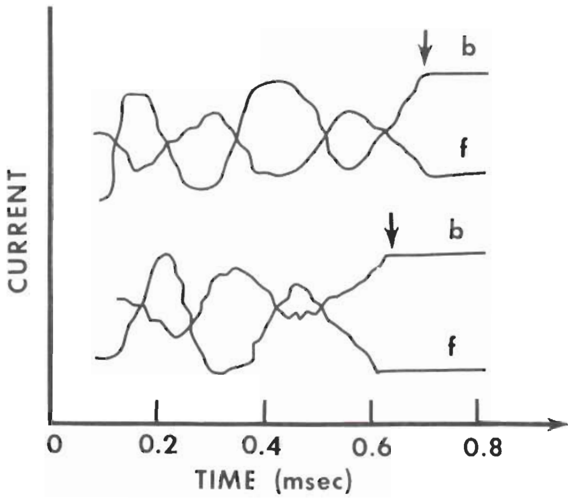


Figure 3. Measured currents to front (f) and back (b) plates of beam analyzer. Arrows indicate abrupt transitions from non-helical to helical orbits. Lower example is for a lower axial field value than upper example, so that transition occurs at higher value of wiggler field.

current waveforms to the analyzer plates as a function of time following firing of the wiggler field spark gap. The examples are for two different axial field values (lower for the bottom example than for the top). One sees the beam gyrate wildly back and forth between the two plates until a certain time, denoted by the arrows, when the wiggler field has decayed to a specific value. The transition to beam collection by the back plate alone (i.e. paraxial helical orbits) is seen to be abrupt. Values of wiggler field were noted at each transition point observed when axial field and beam energy were varied. These values are plotted in Fig. 4 as a function of the independent variable

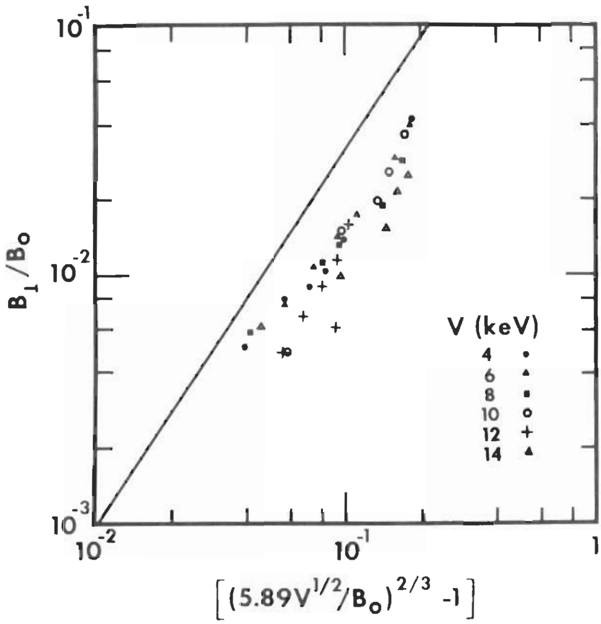


Figure 4. Measured values of $B_\perp/B_0$ at which transitions from stable to unstable orbits were observed, for electron energies between 4-14 keV. Solid line is theoretical prediction.

$(5.89V^{1/2}/B_0)^{2/3} - 1$, as suggested by Eq. (6) for $x_0$ = 3.6 cm. The straight line in Fig. 4 is this same variable raised to the three-halves power.

Transitions from unstable to stable orbits have been observed for wiggler fields as low as 2 gauss (lowest datum in Fig. 4).

## Discussion

Magnetic wigglers for free electron laser applications produce helical electron orbits in the absence of an axial guide field, but may produce strongly non-helical orbits if an axial field is present. One predicted (8) consequence of this phenomenon is an abrupt jump in the orbit from non-helical to helical once the magnetic wiggler field strength falls below a critical value, for fixed axial field and beam energy. This behavior has been observed experimentally over a wide range of (non-relativistic) beam energies and axial field strengths. The data follow an approximate three-halves power law in the variable $(8\pi^2 mV/eB_0^2 \ell_0^2)^{1/3} - 1$, as suggested by the theory. The data fall systematically about 10-20% higher in this variable than is predicted (corresponding to about a factor-of-two smaller value of $B_\perp/B_0$ than is predicted). An overestimate in measured electron beam energies could explain the discrepancy between theory and experiment, but measurement accuracies are believed sufficient to rule this out. Finite geometry effects, due either to off-axis departures of the wiggler field from Eq. (1), or from the finite spatial resolution of the analyzer, could also contribute to the apparent discrepancy.

However, the crucial points for users of magnetic wigglers in axial guide magnetic fields are (1), the care required in wiggler construction (especially at the "first" turn, and within a gradual transition region) in order to observe a paraxial helical orbit <u>at all</u>; and (2), the clear observation of an abrupt transition between stable and unstable orbits at (sometimes <u>very low</u>) critical wiggler fields, much as had been predicted by theory.

It may be that the non-helical orbits will be of utility, although it would be easy to despair in attempting to formulate a theory for free electron laser operation with such a complex equilibrium state. These orbits can possess large amplitude harmonic overtones (10) which should radiate <u>incoherent</u> radiation at wavelengths a few time shorter than $\ell_0/2\gamma^2$. It may even be possible to observe <u>coherent</u> amplification on such a spatial overtone of the fundamental wiggler period; but speculation carries risks....

## Acknowledgments

The authors have benefited from instructive discussions with L. Friedland. A critical reading of the manuscript by S. Y. Park is appreciated.

## References and Footnotes

* This research was sponsored by the U.S. Office of Naval Research, and by the U.S.-Israel Binational Science Foundation.

† Also at Mason Laboratory, Yale University, P.O. Box 2159 Yale Station, New Haven, Connecticut 06520.

1. I. B. Bernstein and J. L. Hirshfield, Phys. Rev. A 20, 1661 (1979).

2. See, for example, C. A. Brau, Laser Focus 17, 48 (1981).

3. See, for example, E. D. Shaw and C. K. N. Patel, Workshop on Free-Electron Generators of Coherent Radiation, Telluride, Colorado, Aug. 1979.

4. D. A. G. Deacon, L. R. Elias, J. M. J. Madey, G. J. Ramien, H. A. Schwettman, and T. I. Smith, Phys. Rev. Lett. 38, 892 (1977).

5. P. Sprangle and V. L. Granatstein, Phys. Rev. A 17, 1792 (1978).

6. L. Friedland and J. L. Hirshfield, Phys. Rev. Lett. 44, 1456 (1980).

7. I. B. Bernstein and L. Friedland, Phys. Rev. A 23, 816 (1981).

8. L. Friedland, Phys. Fluids 23, 2376 (1980).

9. Of course, these considerations also to apply to other applications for wigglers in axial fields, such as in tailoring solid electron beams for gyrotrons.

10. S. Y. Park, J. M. Baird, R. A. Smith, and J. L. Hirshfield, to be published.