Collective Argumentation and Disjunctive Logic Programming

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Abstract

An extension of an abstract argumentation framework is introduced that provides a direct representation of global conflicts between sets of arguments. The extension, called collective argumentation, turns out to be suitable for representing semantics of disjunctive logic programs. Collective argumentation theories are shown to possess a four-valued semantics, and are closely related to multiple-conclusion (Scott) consequence relations. Two special kinds of collective argumentation, positive and negative argumentation, are considered in which the opponents can share their arguments. Negative argumentation turns out to be especially appropriate for analyzing stable sets of arguments. Positive argumentation generalizes certain alternative semantics for logic programs.

Introduction

The general argumentation theory [Dun95b, BDKT97] has proved to be a powerful framework for representing nonmonotonic formalisms in general, and semantics for normal logic programs, in particular. Thus, it has been shown that the main semantics for the latter are naturally representable in this framework (see below).

The theory of logic programming has also suggested, however, an extension of the notion of a normal program to that of a disjunctive program in which the program rules may have disjunctive heads representing alternative possible conclusions. Such rules have turned out to be strictly more expressive than normal program rules, and they have allowed to give a direct description of indeterminate, or partial, information about the world. As a result, the emerged disjunctive logic programming has developed into a powerful general-purpose reasoning system that has given a computational representation for a broader range of problems arising in Knowledge Representation and Artificial Intelligence - see, e.g., [LMR92, BG94] for an overview of disjunctive logic programming and its applications to knowledge representation.

Unfortunately, the general argumentation theory has turned out to be insufficient for representing disjunctive logic programs and their semantics. Accordingly, in this paper we will suggest an extension of the abstract argumentation theory suitable for disjunctive logic programs. The extension, called collective argumentation, will be obtained by defining the attack relation directly among sets of arguments, instead of individual arguments. In other words, we will permit situations in which a set of arguments 'collectively' attacks another set of arguments in a way that is not reducible to attacks among particular arguments from these sets.

Due to the tight connection between collective argumentation and disjunctive programming, the semantics suggested for disjunctive logic programs will serve in this paper as the main source of intuitions in developing the general principles behind collective argumentation. It should be stressed, however, that the scope and importance of collective argumentation is much wider than logic programming. Taken in a broader perspective, collective argumentation is purported to describe reasoning situations in which the conflict between incompatible views or theories is global and cannot be reduced to particular claims made by these theories. As is well known, such conflicts constitute a primary subject of the modern philosophy of science. In addition, collective argumentation will be shown to suggest a natural setting for studying kinds of argumentation in which the opponents can provisionally share their arguments in order to disprove their adversaries. Though related, such an argumentation will be quite different from (and more complex than) the forms of argumentation definable in Dung's argumentation theory.

The plan of the paper is as follows. After a brief and biased description of the abstract argumentation theory, we will suggest its generalization in which the attack relation is defined as holding primarily between sets of arguments. The suggested collective argumentation theory will be shown to be adequate, in principle, for representing any 'well-behaved' semantics for disjunctive logic programs. We will describe also a certain four-valued semantics that will be adequate for such argumentation theories. Moreover, it will be shown that the latter are representable also in terms of multiple-conclusion (Scott) consequence relations.

As an application of the general theory, we will consider two special kinds of collective argumentation called, respectively, negative and positive argumentation. On the semantic level, these two kinds of argumentation will correspond to two main ways of restricting a four-valued reasoning to a three-valued one. For both these kinds of argumentation, the attack relation will be defined by 'borrowing' arguments of the other side in order to disprove the latter. Negative argumentation will be shown to be especially appropriate for analyzing stable sets of arguments, since it will give a very simple description of such sets. Positive argumentation will also generalize some known semantics suggested for logic programs.

1 Abstract Argumentation Theory

To begin with, we give a brief description of Dung's argumentation theory [Dun95b].

Definition 1.1. An *abstract argumentation theory* is a pair $\langle \mathcal{A}, \hookrightarrow \rangle$, where $\mathcal{A}$ is a set of *arguments*, while $\hookrightarrow$ is a binary *attack* relation on $\mathcal{A}$.

A general task of argumentation theory consists in determining 'good' sets of arguments that are safe in some sense with respect to the attack relation. To this end, we should extend first the attack relation to sets of arguments: if Γ, Δ are sets of arguments, then $\Gamma \hookrightarrow \Delta$ is defined to hold if $\alpha \hookrightarrow \beta$, for some $\alpha \in \Gamma$, $\beta \in \Delta$.

An argument α will be called *allowable* for the set of arguments Γ, if Γ does not attack α. For any set of arguments Γ, we will denote by $[\Gamma]$ the set of all arguments allowable by Γ, that is

$$[\Gamma] \;=\; \{\alpha \mid \Gamma \not\hookrightarrow \alpha\}$$

An argument α will be said to be *acceptable* for the set of arguments Γ, if Γ attacks any argument against α. As can be easily checked, the set of arguments that are acceptable for Γ coincides with $[[\Gamma]]$.

Using the above notions, we can give a quite simple characterization of the basic objects of an abstract argumentation theory.

Definition 1.2. A set of arguments Γ will be called

- *conflict-free* if $\Gamma \subseteq [\Gamma]$;
- *admissible* if it is conflict-free and $\Gamma \subseteq [[\Gamma]]$;
- a *complete extension* if it is conflict-free and $\Gamma = [[\Gamma]]$;
- a *preferred extension* if it is a maximal complete extension;
- a *stable extension* if $\Gamma = [\Gamma]$.

A set of arguments Γ is conflict-free if it does not attack itself. A conflict-free set Γ is admissible iff any argument from Γ is also acceptable for Γ, and it is a complete extension if it coincides with the set of arguments that are acceptable with respect to it. Finally, a stable extension is a conflict-free set of arguments that attacks any argument outside it. Any stable extension is also a preferred extension, any preferred extension is a complete extension, and any complete extension is an admissible set. Moreover, as has been shown in [Dun95b], any admissible set is included in some complete extension. Consequently, preferred extensions coincide with maximal admissible sets. In addition, the set of complete extensions forms a complete lower semi-lattice: for any set of complete extensions, there exists a unique greatest complete extension that is included in all of them. In particular, there always exists a least complete extension of an argumentation theory.

As has been shown in [Dun95a], under a suitable translation, the above objects correspond to well-known semantics suggested for normal logic programs. Thus, stable extensions correspond to stable models (answer sets), complete extensions correspond to partial stable models, preferred extensions correspond to regular models, while the least complete extension corresponds in this sense to the well-founded semantics (WFS). These results have shown, in effect, that the abstract argumentation theory successfully captures the essence of logical reasoning behind normal logic programs.

Unfortunately, the above argumentation theory cannot be extended directly to disjunctive logic programs. The reasons for this shortcoming, as well as a way of modifying the argumentation theory are discussed in what follows. As we will see, however, only a small part of the above nice and well-organized structure can be transferred into a more general framework of collective argumentation.

2 Collective Argumentation

We will begin with pointing out a peculiar discrepancy between the abstract argumentation theory, on the one hand, and the general abductive framework used for interpreting semantics for logic programs, on the other hand (see, e.g., [Dun95a, KT99]). The main objects of the abductive argumentation theory are sets of assumptions (abducibles) of the form $\mathbf{not}\, p$ that play the role of arguments in the associated argumentation theory. In addition, the attack relation is defined in this framework as a relation between sets of abducibles and particular abducibles they attack. For example, the program rule $r \leftarrow \mathbf{not}\, p, \mathbf{not}\, q$ is interpreted as saying that the set of assumptions $\{\mathbf{not}\, p, \mathbf{not}\, q\}$ attacks the assumption $\mathbf{not}\, r$.

The above attack relation is employed for defining the basic models of the abductive framework. The abstract argumentation theory defines its main objects, however, as *sets* of arguments. Consequently, they should correspond to *sets of sets* of assumptions in the abductive framework, though the latter defines such objects as certain plain sets of assumptions! In other words, we have a certain discrepancy between the levels of representations of intended objects in these two theories.

The above discrepancy will disappear once we notice that all the basic objects of the abstract argumentation theory are definable, in effect, in terms of the derived attack relation $\Gamma \hookrightarrow \alpha$ between sets of arguments and particular arguments. Indeed, only the latter relation was used in defining the above operator $[\Gamma]$. As a result, the abductive argumentation theory can be constructed in the same way as the abstract theory, with the only distinction that the attack relation $\Gamma \hookrightarrow \alpha$ between sets of arguments and particular arguments is not reducible to the attack relation among individual arguments.

The abductive argumentation naturally suggests that assumptions, or abducibles, can be considered as full-fledged arguments, while the attack relation is best describable as a relation among sets of such arguments. Indeed, once we allow for a possibility that a set of arguments can produce a nontrivial attack that is not reducible to attacks among particular arguments, it is only natural to allow also for a possibility that a set of arguments could be attacked in such a way that we cannot single out a particular argument in the attacked set that could be blamed for it. For instance, in a quite common case we can disprove some conclusion jointly supported by a disputed set of arguments, though no particular argument in the set could be held responsible for this outcome. The following generalization of the abstract argumentation framework reflects this idea.

Definition 2.1. A *collective argumentation theory* is a pair $\langle \mathcal{A}, \hookrightarrow \rangle$, where $\mathcal{A}$ is a set of arguments, and $\hookrightarrow$ is an attack relation on finite subsets of $\mathcal{A}$ satisfying the following *monotonicity condition*:

(**Monotonicity**) If $\Gamma \hookrightarrow \Delta$, then $\Gamma \cup \Gamma' \hookrightarrow \Delta \cup \Delta'$.

Though the attack relation is defined above only on finite sets of arguments, it can be naturally extended to arbitrary such sets by imposing the following *compactness constraint*:

(**Compactness**) $\Gamma \hookrightarrow \Delta$ only if $\Gamma' \hookrightarrow \Delta'$, for some finite $\Gamma' \subseteq \Gamma$, $\Delta' \subseteq \Delta$.

As the reader may notice, the suggested argumentation framework has many properties in common with ordinary sequent calculus, or consequence relations. Moreover, in order to highlight the similarity, we will use in what follows the same agreements for the attack relation as that commonly accepted for consequence relations. Thus, $\Gamma, \Gamma_1 \hookrightarrow \Delta, \Delta_1$ will have the same meaning as $\Gamma \cup \Gamma_1 \hookrightarrow \Delta \cup \Delta_1$. Similarly, $\alpha, \Gamma \hookrightarrow \Delta, \beta$ will be an alternative notation for $\{\alpha\} \cup \Gamma \hookrightarrow \{\beta\} \cup \Delta$, etc. Actually, it will be shown below that the similarity is not accidental, since the collective attack relation is representable as a certain consequence relation in an extended language.

The argumentation theory from [Dun95a] satisfies all the above properties of collective argumentation. Moreover, the above modification of the abstract argumentation theory has already been suggested, in effect, in [KT99]. However, the attack relation in the latter paper was (implicitly) required to satisfy a couple of further properties described in the following definition.

Definition 2.2. A collective argumentation theory will be called

- *affirmative* if no set of arguments attacks the empty set $\emptyset$;
- *local* if it satisfies the following condition:

  (**Locality**) If $\Gamma \hookrightarrow \Delta, \Delta'$, then either $\Gamma \hookrightarrow \Delta$ or $\Gamma \hookrightarrow \Delta'$.

- *normal* if it is both affirmative and local.

If a collective argumentation theory is normal, then it can be easily shown that $\Gamma \hookrightarrow \Delta$ will hold if and only if $\Gamma \hookrightarrow \alpha$, for some $\alpha \in \Delta$. Consequently, the attack relation in such argumentation theories is reducible to the relation $\Gamma \hookrightarrow \alpha$ between sets of arguments and single arguments, and the resulting theory will coincide with that given already in [Dun95a].

It turns out, however, that the general, non-local framework of collective argumentation is precisely what is needed for representing semantics of disjunctive logic programs.

3 Semantics and consequence relations

Collective argumentation can be given a four-valued semantics that will be instructive in describing the meaning of the attack relation, as well as for imposing plausible constraints on argumentation theories and their models.

A natural understanding of the attack $\Gamma \hookrightarrow \Delta$ is

> *at least one of the arguments in Δ should be rejected whenever all the arguments from Γ are accepted.*

The expressive capabilities of the argumentation theory depend, however, on the absence of usual 'classical' constraints on the acceptance and rejection of arguments, so it permits situations in which an argument is both accepted and rejected, or, alternatively, neither accepted, nor rejected. Such an understanding can be captured formally by assigning to any argument an arbitrary subset of the set $\{\mathbf{t}, \mathbf{f}\}$, where $\mathbf{t}$ denotes acceptance (truth), while $\mathbf{f}$ denotes rejection (falsity) (cf. a similar description in [JV99]). Then the assignment of $\{\mathbf{t}\}$ means that the argument is accepted (and not rejected), $\{\mathbf{f}\}$ means that the argument is plainly rejected, $\{\mathbf{t}, \mathbf{f}\}$ means that it is both accepted and rejected, and $\emptyset$ means that it is neither accepted, nor rejected. This interpretation is nothing other than the well-known *Belnap's interpretation* of four-valued logic (see [Bel77]). As a result, collective argumentation acquires a natural four-valued semantics that is described in the following definitions.

Definition 3.1.
- A (four-valued) *interpretation* of an argumentation theory $\mathbb{A}$ is a function $v : \mathcal{A} \mapsto 2^{\{\mathbf{t},\mathbf{f}\}}$ assigning each argument a subset of $\{\mathbf{t}, \mathbf{f}\}$.

- An attack $\Gamma \hookrightarrow \Delta$ will be said to *hold* in an interpretation v if either $\mathbf{t} \notin v(\gamma)$, for some $\gamma \in \Gamma$, or $\mathbf{f} \in v(\delta)$, for some $\delta \in \Delta$.

- An interpretation v is a *model* of a collective argumentation theory $\mathbb{A}$ if every attack from $\mathbb{A}$ holds in v.

It turns out to be convenient to identify any four-valued interpretation v with a pair (Π^v_+, Π^v_-) such that Π^v_+ is the set of accepted arguments, while Π^v_- is the set of all arguments that are not rejected:

$$\Pi^v_+ = \{\pi | \, \mathbf{t} \in v(\pi)\} \qquad \Pi^v_- = \{\pi | \, \mathbf{f} \notin v(\pi)\}$$

Clearly, the source interpretation can be restored from the above pair of sets. Moreover, the models of an argumentation theory correspond in this sense to bitheories described in the next definition.

Definition 3.2. A pair (Π_+, Π_-) of sets of arguments will be called a *bitheory* of an argumentation theory if $\Pi_+ \not\hookrightarrow \Pi_-$.

For a set V of interpretations, we will denote by $\hookrightarrow_V$ the set of all attacks that hold in each interpretation from V. Then the following result is actually a representation theorem showing that the four-valued semantics is adequate for collective argumentation.

Theorem 3.1. $(\mathcal{A}, \hookrightarrow)$ *is a collective argumentation theory if and only if* $\hookrightarrow = \hookrightarrow_V$*, for some set of four-valued interpretations* V*.*

Proof. For any set of interpretations V, the attack relation $\hookrightarrow_V$ satisfies Monotonicity, so it determines a collective argumentation theory. Now, if $\mathbb{A}$ is a collective argumentation theory, let $V_{\mathbb{A}}$ denote the set of all interpretations corresponding to bitheories of $\mathbb{A}$. We will show that $\hookrightarrow_{V_{\mathbb{A}}} = \hookrightarrow$.

Since bitheories correspond to models of an argumentation theory, any attack $\Gamma \hookrightarrow \Delta$ of the theory will hold in every interpretation from $V_{\mathbb{A}}$. Consequently $\hookrightarrow \subseteq \hookrightarrow_{V_{\mathbb{A}}}$. Now, if $\Gamma \not\hookrightarrow \Delta$ in $\mathbb{A}$, then (Γ, Δ) is a bitheory of $\mathbb{A}$, and hence its corresponding interpretation, say v, belongs to $V_{\mathbb{A}}$. But $\Gamma \hookrightarrow \Delta$ clearly does not hold in v, so $\Gamma \not\hookrightarrow_{V_{\mathbb{A}}} \Delta$. This completes the proof. □

3.1 Argumentation theories vs. consequence relations

We have suggested elsewhere the notion of a *biconsequence relation* as a syntactic formalism of four-valued reasoning based on the Belnap's interpretation (see, e.g., [Boc98a]). It has been shown, in particular, that a certain natural negation connective allows to transform biconsequence relations into a sequent calculus. This negation connective will allow us now to obtain the same translation for argumentation theories.

Let us extend the language of collective argumentation with a new negation connective $\sim$ on arguments having the following semantic interpretation:

$$\sim \alpha \text{ is accepted iff } \alpha \text{ is rejected}$$
$$\sim \alpha \text{ is rejected iff } \alpha \text{ is accepted}$$

As has been shown in [Boc98b], this negation captures the monotonic (logical) content of the default negation **not** used in logic programming. For argumentation theories, it is completely characterized by the following set of rules:

$$\begin{gathered} \alpha \hookrightarrow \sim \alpha \qquad \sim \alpha \hookrightarrow \alpha \\ \text{If } \Gamma \hookrightarrow \alpha, \Delta \text{ and } \Gamma, \sim \alpha \hookrightarrow \Delta \text{ then } \Gamma \hookrightarrow \Delta \\ \text{If } \Gamma, \alpha \hookrightarrow \Delta \text{ and } \Gamma \hookrightarrow \Delta, \sim \alpha \text{ then } \Gamma \hookrightarrow \Delta \end{gathered} \tag{AN}$$

The resulting collective argumentation theories will be called *N-argumentation theories*. As we are going to show, such argumentation theories are interdefinable with certain consequence relations.

By a *Belnap consequence relation* in the language with a negation $\sim$ we will mean a Scott (multiple-conclusion) consequence relation $\Vdash$ satisfying the following Double Negation rules for $\sim$:

$$\alpha \Vdash \sim\sim \alpha \qquad \sim\sim \alpha \Vdash \alpha$$

For any set of propositions Γ of the language, we will denote by $\sim \Gamma$ the set $\{\sim \alpha \mid \alpha \in \Gamma\}$. Now, for a given argumentation theory, we can define the following consequence relation:

$$\Gamma \Vdash \Delta \equiv \; \Gamma \hookrightarrow \sim \Delta \tag{CA}$$

Similarly, for any Belnap consequence relation we can define the corresponding argumentation theory as follows:

$$\Gamma \hookrightarrow \Delta \equiv \; \Gamma \Vdash \sim\Delta \tag{AC}$$

Then the following result shows an exact equivalence of the two notions.

Theorem 3.2.

- *If $\mathbb{A}$ is an N-argumentation theory, then (CA) determines a Belnap consequence relation; its corresponding argumentation theory given by (AC) coincides with $\mathbb{A}$.*
- *If $\mathbb{B}$ is a Belnap consequence relation, then the argumentation theory determined by (AC) is an N-argumentation theory; its corresponding consequence relation given by (CA) coincides with $\mathbb{B}$.*

The proof presents no difficulty and amounts to a straightforward check of the corresponding rules for Belnap consequence relations and N-argumentation theories, respectively. The result allows, in particular, to translate any claim about collective argumentation into a certain assertion about Belnap consequence relations, and vice versa. It should be kept in mind, however, that the four-valued semantics reflects only the *monotonic* (logical) content of argumentation theories. The latter give preference to acceptance of arguments, as opposed to their rejection, so arguments of the form $\sim\alpha$ cannot be treated in the same way as plain arguments α. The task of defining the intended (nonmonotonic) models of argumentation theories will be our subject in subsequent sections. As a first step, we will consider the models suggested for disjunctive logic programs.

4 Collective Argumentation and Disjunctive Logic Programming

Despite an obvious success, the abstract theory of argumentation is still not abundant with intuitions and principles that could guide its development independently of applications. In this respect, logic programming and its semantics constitute one of the crucial sources and driving forces behind the development of argumentation theories. Consequently, as a first step in studying collective argumentation, we consider its representation capabilities in describing semantics for disjunctive logic programs.

In what follows, given a set of propositional atoms C, we will denote by $\overline{C}$ the complement of C in the set of all atoms. In addition, $\mathbf{not}\,C$ will denote the set of all negative literals (abducibles) $\mathbf{not}\,p$, for $p \in C$.

By the general correspondence between normal logic programs and abductive argumentation, a set of abducibles $\mathbf{not}\,C$ attacks an abducible $\mathbf{not}\,p$ in the abductive theory associated with a normal logic program P if P, taken together with $\mathbf{not}\,C$ as a set of additional assumptions, allows to derive p.

The above description immediately suggests a generalization according to which any disjunctive logic program P determines an attack relation among sets of abducibles as follows:

$$\mathbf{not}\, C \text{ attacks } \mathbf{not}\, D \ \text{ iff } \ P \cup \mathbf{not}\, C \text{ derives } \bigvee D.$$

As can be easily verified, the above defined attack relation satisfies all the properties of collective argumentation. However, it is in general not local: $P \cup \mathbf{not}\, C$ may derive $p \vee q$ without deriving either p or q. Still, it will be affirmative for disjunctive logic programs without constraints.

Example. The following disjunctive program gives a (simplified) representation for the well-known Poole's example (see [Poo91]:

$$left{-}hand{-}broken \vee right{-}hand{-}broken \leftarrow$$
$$left{-}hand{-}usable \leftarrow \mathbf{not}\, left{-}hand{-}broken$$
$$right{-}hand{-}usable \leftarrow \mathbf{not}\, right{-}hand{-}broken$$

This example is interesting due to the fact that its standard formalization in default logic (when $\vee$ in the first rule is understood as a classical disjunction) gives a non-intuitive result that both hands are usable.

If $\alpha_1, \alpha_2, \beta_1, \beta_2$ denote, respectively, the abducibles $\mathbf{not}\, left{-}hand{-}broken$, $\mathbf{not}\, right{-}hand{-}broken$, $\mathbf{not}\, left{-}hand{-}usable$ and $\mathbf{not}\, right{-}hand{-}usable$, then the above program is translatable into the following collective argumentation theory:

$$\emptyset \hookrightarrow \alpha_1, \alpha_2$$
$$\alpha_1 \hookrightarrow \beta_1$$
$$\alpha_2 \hookrightarrow \beta_2$$

In this argumentation theory, the set $\{\alpha_1, \alpha_2\}$ is attacked by any set of arguments, though there are no 'specific' attacks on either α_1 or α_2, taken alone. And the problem here is that β_1 and β_2 cannot be rejected simultaneously (since this would mean that both hands are usable), though we should reject at least one of them.

The appropriateness of the original argumentation theory for representing semantics of normal logic programs was based, ultimately, on the fact that these semantics are completely determined by rules of the form $p \leftarrow \mathbf{not}\, C$ that are derivable from a program. A similar principle, called *the principle of partial deduction, or evaluation* is valid also for the majority of semantics suggested for disjunctive logic programs. According to this principle, semantics of such programs should be completely determined by rules $C \leftarrow \mathbf{not}\, D$ without positive atoms in bodies that are derivable from the source program[1].

The above considerations indicate that practically all 'respectable' semantics for disjunctive programs should be expressible in terms of collective argumentation theories associated with such programs. This theoretical possibility does not imply, however, that the actual representation of such semantics must be an easy task. In fact, it turns out that the actual semantics suggested for disjunctive programs do not fit easily into the general constructions of Dung's argumentation theory. A most immediate reason for this is that the operator $[\Gamma]$ of the abstract argumentation theory is no longer suitable for capturing the main content of collective argumentation.

4.1 Stable, Partial Stable and Admissible Sets

For any set of arguments Γ, we will denote by $\langle\Gamma\rangle$ the set of all maximal sets Δ such that $\Gamma \not\hookrightarrow \Delta$:

$$\langle\Gamma\rangle = \{\Delta \mid \Gamma \not\hookrightarrow \Delta \wedge (\forall \Delta')(\Delta \subset \Delta' \rightarrow \Gamma \hookrightarrow \Delta')\}$$

The operator $\langle\Gamma\rangle$ will play in what follows the same role as the allowability operator $[\Gamma]$ in Dung's argumentation theory. Note that $\Delta \in \langle\Gamma\rangle$ holds if and only if (Γ, Δ) is a *negatively minimal* bitheory in the sense that the corresponding model contains a minimal number of rejected arguments (for a given set Γ of accepted ones).

Now we can give a rather simple description of stable and partial stable models for disjunctive programs (see [GL91, Prz91]) in terms of collective argumentation.

Definition 4.1. • A set Γ will be said to be *stable* if $\Gamma \in \langle\Gamma\rangle$.

[1] See [Boc98b] for the role of this principle in determining semantics of logic programs of a most general kind.

- A bitheory (Γ, Δ) will be called *p-stable* if $\Gamma \subseteq \Delta$, $\Gamma \in \langle\Delta\rangle$, and $\Delta \in \langle\Gamma\rangle$.

For p-stable bitheories, the inclusion $\Gamma \subseteq \Delta$ reflects the consistency requirement that no argument can be both accepted and rejected in such models (see below). Note also that stable sets correspond precisely to p-stable bitheories of the form (Γ, Γ).

The following lemmas give more direct, and often more convenient, descriptions of the above objects. The proofs are immediate, so we omit them.

Lemma 4.1. Γ *is stable iff* $\Gamma = \{\alpha \mid \Gamma \not\hookrightarrow \Gamma, \alpha\}$.

The above equation says that a stable set is a set Γ consisting of all arguments α such that Γ does not attack $\Gamma \cup \{\alpha\}$. A similar description can be given for partial stable sets:

Lemma 4.2. (Γ, Δ) *is p-stable iff* $\Gamma = \{\alpha \mid \Delta \not\hookrightarrow \Gamma, \alpha\}$, $\Delta = \{\alpha \mid \Gamma \not\hookrightarrow \Delta, \alpha\}$, *and* $\Gamma \subseteq \Delta$.

Recall that normal collective argumentation theories can be identified with abstract Dung's argumentation theories. Moreover, the above descriptions can be used to show that if a collective argumentation theory is normal, then stable argument sets will coincide with stable extensions, while p-stable pairs will correspond exactly to complete extensions. These facts could also be obtained as a by-product of the correspondence between such objects and relevant semantics of disjunctive programs stated below.

Theorem 4.3. *If* $\mathbb{A}_P$ *is a collective argumentation theory corresponding to a disjunctive program* P*, then*

- C *is a stable model of* P *iff* $\mathbf{not}\,\overline{C}$ *is a stable set in* $\mathbb{A}_P$.
- (C, D) *is a p-stable model of* P *iff* $(\mathbf{not}\,\overline{C}, \mathbf{not}\,\overline{D})$ *is p-stable in* $\mathbb{A}_P$.

Example. Let us return to the Poole's example, reproduced below:

$$\hookrightarrow \alpha_1, \alpha_2 \qquad \alpha_1 \hookrightarrow \beta_1 \qquad \alpha_2 \hookrightarrow \beta_2$$

This collective argumentation theory has two stable sets $\{\alpha_1, \beta_2\}$ and $\{\alpha_2, \beta_1\}$, which correspond to the two stable models of the original disjunctive program, namely $\{right{-}hand{-}broken, left{-}hand{-}usable\}$ and $\{left{-}hand{-}broken, right{-}hand{-}usable\}$. As can be seen, these stable models give a quite satisfactory solution to the Poole's problem, mentioned earlier.

4.1.1 P-stable sets and doubling

P-stable models, as used above, have been introduced in [Boc98b] as a slight modification of Przymusinski's partial stable models for disjunctive programs [Prz91]; the reason was that Przymusinski's semantics violated the above-mentioned principle of partial deduction. In our present context, this means that Przymusinski's semantics is not definable directly in terms of the collective argumentation theory associated with a disjunctive program. Note, however, that the modification has not changed the correspondence with partial stable models for normal logic programs.

Actually, it can be shown that p-stable sets are representable precisely as stable sets of a certain modified argumentation theory. The underlying idea consists in 'doubling' the attack relation in a way familiar from the logic programming literature (see, e.g., [Wal93][2]).

Given a collective argumentation theory $\mathbb{A} = (\mathcal{A}, \hookrightarrow)$, we will define a new argumentation theory $\mathbb{A}_\circ = (\mathcal{A}_\circ, \hookrightarrow_\circ)$ as follows. First, for each argument $\alpha \in \mathcal{A}$, we introduce a new argument α°. For any subset Γ of $\mathcal{A}$, we will denote by Γ° the set $\{\alpha^\circ \mid \alpha \in \Gamma\}$. Now we define $\mathcal{A}_\circ$ as $\mathcal{A} \cup \mathcal{A}^\circ$. Finally, we define a new attack relation on $\mathcal{A}_\circ$:

$$\Gamma, \Delta^\circ \hookrightarrow_\circ \Phi, \Psi^\circ \;\equiv\; \Gamma \hookrightarrow \Psi \text{ or } \Delta \hookrightarrow \Phi$$

Then we have

Theorem 4.4. *A pair* (Γ, Δ) *is p-stable in an argumentation theory* $\mathbb{A}$ *if and only if* $\Gamma \cup \Delta^\circ$ *is stable in* $\mathbb{A}_\circ$ *and* $\Gamma \subseteq \Delta$*.*

Proof. Due to Lemma 4.1, $\Gamma \cup \Delta^\circ$ is stable in $\mathbb{A}_\circ$ iff

$$\Gamma \cup \Delta^\circ = \{\tilde{\alpha} \mid \Gamma, \Delta^\circ \not\hookrightarrow_\circ \Gamma, \Delta^\circ, \tilde{\alpha}\}$$

This condition can be rewritten as the following two equations:

$$\Gamma = \{\alpha \mid \Gamma, \Delta^\circ \not\hookrightarrow_\circ \Gamma, \Delta^\circ, \alpha\} \quad \Delta = \{\alpha \mid \Gamma, \Delta^\circ \not\hookrightarrow_\circ \Gamma, \Delta^\circ, \alpha^\circ\}$$

Now, by the definition of a new attack, $\Gamma, \Delta^\circ \hookrightarrow_\circ \Gamma, \Delta^\circ, \alpha$ holds iff either $\Gamma \hookrightarrow \Delta$ or $\Delta \hookrightarrow \Gamma, \alpha$. Note, however, that if (Γ, Δ) is p-stable in $\mathbb{A}$, then $\Gamma \not\hookrightarrow \Delta$ and $\Delta \not\hookrightarrow \Gamma$. Also, if $\Gamma \cup \Delta^\circ$ is stable in $\mathbb{A}_\circ$, then it is conflict-free,

[2] A similar reduction of Przymusinski's partial stable semantics has been suggested in [JNSY00].

that is $\Gamma, \Delta^\circ \not\hookrightarrow_\circ \Gamma, \Delta^\circ$, which immediately implies $\Gamma \not\hookrightarrow \Delta$ and $\Delta \not\hookrightarrow \Gamma$. Consequently, $\Gamma, \Delta^\circ \hookrightarrow_\circ \Gamma, \Delta^\circ, \alpha$ holds iff $\Delta \hookrightarrow \Gamma, \alpha$. Similarly, it can be shown that $\Gamma, \Delta^\circ \hookrightarrow_\circ \Gamma, \Delta^\circ, \alpha^\circ$ holds iff $\Gamma \hookrightarrow \Delta, \alpha$. As a result, the above two equations turn out to be equivalent to the two conditions for p-stable pairs stated in Lemma 4.2. This completes the proof. □

The above theorem shows, in effect, that partial stable models are essentially stable models 'in disguise'. Unfortunately, in the case of collective argumentation they lack most of the structural properties they had in the normal case of Dung's theory. Most importantly, they do not form a lower semilattice and, in particular, there may be no least partial stable model. In fact, unlike the normal case, collective argumentation theories may have no partial stable models at all:

Example. The collective argumentation theory

$$\hookrightarrow \alpha, \beta, \gamma \qquad \alpha \hookrightarrow \beta \qquad \beta \hookrightarrow \gamma \qquad \gamma \hookrightarrow \alpha$$

does not have p-stable bitheories. This follows already from the fact that it exactly corresponds to the disjunctive program without partial stable models, presented in [Prz91]:

$$\begin{gathered} Work \lor Tired \lor Sleep \\ Work \leftarrow \mathbf{not}\, Tired \\ Sleep \leftarrow \mathbf{not}\, Work \\ Tired \leftarrow \mathbf{not}\, Sleep \end{gathered}$$

An even simpler argumentation theory without p-stable models is

$$\hookrightarrow \alpha, \beta \qquad \alpha \hookrightarrow \alpha \qquad \beta \hookrightarrow \beta$$

4.1.2 Consistent and admissible sets

Notice first that any collective argumentation theory $\mathbb{A}$ includes a normal argumentation sub-theory that is determined by all attacks of the form $\Gamma \hookrightarrow \alpha$ that hold in $\mathbb{A}$. For the latter, we can define conflict-free and admissible sets, as well as all kinds of extensions just as it was done earlier for Dung's argumentation theory. It should be clear, however, that these objects will not be adequate for representing plausible sets of arguments for the source collective argumentation theory, since they reflect only a very small fragment of collective argumentation. For example, even the definition of a conflict-free set will be insufficient for the collective case, since it precludes only local self-attacks. The following definition suggests a more appropriate notion.

Definition 4.2. A set of arguments Γ will be said to be *consistent* if $\Gamma \not\hookrightarrow \Gamma$.

Clearly, consistency reduces to being conflict-free for normal argumentation, but in general it is stronger than the latter.

Recall now that admissible sets in Dung's argumentation theory are definable as conflict-free sets that counterattack any argument against them. This definition can be transferred to collective argumentation as follows:

Definition 4.3. A consistent set of arguments Γ will be called *admissible* if, for any Δ, if $\Delta \hookrightarrow \Gamma$, then $\Gamma \hookrightarrow \Delta$.

Admissible sets can also be described in terms of the allowability operator. Namely, Γ is admissible if no argument set from $\langle\Gamma\rangle$ attacks Γ. Unfortunately, just as for partial stable sets, in the context of collective argumentation the notion of admissibility behaves in a much less ordered fashion than in Dung's theory. Thus, even stable sets as defined earlier in this section need not be admissible in this sense.

Example. Let us consider an argumentation theory $\{\alpha \hookrightarrow \beta \hookrightarrow \alpha, \beta\}$. As can be seen, $\{\beta\}$ is a stable set of this theory, but it is not admissible: we have $\alpha \hookrightarrow \beta$, though $\beta \not\hookrightarrow \alpha$.

Abusing somewhat our terminology, we will say that a set Γ is a *stable extension* of a collective argumentation theory if it is a stable extension of its normal sub-theory. The connection between such extensions and stable sets is established below. This result will be used later.

Lemma 4.5. *If a stable extension* Γ *is consistent, then it is an admissible and stable set.*

Proof. If $\Delta \hookrightarrow \Gamma$, then $\Delta \nsubseteq \Gamma$ (since Γ is consistent), and therefore there exists $\alpha \in \Delta \setminus \Gamma$. But Γ is a stable extension, so $\Gamma \hookrightarrow \alpha$, and therefore $\Gamma \hookrightarrow \Delta$. This shows that Γ is an admissible set in $\mathbb{A}$. Also, if $\alpha \notin \Gamma$ then $\Gamma \hookrightarrow \alpha$, and hence $\Gamma \hookrightarrow \Gamma, \alpha$. Thus, Γ is a stable set in $\mathbb{A}$. $\square$

The above descriptions and results illustrate the ways in which semantics of disjunctive programs are representable in the framework of collective argumentation. They reveal, however, that the relevant objects are significantly different from the corresponding objects of the abstract argumentation theory. In order to get a further insight into the differences, we will consider below some important special kinds of collective argumentation.

5 Argument sharing

In ordinary disputation and argumentation the parties can provisionally accept some of the arguments defended by their adversaries in order to disprove the latter. Three basic cases of such an 'argument sharing' in attacking the opponents are described in the following definition (see also [BDKT97]).

Definition 5.1.

- Γ *classically attacks* Δ (notation $\Gamma \hookrightarrow^c \Delta$) if $\Gamma, \Delta \hookrightarrow \Gamma, \Delta$;
- Γ *negatively attacks* Δ (notation $\Gamma \hookrightarrow^- \Delta$) if $\Gamma \hookrightarrow \Gamma, \Delta$;
- Γ *positively attacks* Δ (notation $\Gamma \hookrightarrow^+ \Delta$) if $\Gamma, \Delta \hookrightarrow \Delta$.

In a classical attack, the proponent shows, in effect, that her arguments are incompatible with that of the opponent. In a positive attack, the proponent temporarily accepts opponent's arguments in order to disprove the latter, while in a negative attack she shows that her arguments are sufficient for challenging an addition of opponent's arguments. Clearly, if Γ attacks Δ directly, then it attacks the latter classically, positively and negatively, though the reverse implications do not hold.

As can be seen, $\hookrightarrow^c$, $\hookrightarrow^-$ and $\hookrightarrow^+$ are also collective attack relations. Accordingly, given an argumentation theory $\mathbb{A} = (\mathcal{A}, \hookrightarrow)$, we will denote by $\mathbb{A}^c$ (respectively, $\mathbb{A}^-$, $\mathbb{A}^+$) the argumentation theory $(\mathcal{A}, \hookrightarrow^c)$ (resp. $(\mathcal{A}, \hookrightarrow^+)$, $(\mathcal{A}, \hookrightarrow^+)$). It turns out that such 'extended' argumentation theories can be given an invariant structural characterization in terms of additional rules imposed on the attack relation. These additional constraints will make the overall structure of such theories simpler and more regular. For explanatory reasons, we will describe first a simplest such kind, namely the classical argumentation. Negative and positive argumentation will be our subject in subsequent sections.

5.1 Classical argumentation

Classical argumentation can be seen as an 'upper bound' of collective argumentation; it is a simplest kind of argumentation which amounts, in effect, to classical consistency reasoning.

Definition 5.2. A collective argumentation theory will be called *classical* if $\Gamma, \Delta \hookrightarrow \Gamma, \Delta$ always implies $\Gamma \hookrightarrow \Delta$.

It can be easily verified that an argumentation theory based on a classical attack $\hookrightarrow^c$ will be classical. Moreover, the latter determines a least classical 'closure' of the source attack relation:

Lemma 5.1. *$\mathbb{A}^c$ is a least classical argumentation theory containing $\mathbb{A}$.*

Proof. Clearly, $\mathbb{A}^c$ includes $\mathbb{A}$. In addition, if $\hookrightarrow_1$ is a classical attack relation including $\hookrightarrow$, and $\Gamma \hookrightarrow^c \Delta$, then $\Gamma, \Delta \hookrightarrow \Gamma, \Delta$, and hence $\Gamma, \Delta \hookrightarrow_1 \Gamma, \Delta$. But $\hookrightarrow_1$ is classical, so $\Gamma \hookrightarrow_1 \Delta$. Thus, $\hookrightarrow_1$ should include $\hookrightarrow^c$, which shows that $\mathbb{A}^c$ is a least classical argumentation theory containing $\mathbb{A}$. □

An immediate consequence of the above lemma is that classical argumentation theories are precisely theories of the form $\mathbb{A}^c$. In other words, classical argumentation theories provide an invariant description of argumentation theories based on a classical attack.

The following result shows that the classical attack is fully symmetric:

Lemma 5.2. *An argumentation theory is classical if and only if it satisfies:*

(Symmetry) $\Gamma \hookrightarrow \Delta, \Phi$ *iff* $\Gamma, \Delta \hookrightarrow \Phi$.

Proof. If $\Gamma \hookrightarrow \Delta, \Phi$, then $\Gamma, \Delta, \Phi \hookrightarrow \Gamma, \Delta, \Phi$ by monotonicity, and hence $\Gamma, \Delta \hookrightarrow \Phi$. The reverse implication is similar. Now, if Symmetry holds, and $\Gamma, \Delta \hookrightarrow \Gamma, \Delta$, then $\Gamma, \Delta \hookrightarrow \Delta$, and consequently $\Gamma \hookrightarrow \Delta$. □

As a special case of Symmetry, we have

$$\Gamma \hookrightarrow \Delta \text{ iff } \emptyset \hookrightarrow \Gamma, \Delta \text{ iff } \Gamma, \Delta \hookrightarrow \emptyset$$

This shows that a classical attack amounts to inconsistency in a fully classical sense. As a result, a classical argumentation theory has very simple properties:

Lemma 5.3. *Let $\mathbb{A}$ be a classical argumentation theory. Then*

- *Admissible sets coincide with consistent sets;*
- *Stable sets coincide with maximal consistent sets;*
- *P-stable bitheories coincide with stable ones.*

The last fact above is especially instructive, since it shows that, unlike the very special case of a normal argumentation, partial stable sets need not form a lower semilattice.

5.1.1 Semantic characterization

As could be anticipated, classical argumentation theories can be characterized semantically by restricting the set of four-valued interpretations to classical two-valued ones. More exactly, a four-valued interpretation v will be called *classical* if it assigns only **t** and **f** to the arguments. This means that any argument is either accepted or rejected in an interpretation, but not both.

Theorem 5.4. *An argumentation theory is classical if and only if it is determined by a set of classical interpretations.*

Proof. As can be verified, any set of classical interpretations determines a classical argumentation theory. Now, if $\mathbb{A}$ is a classical argumentation theory, let V_c denote the set of (classical) interpretations corresponding to bitheories of the form (Γ, Γ). If $\Gamma \not\hookrightarrow \Delta$ in $\mathbb{A}$, then $\Gamma, \Delta \not\hookrightarrow \Gamma, \Delta$, and therefore the interpretation v corresponding to a bitheory $(\Gamma \cup \Delta, \Gamma \cup \Delta)$ belongs to V_c. But $\Gamma \hookrightarrow \Delta$ clearly does not hold in v, so $\Gamma \not\hookrightarrow_{V_c} \Delta$. This shows that $\mathbb{A}$ is determined by the set of classical interpretations V_c. □

Finally, our last result here shows that classical argumentation can be seen as a combination of positive and negative argumentation. The proof follows immediately from the definitions of respective attack relations.

Lemma 5.5. *For any argumentation theory* $\mathbb{A}$, $(\mathbb{A}^-)^+ = (\mathbb{A}^+)^- = \mathbb{A}^c$.

The above result says, in effect, that positive and negative argumentation are incompatible on pain of collapsing to classical reasoning. This fact will have important implications in what follows.

6 Negative Argumentation

The definition below provides a general description of collective argumentation based on a negative attack.

Definition 6.1. A collective argumentation theory will be called *negative* if $\Gamma \hookrightarrow \Gamma, \Delta$ always implies $\Gamma \hookrightarrow \Delta$.

To begin with, it can be easily verified that any collective argumentation theory of the form $\mathbb{A}^-$ will be negative. Moreover, the latter determines a least negative 'closure' of the source attack relation:

Lemma 6.1. *$\mathbb{A}^-$ is a least negative argumentation theory containing $\mathbb{A}$.*

Proof. Clearly, $\mathbb{A}^-$ is a negative argumentation theory containing $\mathbb{A}$. In addition, if $\hookrightarrow_1$ is a negative attack relation including $\hookrightarrow$, and $\Gamma \hookrightarrow^- \Delta$, then $\Gamma \hookrightarrow \Gamma, \Delta$, and hence $\Gamma \hookrightarrow_1 \Gamma, \Delta$. But $\hookrightarrow_1$ is negative, so $\Gamma \hookrightarrow_1 \Delta$. Thus, $\hookrightarrow_1$ should include $\hookrightarrow^-$, which shows that $\mathbb{A}^-$ is a least negative argumentation theory containing $\mathbb{A}$. □

An immediate consequence of the above lemma is that negative argumentation theories are precisely theories of the form $\mathbb{A}^-$. In other words, negative argumentation theories provide an invariant description of argumentation theories based on a negative attack.

The following result gives an important alternative characterization of negative argumentation; it establishes a correspondence between negative argumentation and shift operations from logic programming (see below).

Lemma 6.2. *An argumentation theory $\mathbb{A}$ is negative iff it satisfies:*

(Import) *If $\Gamma \hookrightarrow \Delta, \Phi$, then $\Gamma, \Delta \hookrightarrow \Phi$.*

Proof. If $\mathbb{A}$ is negative and $\Gamma \hookrightarrow \Delta, \Phi$, then $\Gamma, \Delta \hookrightarrow \Gamma, \Delta, \Phi$ by monotonicity, and hence $\Gamma, \Delta \hookrightarrow \Phi$. The reverse implication is immediate. □

As an important special case of Import, we have that if $\Gamma \hookrightarrow \Delta$, then $\Gamma, \Delta \hookrightarrow \emptyset$. Thus, any negative argumentation theory is bound to be non-affirmative. Furthermore, this implies that inconsistent argument sets attack any argument:

$$\text{If } \Delta \hookrightarrow \Delta, \text{ then } \Delta \hookrightarrow \Gamma$$

This feature will be responsible for the fact that only stable sets will constitute a reasonable semantics for negative argumentation.

6.1 Semantic characterization

Negative argumentation theories can be characterized semantically by restricting the set of possible four-valued interpretations to *consistent* ones that do not assign the set $\{\mathbf{t}, \mathbf{f}\}$ to the arguments. This means that no argument can be both accepted and rejected in an interpretation.

Theorem 6.3. *An argumentation theory is negative if and only if it is determined by a set of consistent interpretations.*

Proof. If an attack $\Gamma \hookrightarrow \Delta$ does not hold in a consistent interpretation v, then all arguments in Γ are accepted, and no argument from Δ is rejected in v. Since v is consistent, no argument in $\Gamma \cup \Delta$ is rejected in v, so the attack $\Gamma \hookrightarrow \Gamma, \Delta$ also does not hold in v. This shows that any argumentation theory determined by consistent interpretations is bound to be negative. In the other direction, if an argumentation theory $\mathbb{A}$ is negative, and $\Gamma \not\hookrightarrow \Delta$, then $(\Gamma, \Gamma \cup \Delta)$ is a bitheory (since $\Gamma \not\hookrightarrow \Gamma, \Delta$). Moreover, $\Gamma \hookrightarrow \Delta$ does not hold in the (consistent) interpretation corresponding to this bitheory. As follows from the proof of the main representation theorem, this means that any negative argumentation theory is determined by a set of consistent interpretations. □

Thus, negative argumentation theories describe argumentation situations in which arguments can be accepted, rejected, or neither accepted, nor rejected. A direct expression of this restriction can be given for N-argumentation theories. Namely, it can be shown that for such theories the rule Import is equivalent to the condition

$$\alpha, \sim\alpha \hookrightarrow \emptyset.$$

6.2 Negatively admissible and stable sets

To begin with, the following result shows that stable sets of a collective argumentation theory are precisely stable extensions of its negative closure.

Lemma 6.4. *A set of arguments is stable in a collective argumentation theory $\mathbb{A}$ if and only if it is a stable extension in $\mathbb{A}^-$.*

Proof. Using the definition of a negative attack, the characteristic condition of stable sets from Lemma 4.1 can be rewritten as follows:

$$\Gamma = \{\alpha \mid \Gamma \not\hookrightarrow^- \alpha\}$$

As can be seen, the latter condition coincides with the definition of stable extensions with respect to the normal sub-theory of $\mathbb{A}^-$. $\square$

Thus, stable extensions of an abstract argumentation theory and stable sets of collective argumentation are indeed close relatives. A further insight on the role of negative argumentation will be obtained using the notion of a negatively admissible set.

Definition 6.2. A consistent set of arguments Γ will be called *negatively admissible* if, for any set Δ, if $\Delta \hookrightarrow \Delta, \Gamma$, then $\Gamma \hookrightarrow \Delta, \Gamma$.

As can be seen from the definition, negatively admissible sets of $\mathbb{A}$ are precisely admissible sets with respect to the negative attack relation $\hookrightarrow^-$. Consequently, they coincide with admissible sets of $\mathbb{A}^-$. The main features of such sets can be discerned from the following quite surprising results about negative argumentation:

Theorem 6.5. *Let $\mathbb{A}$ be a negative argumentation theory. Then*

1. *If Γ is admissible in $\mathbb{A}$, $\Gamma \subseteq \Delta$ and Δ is consistent, then Δ is also admissible in $\mathbb{A}$.*

2. *Stable sets of $\mathbb{A}$ coincide with maximal admissible sets.*

3. *Stable sets of $\mathbb{A}$ coincide with stable extensions of $\mathbb{A}$.*

Proof. (1) Assume that Γ is admissible, $\Gamma \subseteq \Delta$ and $\Phi \hookrightarrow \Delta$. Then $\Phi \hookrightarrow \Delta, \Gamma$, and hence $\Phi, \Delta \hookrightarrow \Gamma$ by Import. Since Γ is admissible, we obtain $\Gamma \hookrightarrow \Phi, \Delta$, and hence $\Gamma, \Delta \hookrightarrow \Phi$ by Import. But the latter amounts to $\Delta \hookrightarrow \Phi$, which shows that Δ is also admissible.

(2) Let Γ be a stable set, and $\Delta \hookrightarrow \Gamma$. Then Δ is not a subset of Γ (since Γ is consistent) and hence $\Gamma \hookrightarrow \Gamma, \Delta$, since Γ is a maximal set that is not attacked by Γ. Applying Import, we obtain $\Gamma \hookrightarrow \Delta$, which shows that any stable set is admissible in negative argumentation theories. Moreover, any superset of a stable set will not already be consistent. Consequently stable sets are maximal admissible sets.

If Γ is a maximal admissible set and $\alpha \notin \Gamma$, then $\Gamma \cup \{\alpha\}$ will not be consistent by the preceding claim, and hence $\Gamma, \alpha \hookrightarrow \Gamma, \alpha$. Consequently $\Gamma, \alpha \hookrightarrow \Gamma$, and therefore $\Gamma \hookrightarrow \Gamma, \alpha$ (since Γ is admissible). This shows that Γ is actually a stable set.

(3) If $\mathbb{A}$ is a negative theory, then $\mathbb{A} = \mathbb{A}^-$, and the third claim follows from Lemma 6.4. $\square$

The above theorem demonstrates that the structure of admissible sets in negative argumentation theories is very simple; they behave much like logically consistent sets. There is, however, a crucial difference: the empty set $\emptyset$ is not, in general, admissible in negative argumentation. Moreover, a negative argumentation theory may have no admissible sets at all; this happens precisely when it has no stable sets.

Example. Consider the following collective argumentation theory $\mathbb{A}$:

$$\alpha, \beta \hookrightarrow \alpha, \beta \qquad \gamma \hookrightarrow \delta$$

Its negative counterpart $\mathbb{A}^-$ is a theory:

$$\alpha, \beta \hookrightarrow \emptyset \qquad \gamma \hookrightarrow \delta$$

As can be seen, the set $\{\alpha, \beta\}$ is inconsistent, so it attacks any set of arguments, though no consistent set of arguments attacks it. As a result, such a theory has no (negatively) admissible sets.

An important consequence of the above theorem is the following

Corollary 6.6. *For any collective argumentation theory* $\mathbb{A}$*, stable sets of* $\mathbb{A}$ *coincide with the stable sets of* $\mathbb{A}^-$*.*

Proof. Immediate from the third claim of the theorem and Lemma 6.4. □

The above corollary implies that Import is a rule that preserves stable sets of collective argumentation theories. Consequently, negative argumentation suggests itself as a natural and very simple framework for describing stable sets, since it identifies such sets with maximal admissible sets. Moreover, it guarantees that any admissible set is included in some stable one.

An additional consequence of the above results is the eventual reduction of stable sets to stable extensions of Dung's argumentation theory. Lemma 6.4 says, in effect, that, after extending a given collective argumentation theory to a negative one (by closing it with respect to Import), we can restrict ourselves to its normal sub-theory; stable extensions of the resulting normal argumentation theory will coincide with stable sets of the original collective argumentation theory.

Negative argumentation in logic programming. The rule Import corresponds to a *shift operation* in disjunctive logic programming that transforms a disjunctive rule of the form $A, C \leftarrow \mathbf{not}\, B$ into the rule $A \leftarrow \mathbf{not}\, B, \mathbf{not}\, C$. Shift operations have been studied extensively in the literature – see, e.g., [Dun92, DGM94, Sch95]. It has been shown, in particular, that this operation may change stable sets when applied to arbitrary program rules. However, the particular shift operation corresponding to Import is restricted to program rules without positive literals in bodies; only this operation has been shown to preserve stable sets. Moreover, Lemma 6.4 implies that stable models of a disjunctive program P coincide with stable models of a certain normal logic program. This program is obtainable by first reducing P to a program without positive atoms in bodies (using the principle of partial evaluation), and then applying shift operations for transferring head atoms of disjunctive rules into their bodies. Note, however, that a rule $A \leftarrow \mathbf{not}\, B$ can be 'shifted' also to a *constraint* $\leftarrow \mathbf{not}\, A, \mathbf{not}\, B$. This latter constraint derives every normal rule of the form $p \leftarrow \mathbf{not}\, A, \mathbf{not}\, B$. The adequacy of our translation of disjunctive programs into normal ones depends essential on such derived rules. Coupled with the fact that the other transformations used in the translation are also computationally expensive, this means that such a translation does not suggests itself as a reasonable computational procedure. Nevertheless, it seems to be important for studying (and justifying) semantics for disjunctive logic programs.

The results of this section hopefully demonstrate usefulness of negative argumentation in studying the stable semantics. But they also strongly suggest that negative argumentation is inappropriate for semantics beyond the stable one. One of the main incentives for introducing partial stable and well-founded models for normal programs was the desire to avoid taking stance on each and every literal and argument. In negative argumentation theories, however, self-contradictory arguments attack any argument whatsoever, so any admissible set must counter-attack any such argument. In particular, if $\Delta \hookrightarrow \Phi$, then any admissible set should attack $\Delta \cup \Phi$. This means that the well-founded semantics (and partial stable models) are no longer viable for such argumentation systems[3].

[3]despite some attempts made in this direction – see, e.g., [YYG00]. Actually, the same difficulty plagues attempts to define partial stable semantics for default logic.

7 Positive Argumentation

In this section we will provide some initial steps in studying positive argumentation.

Definition 7.1. A collective argumentation theory will be called *positive* if $\Gamma, \Delta \hookrightarrow \Delta$ always implies $\Gamma \hookrightarrow \Delta$.

Any collective argumentation theory $\mathbb{A}^+$ based on the positive attack relation will be positive. Moreover, the latter determines a least positive extension of the source attack relation.

Lemma 7.1. *$\mathbb{A}^+$ is a least positive argumentation theory containing $\mathbb{A}$.*

The proof is quite similar to the case of negative argumentation, so we will omit it. The lemma implies that positive argumentation theories are precisely theories of the form $\mathbb{A}^+$, and hence they give an invariant description of argumentation theories based on a positive attack.

Similarly to negative argumentation, positive argumentation can be characterized by the 'exportation' property described in the lemma below:

Lemma 7.2. *An argumentation theory is positive iff it satisfies:*

(Export) *If* $\Gamma, \Delta \hookrightarrow \Phi$*, then* $\Gamma \hookrightarrow \Delta, \Phi$.

The rule Export implies, in particular, that inconsistent arguments are attacked by any argument:

(**Safety**) If $\Delta \hookrightarrow \Delta$, then $\Gamma \hookrightarrow \Delta$.

So, in positive argumentation we are relieved, in effect, from the obligation to refute self-contradictory arguments. This feature can be seen as one of the main advantages of the positive argumentation.

Weak positivity. It can be shown that Safety is a weaker property than Export. Actually, Safety exactly corresponds to the following notion of an attack, introduced in [BDKT97]:

$$\Gamma \hookrightarrow^{\oplus} \Delta \ \equiv\ \Gamma \hookrightarrow \Delta \text{ or } \Delta \hookrightarrow \Delta$$

Let us say that an argumentation theory is *weakly positive*, if it satisfies Safety. Then it can be easily shown that weakly positive theories are precisely argumentation theories based on the above notion of an attack $\hookrightarrow^{\oplus}$.

7.1 Semantics

Positive argumentation can also be characterized semantically by restricting the set of possible four-valued interpretations to *complete* ones, namely to interpretations that do not assign $\emptyset$ to arguments. This means that every argument is either accepted or rejected in an interpretation (or both).

The proof of the following theorem is quite similar to the two earlier representation theorems, so we omit it.

Theorem 7.3. *An argumentation theory is positive if and only if it is determined by a set of complete interpretations.*

For N-argumentation theories, the rule Export is equivalent to the following condition:

$$\emptyset \hookrightarrow \alpha, \sim\alpha$$

The condition explicitly says that any argument should be either rejected or accepted. As could be expected, it is equivalent also to the principle of *reasoning by cases*:

(**Factoring**) If $\Gamma, \alpha \hookrightarrow \Delta$ and $\Gamma, \sim\alpha \hookrightarrow \Delta$, then $\Gamma \hookrightarrow \Delta$.

Reasoning by cases was actually the basis of an alternative semantics for normal logic programs suggested in [Sch92]. Further details about this semantics can be found in [Boc96]. Note, however, that the above rule Factoring was applied in [Sch92] to arbitrary program rules, so the resulting semantics did not satisfy the principle of partial evaluation. Our rule Export is effectively weaker in this sense, since in our 'reconstruction' of logic programs it should be applied only to the reduced program without positive atoms in bodies of its rules.

7.2 Positively admissible sets

As we mentioned, in positive argumentation we are relieved from the obligation to refute self-contradictory arguments. This also means, however, that positive argumentation is inadequate for describing stable sets. Actually, the following trivialization result holds:

Lemma 7.4. *If $\mathbb{A}$ is a positive argumentation theory, then*

- *negatively admissible sets coincide with consistent sets;*
- *stable sets coincide with maximal consistent sets.*

Proof. Negatively admissible sets of $\mathbb{A}$ coincide with admissible sets of $\mathbb{A}^-$. But since $\mathbb{A}$ is positive, $\mathbb{A}^-$ will coincide with the classical closure $\mathbb{A}^c$ (see Lemma 5.5), while in the latter admissible sets will coincide with consistent ones. Moreover, it has been established earlier that stable sets are maximal negatively admissible sets. Hence such sets will coincide now with maximal consistent sets. □

For the time being, the above results leave us with (positively) admissible sets as the only plausible kind of models for positive argumentation.

Definition 7.2. A consistent set of arguments Γ will be called *positively admissible* if $\Gamma \hookrightarrow^+ \Delta$ whenever $\Delta \hookrightarrow^+ \Gamma$.

Just as for negatively admissible sets, positively admissible sets of an argumentation theory $\mathbb{A}$ will coincide with admissible sets of its positive closure $\mathbb{A}^+$. Unfortunately, little is known about the properties of such sets in the general case. One such property is that the definition of such sets can be simplified in the affirmative case by dropping the requirement of consistency (cf. [KT99]).

Lemma 7.5. *If a positive argumentation theory is affirmative, then* Γ *is an admissible set if and only if* $\Gamma \hookrightarrow \Delta$*, for any* Δ *such that* $\Delta \hookrightarrow \Gamma$*.*

Proof. Assume that Γ is inconsistent, that is $\Gamma \hookrightarrow \Gamma$. Then $\emptyset \hookrightarrow \Gamma$ by Export, and hence $\Gamma \hookrightarrow \emptyset$, contrary to the assumption that the argumentation theory is affirmative. Hence Γ is consistent, and therefore it is an admissible set. □

Some further properties of positively admissible sets will be described below for an interesting special kind of positive argumentation theories.

7.3 Normal Positive Argumentation

As a matter of fact, positively admissible sets were introduced for normal programs in [KM91] under the name *weakly stable sets*; maximal such sets were termed *stable theories*. Indeed, our definition turns out to be equivalent (even for general collective argumentation) to the characterization of weakly stable sets, given in [KT99]:

For any Δ, if $\Delta \hookrightarrow \Gamma$, then $\Gamma, \Delta \hookrightarrow \Delta \backslash \Gamma$.

Accordingly, a study of weakly stable sets amounts to a study of admissible sets in positive argumentation theories of the form $\mathbb{N}^+$ such that $\mathbb{N}$ is a normal argumentation theory.

The following definition will provide a characterization of such argumentation theories.

Definition 7.3. A positive argumentation theory will be called *n-positive* if it is affirmative and satisfies

(**Semi-locality**) If $\Gamma \hookrightarrow \Delta, \Phi$, then either $\Gamma, \Delta \hookrightarrow \Phi$ or $\Gamma, \Phi \hookrightarrow \Delta$

Semi-locality could be seen as a 'non-deterministic' variant of the Import rule for negative argumentation. An interesting property of n-positive systems is that the union of two sets of arguments is inconsistent if and only if one of the sets attacks the other:

$$\text{If } \Gamma, \Delta \hookrightarrow \Gamma, \Delta, \text{ then either } \Gamma \hookrightarrow \Delta \text{ or } \Delta \hookrightarrow \Gamma$$

The following result shows that n-positive argumentation theories are precisely positive closures of normal theories.

Theorem 7.6. *A collective argumentation theory is n-positive iff it is a theory of the form* $\mathbb{N}^+$*, for some normal argumentation theory* $\mathbb{N}$*.*

Proof. Let $\mathbb{N}$ be a normal argumentation theory. Then $\mathbb{N}^+$ is clearly both positive and affirmative. Assume that $\Gamma \hookrightarrow^+ \Delta, \Phi$, that is, $\Gamma, \Delta, \Phi \hookrightarrow \Delta, \Phi$. By Locality of $\hookrightarrow$, we infer that either $\Gamma, \Delta, \Phi \hookrightarrow \Delta$ or $\Gamma, \Delta, \Phi \hookrightarrow \Phi$. In the first case we have $\Gamma, \Phi \hookrightarrow^+ \Delta$, while in the second one $\Gamma, \Delta \hookrightarrow^+ \Phi$. Hence $\mathbb{N}^+$ is also semi-local, and therefore it is n-positive.

Assume now that $\mathbb{A}$ is an n-positive argumentation theory, and let $\mathbb{N}_A$ be its normal sub-theory. We will show that $\mathbb{A}$ coincides with $\mathbb{N}_A^+$.

Note first that $\Gamma \hookrightarrow^+ \Delta$ holds in $\mathbb{N}_A$ iff $\Gamma, \Delta \hookrightarrow \alpha$ in $\mathbb{A}$, for some $\alpha \in \Delta$. Now, the latter holds only if $\Gamma, \Delta \hookrightarrow \Delta$, which implies $\Gamma \hookrightarrow \Delta$ by positivity. In the other direction, if $\Gamma \hookrightarrow \Delta$ in $\mathbb{A}$, and Δ is finite, then by applying Semi-locality a sufficient number of times, we will obtain that $\Gamma, \Delta \hookrightarrow \alpha$, for some $\alpha \in \Delta$. Thus, $\Gamma, \Delta \hookrightarrow \Delta$ in $\mathbb{N}_A$, and therefore $\Gamma \hookrightarrow \Delta$ in $\mathbb{N}_A^+$. This concludes the proof. □

Due to the above result, stable theories from [KM91] are exactly representable as maximal admissible sets in n-positive argumentation theories. Actually, a large part of Dung's argumentation theory can be reproduced in this setting.

7.3.1 Complete argument sets

Let us say that a set Δ is *acceptable* to a set of arguments Γ if $\Gamma \hookrightarrow \Phi$, for any Φ such that $\Phi \hookrightarrow \Delta$. Clearly, Γ is admissible if and only if it is acceptable with respect to itself. Also, if Δ is acceptable with respect to Γ, then any argument from Δ will also be acceptable. Accordingly, we will denote by $\langle\langle\Gamma\rangle\rangle$ the set of all arguments that are acceptable with respect to Γ. As can be verified, this set coincides with the union of all sets of arguments that are acceptable with respect to Γ[4].

Theorem 7.7. *Let $\mathbb{A}$ be an n-positive argumentation theory.*

- *If Γ is an admissible set, and Δ is acceptable with respect to Γ, then $\Gamma \cup \Delta$ is also admissible.*
- *If Γ is admissible, then $\langle\langle\Gamma\rangle\rangle$ is also admissible and $\Gamma \subseteq \langle\langle\Gamma\rangle\rangle$.*

Proof. (1) If $\Phi \hookrightarrow \Gamma, \Delta$, for some Φ, then by Semi-locality either $\Phi, \Gamma \hookrightarrow \Delta$ or $\Phi, \Delta \hookrightarrow \Gamma$. We will consider these two cases separately.

(a) If $\Phi, \Gamma \hookrightarrow \Delta$, then $\Gamma \hookrightarrow \Gamma, \Phi$, since Δ is acceptable with respect to Γ. Consequently $\emptyset \hookrightarrow \Gamma, \Phi$ by positivity, and therefore, by Semi-locality, either $\Gamma \hookrightarrow \Phi$, or $\Phi \hookrightarrow \Gamma$. In the first case we have $\Gamma, \Delta \hookrightarrow \Phi$, as required, while in the second case we have again $\Gamma \hookrightarrow \Phi$ by the admissibility of Γ, so it reduces to the first case.

(b) If $\Phi, \Delta \hookrightarrow \Gamma$, then $\Gamma \hookrightarrow \Phi, \Delta$ by admissibility of Γ, and hence by Semi-locality either $\Gamma, \Delta \hookrightarrow \Phi$, or $\Phi, \Gamma \hookrightarrow \Delta$. In the first case we are done, while the second case amounts to (a) above.

(2) If $\Phi \hookrightarrow \langle\langle\Gamma\rangle\rangle$, then by compactness there is a finite set $\{\Delta_1, \dots, \Delta_n\}$ of acceptable sets wrt Γ such that $\Phi \hookrightarrow \Delta_1, \dots, \Delta_n$. By applying the first claim a finite number of times, we have that $\Gamma \cup \Delta_1 \cup \dots \cup \Delta_n$ is an admissible set, and therefore it should counter-attack Φ. Moreover, since Γ is admissible, it should be acceptable wrt Γ, and hence it is included in $\langle\langle\Gamma\rangle\rangle$. Consequently, $\langle\langle\Gamma\rangle\rangle \hookrightarrow \Phi$, and hence $\langle\langle\Gamma\rangle\rangle$ is admissible. □

Let us introduce now the following definition.

Definition 7.4. An admissible set Γ will be said to be *complete* if $\Gamma = \langle\langle\Gamma\rangle\rangle$.

[4]For positively admissible sets, it coincides also with the set $+(\Gamma)$ of *direct implicit additions*, as defined in [KM91].

Complete sets are admissible sets that include all argument sets that are acceptable with respect to it. Accordingly, they can be seen as a generalization of complete extensions in Dung's argumentation theory. The analogy is supported also by the fact that complete sets are actually fixed points of the monotonic operator $\langle\langle\Gamma\rangle\rangle$, so the structure of such sets is quite similar to that of complete extensions (that are fixed points of the operator $[[\Gamma]]$). Moreover, the following consequence of Theorem 7.7 shows that maximal admissible sets (= stable theories) play the role of perfect extensions in this setting.

Corollary 7.8. *Maximal admissible sets of an n-positive argumentation theory coincide with maximal complete sets.*

In addition, for n-positive argumentation theories there always exists a least complete set (= a least fixed point of $\langle\langle\Gamma\rangle\rangle$); it can be seen as a counterpart of the well-founded semantics for positive theories (see [KM91]). Note, however, that the structure of complete sets is a little bit more complex than the structure of complete extensions in Dung's theory. Thus, the following example shows that complete sets do not form a lower semi-lattice.

Example. Consider the following normal argumentation theory:

$$\alpha,\beta,\gamma \hookrightarrow \alpha \quad \alpha,\beta,\gamma \hookrightarrow \beta \quad \delta \hookrightarrow \gamma \quad \epsilon \hookrightarrow \gamma \quad \delta \hookrightarrow \epsilon \quad \epsilon \hookrightarrow \delta$$

The positive closure of this theory has five complete sets, namely $\{\alpha,\beta,\delta\}$, $\{\alpha,\beta,\epsilon\}$, $\{\alpha\}$, $\{\beta\}$ and $\emptyset$. As can be seen, $\{\alpha\}$ and $\{\beta\}$ are *two* maximal complete sets that are included in $\{\alpha,\beta,\delta\}$ as well as in $\{\alpha,\beta,\epsilon\}$.

Our final observation describes stable extensions of n-positive argumentation theories as sets that possess all the good properties argument sets could possibly have.

Lemma 7.9. Γ *is a stable extension of an n-positive argumentation theory if and only if it is an admissible and maximal consistent set.*

Proof. Note first that Γ is consistent in an n-positive argumentation theory iff it is conflict free (indeed, if $\Gamma \hookrightarrow \Gamma$, then by semi-locality there is $\alpha \in \Gamma$ such that $\Gamma \hookrightarrow \alpha$). Now Lemmas 4.5 and 7.4 imply that any stable extension in our context will be both admissible and maximal consistent. In the other direction, assume that Γ is an admissible and maximal consistent set. If $\alpha \notin \Gamma$, then $\Gamma \cup \{\alpha\}$ will already be inconsistent, that is $\Gamma,\alpha \hookrightarrow \Gamma,\alpha$. Then

$\emptyset \hookrightarrow \Gamma, \alpha$ by positivity, and hence by semi-locality either $\Gamma \hookrightarrow \alpha$, or $\alpha \hookrightarrow \Gamma$. But in the second case we also have $\Gamma \hookrightarrow \alpha$ (due to admissibility of Γ), so $\Gamma \hookrightarrow \alpha$ will hold in any case. This shows that Γ is a stable extension. □

8 Ordered argumentation

Finally, we will briefly describe an argumentation system that constitutes a common ground for both positive and negative argumentation.

Definition 8.1. A collective argumentation theory will be called *ordered* if it satisfies the following rule:

(**Ordering**) If $\Gamma \hookrightarrow \Delta, \Phi$ and $\Gamma, \Psi \hookrightarrow \Delta$, then $\Gamma, \Phi \hookrightarrow \Delta, \Psi$.

It is easy to verify that Ordering is a consequence of both Import and Export rules. Hence it is a valid rule for both positive and negative argumentation. Moreover, the following result shows that it covers precisely the common ground of these two kinds of argumentation.

Theorem 8.1. *A collective argumentation theory is ordered if and only if it is an intersection of a positive and a negative argumentation theory.*

Proof. The implication from right to left follows from the fact that Ordering is valid for both positive and negative argumentation theory. In the other direction, we will prove that if $\mathbb{A} = (\mathcal{A}, \hookrightarrow)$ is an ordered argumentation theory, it should coincide with the intersection of $\mathbb{A}^-$ and $\mathbb{A}^+$. Indeed, if $\Gamma \hookrightarrow^+ \Delta$ and $\Gamma \hookrightarrow^- \Delta$, then $\Gamma, \Delta \hookrightarrow \Delta$ and $\Gamma \hookrightarrow \Gamma, \Delta$. By Ordering we conclude $\Gamma \hookrightarrow \Delta$, which shows that the intersection of $\mathbb{A}^-$ and $\mathbb{A}^+$ is included in $\mathbb{A}$. The reverse inclusion is immediate. □

As a consequence of the above result, ordered argumentation can also be given an obvious semantics:

Corollary 8.2. *An argumentation theory is ordered if and only if it is determined by a set of interpretations that are either consistent, or complete.*

It remains to be seen whether this kind of collective argumentation combines the virtues of both negative and positive argumentation, or preserves the shortcomings of each.

9 Conclusions and perspectives

Collective argumentation suggests itself as a natural extension of the abstract argumentation theory that allows to describe kinds of argumentation based on combined and shared arguments. This form of argumentation possesses a four-valued semantics, and is shown to be expressible in the framework of Belnap consequence relations.

Collective argumentation theories naturally correspond to disjunctive logic programs. Moreover, they have been shown to be capable of representing practically any semantic suggested for such programs, though in this study we have restricted ourselves mainly to the stable semantics and semantics based on admissibility. It has been shown, in particular, that a special kind of negative argumentation is especially suitable for the stable semantics, while the complementary positive argumentation seems plausible for studying admissible sets of arguments.

One of the main open issues in our study consists in determining general acceptability principles underlying collective argumentation. This task is also intimately connected with an applied problem of extending the collective argumentation approach to other, weaker semantics suggested for disjunctive programs. One of the key principles behind the latter is that arguments are acceptable not so much because they counterattack any argument against them, but rather because in any larger context of argument sets that are attacked, such arguments cannot be blamed for the existence of an attack. As an initial instantiation of this idea, let us introduce the following notion:

Definition 9.1. An argument α is *strongly acceptable* if, for any sets Γ, Δ such that $\Gamma \hookrightarrow \Delta, \alpha$, we also have $\Gamma \hookrightarrow \Delta$.

The above definition says, in effect, that the argument α is inessential (irrelevant) in all attacked argument sets. Consequently, it can be safely accepted. Note, in particular, that any strongly acceptable argument will belong to every stable set (and even to any partial stable bitheory). Accordingly, such arguments could be seen as part of a certain *well-founded* set of arguments that should be acceptable in any stronger model of collective argumentation. Such argument sets deserve a further study.

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