Elena Afanas'eva†, Anatoly Golberg††,
Ruslan Salimov†††

† Institute of Applied Mathematics and Mechanics
Slovyansk, Ukraine
e-mail: es.afanasjeva@gmail.com

††: Holon Institute of Technology
Holon, Israel
e-mail: golberga@hit.ac.il

†††: Institute of Mathematics of NAS of Ukraine
Kiev, Ukraine
e-mail: ruslan.salimov1@gmail.com

Finite mean oscillation in upper regular metric spaces

In this talk, we discuss the boundary behavior of mappings with integrally restricted conformal moduli in the general metric spaces with measures [1]. Recall that the space (X, d, μ) is called *(Ahlfors) α-regular*, if there exists a constant $C \geqslant 1$ such that

$$C^{-1}\varepsilon^{\alpha} \leqslant \mu(B(x_0, \varepsilon)) \leqslant C\varepsilon^{\alpha}$$

for all balls $B(x_0, \varepsilon)$ centered at $x_0 \in X$ of radius $\varepsilon < \operatorname{diam} \mathrm{X}$.

We consider much general condition. Let $\lambda : \mathbb{R}^+ \to \mathbb{R}^+$ be continuous increasing function satisfying $\lambda(0) = 0$. We say that the space (X, d, μ) is *upper $\lambda(\varepsilon)$-regular at a point $x_0 \in X$*, if

$$\mu(B(x_0, \varepsilon)) \leqslant \lambda(\varepsilon) \tag{1}$$

for all balls $B(x_0, \varepsilon)$ centered at $x_0 \in X$ with radius $\varepsilon < \varepsilon_0$ for some $\varepsilon_0 > 0$. The space (X, d, μ) is called *upper $\lambda(\varepsilon)$-regular* if (1) holds at every $x_0 \in X$. Assuming $\lambda(\varepsilon) = C\varepsilon^{\alpha}$, we arrive at the upper α-regular spaces.

Let D be a domain in an upper $\lambda(\varepsilon)$-regular space (X, d, μ). We say that a function $\varphi : D \to \mathbb{R}$ has a *finite mean oscillation at a point $x_0 \in \overline{D}$* if

$$\overline{\lim_{\varepsilon \to 0}} \; \frac{1}{\mu(D(x_0, \varepsilon))} \int_{D(x_0,\varepsilon)} |\varphi(x) - \overline{\varphi}_{\varepsilon}| \; d\mu(x) < \infty \,, \tag{2}$$

where

$$\overline{\varphi}_{\varepsilon} = \frac{1}{\mu(D(x_0, \varepsilon))} \int_{D(x_0,\varepsilon)} \varphi(x) \; d\mu(x) = \frac{1}{\mu(D(x_0, \varepsilon))} \int\limits_{D(x_0,\varepsilon)} \varphi(x) \; d\mu(x)$$

means the mean value of $\varphi(x)$ over the set $D(x_0, \varepsilon) = \{x \in D : d(x, x_0) < \varepsilon\}$ with respect to the measure μ. The condition (2) assumes that φ is μ-integrable over $D(x_0, \varepsilon)$, $\varepsilon > 0$.

Our main result states:

Theorem. *Suppose that* $\widetilde{\lambda}(s)=\lambda(s)/s$ *is a continuous increasing function on* $(0;1]$*, and* λ *satisfies*

$$(3)\qquad \int\limits_{\log_2^{-1}\frac{1}{\varepsilon}}^{1}\frac{\lambda(s)}{s^3}\,ds=o\left(\int\limits_{\varepsilon}^{\varepsilon_0}\left(\frac{\lambda\left(\log_2^{-1}\frac{1}{t}\right)}{\lambda(2t)}\right)^{\frac{1}{\alpha}}dt\right)\quad as\quad \varepsilon\to 0\,.$$

Let D *be a locally linearly connected domain in an upper* $\lambda(\varepsilon)$*-regular space* (X,d,μ)*, which satisfies* $\mu(D\cap B(x_0,2r))\leqslant\gamma\cdot\mu(D\cap B(x_0,r))$*,* $\forall\, r\in(0,r_0)$*, at all boundary points. Let* D' *be a domain in the space* (X',d',μ') *with a weakly flat boundary, and assume that both* D *and* D' *are compacta. If a function* $Q:D\to[0,\infty]$ *has a finite mean oscillation at all boundary points then any ring* Q*-homeomorphism* $f:D\to D'$ *can be extended to a homeomorphism* $\overline{f}:\overline{D}\to\overline{D'}$*.*

References

[1] E. Afanas'eva, A. Golberg, R. Salimov. *Finite mean oscillation in upper regular metric spaces.* Lobachevskii J. Math. 38 (2017), no. 2, 206–212.

Lucian Beznea

Simion Stoilow Institute of Mathematics of the Romanian
Academy and University of Bucharest,
Faculty of Mathematics and Computer Science
Bucharest, Romania
e-mail: lucian.beznea@imar.ro

Connections between the Dirichlet and the Neumann problem

We give a representation of the solution of the Neumann problem for the Laplace operator on the unit ball in $\mathbb{R}^n$, $n\geq 1$, in terms of the solution of an associated Dirichlet problem. The representation is suitable for extensions to other operators besides the Laplacian, smooth planar domains, and the infinite dimensional case. It also holds in the case of integrable boundary data. We derive an explicit formula for the Dirichlet-to-Neumann operator, and provide an explicit solution of the generalized solution of the Neumann problem. The talk is based on joint works with **Mihai N. Pascu** and **Nicoale R. Pascu**.

Vladimir Bolotnikov

The College of William and Mary
Williamsburg, VA, USA
e-mail: vladi@math.wm.edu

Carathéodory-Fejér type interpolation problems for Stieltjes functions

Let $\mathcal{P}$ denote the Pick class of analytic self-mappings of the upper half plane. The Stieltjes class $\mathcal{S}$ can be defined as the class of Pick functions $f(z)$ such that $zf(z)$ is also in $\mathcal{P}$. Stieltjes functions turn out to be analytic on $(-\infty, 0)$ and their restrictions to $(-\infty, 0)$ are characterized as nonnegative operator monotone functions on $(-\infty, 0)$. The Carathéodory-Fejér problem $\mathcal{CF}_n(\rfloor S, x_0)$ consists of finding a function $f \in \mathcal{S}$ with prescribed $f^{(j)}(x_0) = c_j$ $(j = 0, \ldots, n-1)$ if $x_0 \in \mathbb{C} \backslash (-\infty, 0)$. If $x_0 \geq 0$, then by $f^{(j)}(x_0)$ we mean the limit of $f^{(j)}(z)$ as z tends to x_0 nontangentially.

Roughly speaking, the problem $\mathcal{CF}_n(\mathcal{S}, x_0)$ has a solution if and only if two *Pick matrices* constructed from interpolation data are positive semidefinite, and the problem is indeterminate if and only if both matrices are positive definite, in which case the solution set is parametrized by a linear fractional formula with a free Stieltjes-class parameter. However, this is exactly the case if either $x_0 \notin \mathbb{R}$ or $x_0 < 0$ and n is even. It is almost the case if $x_0 = 0$ or $x_0 > 0$, n is even and all target values are real. In the talk, we will focus on the two remaining cases: (1) $x_0 < 0$ and n is odd and (2) $x_0 > 0$ and the target values are not necessarily real.

Marek Bożejko

Institute of Mathematics
Polish Academy of Sciences
Poland
bozejko@math.uni.wroc.pl

Generalized Gaussian processes and relations with random matrices and positive definite functions on permutation groups

In the talk we will consider the following subjects:

(1) Definitions and remarks on pair-partitions.
(2) Markov random matrices and function h on pair-partitions – $\mathcal{P}_2(2n)$ related to singleton blocks.
(3) Generalized strong Gaussian processes (fields) $\{G(f),\ f \in \mathcal{H}\}$, $\mathcal{H}$ - real Hilbert space, related to the function $b^{h(V)}$.
(4) New positive positive definite functions on permutation (Coxeter) groups.

(5) Construction of the new generalized Fock spaces and generalized Gaussian processes related to Coxeter groups of type B/C and D.

References

[1] Bożejko M., Bożejko W., Generalized Gaussian processes and relations with random matrices and positive definite functions on permutation groups, Infinite Dimensional Analysis, Quantum Probability and Related Topics Vol. 18, No. 3 (2015), 1550020-1 - 1550020-19. DOI: 10.1142/S0219025715500204

[2] M.Bożejko, R.Speicher, Interpolation between bosonic and fermionic relations given by generalized Brownian motion, Math.Zeit. 222(1996),135-159.

[3] M.Bożejko,H.Yoshida, Generalized q-deformed Gaussian random variables,Banach Center Publications,Warszawa 2008.

[4] W.Bryc,A.Dembo, T.Jiang, Spectral measure of large random Hankel,Markov and Toeplitz matrices, Annals of Prob. 34(2006),1-38.

[5] S.Belinschi, M.Bożejko, F.Lehner, R.Speicher, The normal distribution is free-infinitely divisible, Adv.Math.226(2011),3677-3698.

[6] A.Buchholz, New interpolations between classical and free Gaussian processes, Probability and Mathematical Statistics Vol. 35(2) (2015), 181189.

Arthur A. Danielyan

University of South Florida
Florida, USA
e-mail: adaniely@usf.edu

On bounded analytic functions

By Fatou's fundamental theorem of 1906, a bounded analytic function f in the open unit disc D has radial (even non tangential) limits at the points of the unit circle T except a subset E of measure zero. It is a well-known elementary (topological) fact that the exceptional set E is a $G_{\delta\sigma}$. But if f has unrestricted limits at each point of $T \setminus E$, then obviously E becomes just an F_σ set. We show that the converse statement is true as well. Namely, we prove the following:

Theorem 1. *Let E be a subset on T. There exists a bounded analytic function in D which has no radial limits on E but has unrestricted limits at each point of $T \setminus E$ if and only if E is an F_σ set of measure zero.*

An obvious corollary of Theorem 1 is the Lohwater-Piranian theorem of 1957: If E is an F_σ set of measure zero on T then there exists a bounded analytic function in D which has no radial limits exactly on E. In 1994 Kolesnikov has proved the following remarkable converse of Fatou's theorem: There exists a bounded analytic function in D which has no radial limits exactly on the given set E (on T) if and only if E is a $G_{\delta\sigma}$ set of measure zero. Both Theorem 1 and Kolesnikov's theorem extend the Lohwater-Piranian theorem up to necessary and sufficient results, *but in different directions.* The method of the proof of

Theorem 1 is *completely elementary* and it offers some simplification even for the proof of Kolesnikov's theorem.

Vadim Derkach

Department of Mathematics, Technion
Haifa, Israel
e-mail: vadder@technion.ac.il

Equilibria in Thin Polycrystalline Systems: Surface Diffusion and Mean Curvature Motion

The motion of a thin polycrystalline system is a quite complicated process, involving a large variety of physical effects. To gain intuition, we focus on a relatively classical model in which the exterior surface is taken to evolve by motion by surface diffusion and the grain boundaries, which separate between neighboring grains, are taken to evolve by mean curvature motion. We also take into account the coupled motion of the grain boundaries and the exterior surface along thermal grooves, as well as balance of mechanical forces, continuity of chemical potential, balance of mass flux, and certain symmetry assumptions along the boundaries. In the context of these assumptions, we consider a number of special 3D geometries and demonstrate existence of the steady states under appropriate conditions. Recent results reflect joint work with A. Novick-Cohen and J. McCuan.

References

[1] V. Derkach, J. McCuan, A. Novick-Cohen, A. Vilenkin. Geometric interfacial motion: coupling surface diffusion and mean curvature motion. In: *Mathematics for Nonlinear Phenomena - Analysis and Computation*, Eds: Y. Maekawa, S. Jimbo. Springer Proceedings in Math & Statistics, V. 215, pp.23–46, Springer, 2017.

[2] V.Derkach, A.Novick-Cohen, A.Vilenkin. *Grain boundary migration with thermal grooving effects: a numerical approach.* J. Elliptic Parabol. Equ., V. 2, (2016), pp.389-413.

Sergey Favorov

Karazin's Kharkiv National University
Kharkiv, Ukraine
e-mail: sfavorov@gmail.com

Quasicrystals Fourier and tempered distributions

Theorem 1. *Let μ be a "large" quasicrystal Fourier, i.e., complex measure on $\mathbf{R}^d$ with a uniformly discrete support Λ, such that $|\mu(\{\lambda\})| \geq c > 0$ for all $\lambda \in \Lambda$, and the distributional Fourier transform $\hat{\mu}$ be a measure with a countable support. If $|\hat{\mu}|(B(r)) = O(r^d)$ as $r \to \infty$, then Λ is a finite union of translates of* **several** *full-rank lattices.*

Let

$$f_j = \sum_{\lambda \in \Lambda_j,\ k_1 + \cdots + k_d \leq m(\lambda)} p_{\lambda,k}^{(j)} D^k \delta_\lambda, \quad k \in (\mathbf{N} \cup \{0\})^d, \quad j = 1, 2,$$

be tempered distributions on $\mathbf{R}^d$ with discrete supports Λ_j. It can be easily checked that $\sup_{\lambda \in \Lambda_j} m(\lambda) < \infty$.

Theorem 2. *Suppose that $\inf_{\lambda \in \Lambda_j} \sup_k |p_{\lambda,k}^{(j)}| > 0$ and the set $\{\lambda_1 - \lambda_2 :\ \lambda_1 \in \Lambda_1, \lambda_2 \in \Lambda_2\}$ is discrete. If distributional Fourier transforms $\hat{f}_j$ are discrete complex measures with supports Γ_1, Γ_2 satisfied the condition*

$$(*) \qquad \exists h > 0,\ c > 0 \quad \text{such that} \quad |\gamma - \gamma'| > c \min\{1, |\gamma|^{-h}\} \quad \forall \gamma, \gamma' \in \Gamma,$$

then Λ_1, Λ_2 are finite unions of translates of a **unique** *full-rank lattice.*

The proofs of these theorems are based on generalization of Wiener's Theorem on Fourier series and properties of almost periodic distributions and measures. In particular, we prove

Theorem 3. *Every tempered distribution whose Fourier transform is a discrete complex measure with support satisfied condition $(*)$ is almost periodic.*

Avraham Feintuch

Ben Gurion University of the Negev
Beer Sheva, Israel
e-mail: abie@math.bgu.ac.il

Serial Pursuit, Shift Operators on l^∞, and Borel Summability

Consider a bounded infinite sequence of kinematic points in the complex plane where each point heads toward the sum of the relative displacements to its two neighbours.The study of the asymptotic behaviour of this system requires the study of the shift operator on the Banach space of bounded complex sequences and is connected to the Borel summability of these sequences.

Aimo Hinkkanen

University of Illinois at Urbana-Champaign
Urbana, IL, USA
e-mail: aimo@math.uiuc.edu

Quasiconformal mappings and the skew of triangles

The skew of a topological triangle is the ratio between the largest and smallest Euclidean distances between any two distinct vertices (thus only the vertices matter, not the shape of the sides). This concept was introduced by John Hubbard who proved that if, for a homeomorphism between plane domains, the skew of the image triangle is bounded for all Euclidean triangles in the domain of definition with skew below a certain absolute constant (about 1.53), then the mapping is quasiconformal. Hubbard asked whether it suffices to consider only equilateral triangles. Javier Aramayona and Peter Haïssinsky showed that there exists a constant $\varepsilon > 0$ such that it is sufficient that the skew of the image of every equilateral triangle is at most $1 + \varepsilon$. In joint work with Colleen Ackermann and Haïssinsky, we have proved Hubbard's conjecture, that is, the mapping is quasiconformal if there is a uniform bound for the skew of the image of every equilateral triangle. It is possible to base the proof on the metric or the analytic definition of quasiconformality. We also obtain a bound for the maximal dilatation of the mapping in terms of the bound for the skew.

References

[1] C. Ackermann, P. Haïssinsky, and A. Hinkkanen. *Equilateral triangle skew condition for quasiconformality.* arXiv:1606.06553v1.

[2] J. Aramayona and P. Haïssinsky. *A characterisation of plane quasiconformal maps using triangles.* Publ. Mat., 52 (2), (2008), pp. 459–471.

[3] J.H. Hubbard. *Teichmüller theory and applications to geometry, topology, and dynamics. Vol. 1.* Matrix Editions, Ithaca, NY, USA, 2006.

Ritva Hurri-Syrjänen

University of Helsinki
Helsinki, Finland
e-mail: ritva.hurri-syrjanen@helsinki.fi

On Neumann eigenvalues and quasiconformal mappings

The goal of my talk is to address the question on lower estimates of Neumann eigenvalues of the p-Laplace operator for domains in the Euclidean n-space. The idea is to transfer a known (r, q)- Sobolev-Poincaré inequality in a given domain Ω to an (s, p)-Sobolev-Poincaré inequality in $\tilde{\Omega}$ which is the image of Ω under a given quasiconformal mapping. We do this by using the geometric theory of composition operators induced by this quasiconformal homeomorphism and the quasiconformal mapping theory. As corollaries we obtain lower estimates of Neumann eigenvalues in $\tilde{\Omega}$.

My talk is based on joint work with Vladimir Gol'dshtein and Alexander Ukhlov, as well as with Susan Staples.

References

[1] Vladimir Gol'dshtein, Ritva Hurri-Syrjänen, and Alexander Ukhlov. *Space quasiconformal mappings and Neumann eigenvalues in fractal type domains.* Georgian Math. J. (online April 2018).

[2] Ritva Hurri-Syrjänen and Susan G. Staples *Quasiconformal maps and Poincaré domains. Rocky Mountain J. Math.*, 26 (1996),pp. 1395–1423.

Tomoki Kawahira

Tokyo Institute of Technology
Tokyo, Japan
e-mail: kawahira@math.titech.ac.jp

On dynamical and parametric Zalcman functions

We apply a version of Zalcman's lemma ([5, 6]) to

- dynamics of rational maps on the Riemann sphere; and
- the bifurcation locus of a family of rational maps.

Then we have families of non-constant meromorphic functions that we call the *dynamical* and *parametric Zalcman functions* (after Steinmetz [3]). In this talk, we present some basic properties of these families and give a simple proof of the local similarities between the Julia sets, Mandelbrot set, and the tricorn ([4, 2, 1]) by using interactions of dynamical and parametric Zalcman functions.

References

[1] T. Kawahira. *Quatre applications du lemme de Zalcman à la dynamique complexe.* J. d'Analyse Math., 124, (2014), pp.309–336.

[2] J.E. Rivera-Letelier. *On the continuity of Hausdorff dimension of Julia sets and similarity between the Mandelbrot set and Julia sets.* Fund. Math. 170, (2001) pp.287–317.

[3] N. Steinmetz. *Zalcman functions and rational dynamics.* New Zealand J. Math. 32, (1), (2003), pp.91–104.

[4] Tan L. Similarity between the Mandelbrot set and Julia sets. *Comm. Math. Phys.* 134, (1990), pp.587–617

[5] L. Zalcman. *A heuristic principle in complex function theory.* Amer. Math. Monthly 82(8),(1975), pp.813 – 818.

[6] L. Zalcman. *Normal families: new perspectives.* Bull. Amer. Math. Soc. 35, (1998), pp.215–230.

Eva Kopecká

University of Innsbruck
Innsbruck, Austria
e-mail: eva.kopecka@uibk.ac.at

Orthogonal projections: slow convergence and divergence of their products

Abstract: We investigate when slow convergence of cyclic products of projections implies divergence of random products of projections and vice versa.

Let X and Y be two closed subspaces of a Hilbert space. If we send a point back and forth between them by orthogonal projection, the iterates converge to the projection of the point on the intersection of X and Y. Already on three subspaces X, Y and Z we can project either cyclically as above: $X, Y, Z, X, Y, Z, X, Y, Z, \ldots$, or "randomly", for example: $X, Y, X, Y, Z, Y, X, Y, Z, Y, Z, \ldots$.

It turns out that these two cases possibly result in a completely different (non-)convergence behavior: the cyclic products always converge, the random products may diverge. However, it is well known that cyclic products converge fast if and only if $X^\perp + Y^\perp + Z^\perp$ is closed. Geometrically speaking, if X, Y, and Z "almost touch", the cyclic products converge arbitrarily slowly. We give such a geometric condition for the existence of diverging "random" products.

Yakov Krasnov

Bar-Ilan University
Ramat-Gan, Israel
e-mail: krasnov@math.biu.ac.il

Elements of Spectral Theory in Algebras

In this talk, we study the variety of all nonassociative (NA) algebras from the idempotent point of view. We are interested, in particular, in the spectral properties of idempotents when algebra is generic, i.e. idempotents are in general position. Our main result states that in this case, there exist at least $n-1$ nontrivial obstructions (syzygies) on the Peirce spectrum of a generic NA algebra of dimension n. We also discuss the exceptionality of the eigenvalue $\lambda = \frac{1}{2}$ which appears in the spectrum of idempotents in many classical examples of NA algebras and characterize its extremal properties in metrised algebras.

We establish the basic properties and restrictions of the set of the idempotents in finite dimensional non-associative real and complex algebras using the syzygies among the Peirce numbers as spectral parameters.

We study the reciprocal influence of the Peirce numbers allocation on the geometry of trajectories of ODE's in algebra. The canonical form of generic algebra will be present also. (Joint work with V.Tkatchev).

Samuel L. Krushkal

Bar-Ilan University
Ramat Gan, Israel
e-mail: krushkal@math.biu.ac.il

Quasiconformal features and Fredholm eigenvalues of convex polygons

An important still open problem in geometric complex analysis is to find the algorithms for explicit determination of the basic functionals intrinsically connected with conformal and quasiconformal maps such as their Teichmüller and Grunsky norms, Fredholm eigenvalues and the quasireflection coefficient. It has not even be solved for convex polygons. This case has intrinsic interest in view of the connection of such polygons with the geometry of the universal Teichmüller space.

We provide a new approach based on affine transformations of univalent functions.

Bochen Liu

Bar-Ilan University
Ramat Gan, Israel
e-mail: Bochen.Liu1989@gmail.com

An L^2 identity and pinned distance problem

Given a measure on a subset of Euclidean spaces. The L^2 spherical averages of the Fourier transform of this measure was originally used to attack Falconer distance conjecture, via Mattila's integral. In this talk, we will consider pinned distance problem, a stronger version of Falconer distance problem, and show that spherical averages imply the same dimensional threshold on both problems. In particular, with the best known spherical averaging estimates, we improve a result of Peres and Schlag on pinned distance problem significantly.

The idea is to reduce the pinned distance problem to an integral where spherical averages apply. The key new ingredient is an identity between square functions. Invariant measures on orthogonal groups play an important role.

M. Elena Luna–Elizarrarás

Holon Institute of Technology
Holon, Israel
e-mail: lunae@hit.ac.il

On fractal–like constructions in four dimensions using bicomplex numbers

The set of bicomplex numbers $\mathbb{BC}$ is a commutative ring with a peculiar subset inside it: the set of hyperbolic numbers $\mathbb{D}$. This set plays inside $\mathbb{BC}$ a quite similar role to the that played by the set of real numbers $\mathbb{R}$ inside the complex numbers.

In [2] there was introduced the notion of hyperbolic curves, which are two dimensional surfaces in $\mathbb{R}^4$ but with a very specific orientation. In this talk will be presented how these hyperbolic curves together with some bicomplex functions are useful to get more understanding about the construction and the geometrical meaning of the cartesian product of some classical fractals.

This talk is based on a joint work with Alexander Balankin and Michael Shapiro.

References

[1] A.S. Balankin, J. Bory–Reyes, M.E. Luna–Elizarrarás, M. Shapiro. *Cantor–type sets in hyerbolic numbers.* Fractals 24, (2016).

[2] M. E. Luna-Elizarrarás, M. Shapiro, D. C. Struppa, A. Vajiac. *Bicomplex holomorphic functions: The algebra, geometry and analysis of bicomplex numbers.* Frontiers in Mathematics, Springer Birkhäuser, 2015.

Anthony G. O'Farrell

National University of Ireland, Maynooth
Maynooth, Ireland
e-mail: anthony.ofarrell@mu.ie

Boundary values of holomorphic distributions

We consider the behaviour at a boundary point of an open subset $U \subset \mathbb{C}$ of distributions that are holomorphic on U and belong to what are called negative Lipschitz classes. The result explains the significance for holomorphic functions of Wiener-type series involving Hausdorff contents of dimension between 0 and 1. We begin with a survey about function spaces and capacities that sets the problem in context and reviews the relevant general theory.

Fedor Pakovich

Ben–Gurion University of the Negev
Beer Sheva, Israel
e-mail: pakovich@math.bgu.ac.il

Commuting rational functions revisited

We present new results concerning the classical problem of describing commuting rational functions of one complex variable.

Victor Palamodov

School of Mathematical Sciences,
Tel Aviv University
Tel Aviv, Israel
e-mail: palamodo@post.tau.ac.il

An analytic method of reconstruction for X-ray phase contrast imaging

The method of X-ray phase contrast imaging in the near-field propagation regime is effective for microscale imaging of biological tissues. Using hard coherent beams this method permits better contrast comparing with the routine X-ray tomography but involves a phase retrieval problem since of physical limitation of detectors.

Explicit formulas were proposed for reconstruction of the complex refraction index of an object from intensity distribution of one hologram. The method is based on the classical complex analysis [1].

References

[1] V. Palamodov. *A method of reconstruction for X-ray phase contrast imaging with arbitrary Fresnel number.* arxiv:math.NA 1803.08938, (2018) pp.1-13.

Valerii Pchelintsev

Tomsk Polytechnic University
Tomsk, Russia;
Ben–Gurion University of the Negev
Beer Sheva, Israel
e-mail: vapchelincev@tpu.ru

Applications of the quasiconformal geometry to the Neumann eigenvalue problem

In this talk we give applications of the quasiconformal geometry to the spectral estimates of the p-Laplace operator ($1 < p \leq 2$) with the Neumann boundary condition for a large class of simply connected planar domains. This class includes Lipschitz domains, Gehring domains and some fractal domains (snowflakes).

Our main technique is based on the geometrical theory of composition operators on Sobolev spaces generated by quasiconformal mappings. This study leads to estimates of the Neumann eigenvalues in terms of the quasihyperbolic geometry of domains.

The talk is based on joint works with Vladimir Gol'dshtein and Alexander Ukhlov.

Research was supported by RFBR Grant no 18-31-00011.

References

[1] V. Gol'dshtein, V. Pchelintsev, A. Ukhlov. *Spectral Properties of the Neumann-Laplace Operator in Quasiconformal Regular Domains.* Dedicated to the memory of S. G. Krein: "Krein 100 anniversary AMS volume". (In press).

[2] V. Gol'dshtein, V. Pchelintsev, A. Ukhlov. *Spectral Estimates of the p-Laplace Neumann Operator and Brennan's Conjecture.* Boll. Unione Mat. Ital. (In press).

Vladimir Rabinovich

Instituto Politécnico Nacional, ESIME Zacatenco
Mexico City, Mexico
e-mail: vladimir.rabinovich@gmail.com

Fredholm theory and essential spectrum of quantum graphs with general vertex conditions

We study the Fredholm property and essential spectrum of unbounded operators on $L^2(\Gamma)$ generated by electromagnetic Schrödinger operators S on graphs Γ periodic with respect to a group $\mathbb{G}$ isomorphic to $\mathbb{Z}^n$. We suppose that S is equiped with a general vertex conditions. We associate with $\mathcal{H}$ a family $Lim(\mathcal{H})$ of limit operators $\mathcal{H}^h$ generated by sequences $h : \mathbb{G} \ni h_m \longrightarrow \infty$.

The main results of the paper are:

(i) $\mathcal{H}$ is a Fredholm operator if and only if all limit operators $\mathcal{H}^h$ of $\mathcal{H}$ are invertible;

(ii) Let $\mathcal{H}$ be a self-adjoint operator. Then

$$sp_{ess}\mathcal{H} = \bigcup_{\mathcal{H}^h \in Lim(\mathcal{H})} sp\mathcal{H}^h \tag{4}$$

where $sp_{ess}\mathcal{H}$ is the essential spectrum of $\mathcal{H}$.

We apply formula (4) to the study of the essential spectrum of perturbations of periodic Schrödinger operators by slowly oscillating potentials.

Daniel Reem

Department of Mathematics,
The Technion - Israel Institute of Technology
Haifa, Israel
e-mail: dream@technion.ac.il

Fixed points of Legendre-Fenchel type transforms and polarity type operators

A recent result characterizes the fully order reversing operators acting on the class of lower semicontinuous proper convex functions in a real Banach space as certain linear deformations of the Legendre-Fenchel transform. Motivated by the Hilbert space version of this result and by the well-known result saying that this convex conjugation transform has a unique fixed point, namely the normalized energy function, we investigate the fixed point equation in which the involved operator is a fully order reversing operator acting on the above-mentioned class of functions. It turns out that this nonlinear equation is very sensitive to the involved parameters and can have no solution, a unique solution, or several (possibly infinitely many) ones. We also consider a convex geometry version of this equation and obtain a similar characterization of its set of solutions (in particular, the equation is uniquely solvable by an ellipsoid if a certain key linear operator is positive definite). Our analysis yields a few byproducts, among them ones related to coercive bilinear forms (essentially a quantitative convex analytic converse to the celebrated Lax-Milgram theorem from partial differential equations) and to functional equations and inclusions involving monotone operators.

The talk is based on joint works with Alfredo N. Iusem (IMPA) and Simeon Reich (The Technion).

Vladimir Rovenski

Department of Mathematics, University of Haifa
Haifa, Israel
e-mail: vrovenski@univ.haifa.ac.il

Variations of the Godbillon-Vey invariant of a foliated manifold

The Godbillon-Vey (GV) cohomology class was defined first for codimension-one foliated manifolds, and then extended for foliations of codimension $q > 1$, see survey in [2]. Variations of GV class under infinitesimal deformations of foliations have been investigated in terms of a transversal projective structure, see [1]. In this joint work with P. Walczak, continuing our study in [3], we consider a smooth closed manifold M^{2q+1} equipped with a q-form ω and a q-vector field $\mathbf{T} = T_1 \wedge \ldots \wedge T_q$ such that $\omega(\mathbf{T}) = 1$. We show that a $(2q+1)$-form

$$\eta = \iota_{\mathbf{T}}\, d\omega := d\omega(T_1, \ldots, T_q, \cdot\,)\,.$$

is analogous to that defining the Godbillon-Vey invariant $\int_M \eta \wedge (d\eta)^q$ of a $(q+1)$-dimensional foliation. We express this $(2q+1)$-form in terms of geometry of $\mathbf{T}$ and the distribution $\mathcal{D} = \ker \omega$. For $q = 1$, we express this invariant $\int_M \eta \wedge d\eta$ in terms of the curvature and torsion of normal curves and the non-symmetric second fundamental form of $\mathcal{D}$. Then we explore critical points of associated actions (deducing Euler-Lagrange equations): for variable pair $(\omega, \mathbf{T})$ when $\mathcal{D}$ is integrable, and for variable metric on M compatible with $(\omega, \mathbf{T})$. Finally, we give sufficient conditions for critical points (foliations) when variations are among foliations, and present examples among twisted products. All this done when distributions and forms are defined on a closed manifold outside a “singularity set” under assumption of convergence of certain integrals.

References

[1] Asuke T. *Transverse projective structures of foliations and infinitesimal derivatives of the Godbillon-Vey class*. Int. J. of Math. 26:4, (2015), 1540001 (29 pp.)

[2] Candel A. and Conlon L. *Foliations, II*, Amer. Math. Soc., 2003.

[3] Rovenski V. and Walczak P. *Variations of the Godbillon-Vey invariant of foliated 3-manifolds*, 2018, submitted to “Complex Analysis and Operator Theory”.

Hiroshige Shiga

Tokyo Institute of Technology
Tokyo, Japan
e-mail: shiga@math.titech.ac.jp

The quasiconformal prolongation of Riemann surfaces

In the Teichmüller space of Riemann surfaces, we consider the set of Riemann surfaces which are quasiconformally equivalent to a given Riemann surface. Hence it is nesessary to clarify conditions for two Riemann surfaces to be quasiconformally equivalent. If Riemann surfaces are topologically finite, it is well known that such a condition is given in terms of geometric boundary properties of Riemann surfaces. For Riemann surfaces of topologically infinite type, however, the situation is rather complicated.

In this talk, we shall discuss some conditions for open Riemann surfaces of infinite type to be quasiconformally equivalent.

Mitsuhiro Shishikura

Kyoto University
Kyoto, Japan
e-mail: mitsu@math.kyoto-u.ac.jp

Quasiconformal variation of cross-ratios and applications

A quasiconformal mapping $f : \mathbb{C} \to \mathbb{C}$ defines a deformation of the conformal structure defined by $ds = |dz + \mu_f d\bar{z}|$. We study the deformation of 2-dimensional torus (elliptic curve) via Grötzsch inequality. This gives an estimate on the variation of cross-ratios of 4 points on the Riemann sphere, through the theory of elliptic functions. As an application, we give a simple proof of Teichmüller-Wittich-Belinskiĭ's and Gutlyanskiĭ-Martio's theorems on the conformality of quasiconformal mappings at a given point. As another application, we also give a proof of Ahlfors-Bers's formula on the parametric differentiation of qc-mappings (without going through Calderón-Zygmund inequality).

References

[1] M. Shishikura. *Conformality of qc-mappings at a point*, Preprint, arxiv.org/abs/1802.09137 (2018).

Toshiyuki Sugawa

GSIS, Tohoku University
Sendai, Japan
e-mail: sugawa@math.is.tohoku.ac.jp

Aharonov invariants revisited

Let $f(z)$ be a locally univalent analytic function on a domain in the complex plane. Generalizing the Schwarzian derivative, Aharonov [1] proposed the invariants $\psi_n[f](z)$ by the series expansion

$$\frac{f'(z)}{f(z+w)-f(z)} = \frac{1}{w} - \sum_{n=1}^{\infty} \psi_n[f](z) w^{n-1}$$

for small enough w. Note that $2\psi_1[f] = f''/f'$ and $6\psi_2[f] = (f''/f')' - (f''/f')^2/2$. Aharonov [1] (see also [2]) showed that a nonconstant meromorphic function f on the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ is univalent if and only if

$$\sum_{n=1}^{\infty} n \left| \sum_{k=1}^{n} \binom{n-1}{k-1} (-\bar{z})^{n-k} (1-|z|^2)^{k+1} \psi_{k+1}[f](z) \right|^2 \le 1, \quad z \in \mathbb{D}.$$

Seong-A Kim and the speaker introduced in [3] the projective Schwarzian derivatives of higher order for a non-constant holomorphic map from a projective Riemann surface with a smooth conformal metric into another projective Riemann surface. In the case when the source surface is the unit disk $\mathbb{D}$ and the target surface is the Riemann sphere, the Aharonov condition can be expressed in terms of our projective Schwarzians. We consider also quasiconformal extension properties of univalent functions.

References

[1] D. Aharonov, *A necessary and sufficient condition for univalence of a meromorphic function*, Duke Math. J. **36** (1969), 599–604.
[2] R. Harmelin, *Aharonov invariants and univalent functions*, Israel J. Math. **43** (1982), 244–254.
[3] S. Kim and T. Sugawa, *Geometric invariants associated with projective structures and univalence criteria*, Tohoku Math. J. **63** (2011), 41–57.

Alexander Ukhlov

Ben–Gurion University of the Negev
Beer-Sheva, Israel
e-mail: ukhlov@math.bgu.ac.il

Spectral estimates of degenerate p-Laplace operators in non-convex domains

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain. The Neumann eigenvalue problem for the two-dimensional degenerate p-Laplace operator ($p > 2$)

$$-\operatorname{div}(|\nabla u|^{p-2}\nabla u) = \mu_p |u|^{p-2} u \text{ in } \Omega,\ \frac{\partial u}{\partial n} = 0 \text{ on } \partial\Omega,$$

arises in study of free vibrations of nonelastic membranes. We suggest spectral estimates of the first non-trivial Neumann eigenvalue $\mu_p(\Omega)$ of the p-Laplace operator in the terms of the (quasi)conformal geometry of domains. Let $\Omega \subset \mathbb{R}^2$ be a K-quasidisc. Then

$$\mu_p(\Omega) \geq \frac{Q_p(K)}{|\Omega|^{\frac{p}{2}}} = \frac{Q_p^*(K)}{R_*^p},$$

where R_* is a radius of a disc Ω^* of the same area as Ω and $Q_p^*(K) = Q_p(K)\pi^{-p/2}$ depends on p and a quasiconformality coefficient K only.

(Joint works with Vladimir Gol'dshtein and Valerii Pchelintsev)

References

[1] V. Gol'dshtein, V. Pchelintsev, A. Ukhlov. *Integral estimates of conformal derivatives and spectral properties of the Neumann-Laplacian.* J. Math. Anal. Appl., V. 463, (2018), pp. 19–39.

[2] V. Gol'dshtein, V. Pchelintsev, A. Ukhlov. *On the First Eigenvalue of the Degenerate p-Laplace Operator in Non-Convex Domains.* Integral Equations Operator Theory, (In press).

Alekos Vidras

Dept. of Mathematics and Statistics, Univ. of Cyprus
Nicosia, Cyprus
e-mail: msvidras@ucy.ac.cy

Cauchy-Fantappie formula on tube domain in $\mathbf{C}^2$

On the tube $T_B = \mathbf{R}^2 \times i\{(y_1, y_2) \in \mathbf{R}^2 : y_1^2 + y_2^2 < 1\}$ in $\mathbf{C}^2$, with a defining function $\Phi(\zeta, \bar{\zeta}) = (\frac{\zeta_1 - \bar{\zeta}_1}{2i})^2 + (\frac{\zeta_2 - \bar{\zeta}_2}{2i})^2 - 1$, $\zeta \in \mathbf{C}^2$, we consider the Hardy space $H^2(T_B)$. It is the space of holomorphic functions $f(z) = \int\limits_{\mathbf{R}^2} h(t) e^{2\pi i z \cdot t} dt$, $z \in T_B$, where the function h satisfies the estimate

$$\sup_{y \in B} \int\limits_{\mathbf{R}^2} |h(t)|^2 e^{-4\pi y \cdot t} dt \leq A^2 < +\infty.$$

Our theorem asserts that every function $f \in H^2(T_B)$ is represented by $Cauchy - Fantappiè$ formula supported on the boundary ∂T_B:

$$f(z) = \frac{1}{(2\pi)^2} \int\limits_{\mathbf{R}^2 \times i S^1} \frac{f(\zeta) \partial \Phi(\zeta, \bar{\zeta}) \wedge \bar{\partial} \partial \Phi(\zeta, \bar{\zeta})}{< \nabla \Phi(\zeta, \bar{\zeta}), \zeta - z >^2}, \ z \in T_B.$$

The absence of Stokes' theorem for unbounded domain is superseded by a separation of singularities type theorem allowing to express a function $f \in H^2(T_B)$ as a difference of two holomorphic functions $f_1 \in H^2(T_{(S_H^-)^{int}})$ and $f_2 \in H^2(T_{(S_H^+)^{int}})$, defined on suitable tubes $T_{(S_H^-)^{int}}$ and $T_{(S_H^+)^{int}}$, whose base contains a cone, and satisfying $T_B = T_{(S_H^-)^{int}} \cap T_{(S_H^+)^{int}}$. It is proved that f_i, $i = 1, 2$, are representable by $Cauchy - Fantappiè$ in the corresponding tube $T_{(S_H^{\pm})^{int}})$ (and conversely). The main result then follows as a direct consequence of a duality argument.

References

[1] N.Alexandrou, A.Vidras. *Cauchy-Fantappie integral formula for holomorphic functions on special tube domains in* $\mathbf{C}^2$, preprint.

Rafał Zalas

Technion – Israel Institute of Technology
Haifa, Israel
e-mail: rzalas@technion.ac.il

Regular operators in fixed point problems

In this talk we present a systematic study of weakly, boundedly and linearly regular operators. All three types are relevant since they lead to weak, strong and linear convergence of various iterative methods, respectively. In the main result we show that the type of the regularity is preserved under convex combinations and products of operators. These operations appear in many examples of projection methods. This is a joint work with Andrzej Cegielski and Simeon Reich; see [1].

References

[1] A. Cegielski, S. Reich, R. Zalas. *Regular sequences of quasi-nonexpansive operators and their applications.* SIAM J. Optim., accepted.

Gilles Zemor

Institut de Mathématiques de Bordeaux
Université de Bordeaux
Talence, France
e-mail: zemor@math.u-bordeaux.fr

Computing critical probabilities for percolation on regular tilings of the hyperbolic plane

We introduce a method for deriving upper bounds on critical probabilities for percolation which involves taking finite quotients of the infinite graph and estimating the probability that a random subgraph contains a homologically non-trivial cycle on the quotient. Based on joint work with Nicolas Delfosse.