ABSTRACTS

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On the uniform convergence of $p-$Bieberbach polynomials in domains of complex plane

Let $G \subset \mathbb{C}$ be a finite Jordan domain with $0 \in G$; $L := \partial G$; $w{=}\varphi(z)$ be the conformal mapping of G onto the disk $B(0, \rho_0) := \{w : |w| < \rho_0\}$ normalized by $\varphi(0) = 0$, $\varphi'(0) = 1$. For $p > 0$ let $A^1_p(G)$ denote the set of analytic in G functions $f(z)$ normalized by $f(0) = 0$, $f'(0) = 0$ and such that

$$\|f\|^p_{A^1_p(G)} :=_G |f'(z)|^p \, d\sigma_z < \infty,$$

where $d\sigma_z$ denotes two dimensional Lebesgue measure.

Let us denote $\wp_n$ the class of all polynomials $P_n(z)$, $\deg P_n(z) \leq n$, satisfying the conditions: $P_n(0) = 0$, $P'_n(0) = 1$.

Let us consider the following extremal problem:

$$\left\{ \|\varphi - P_n\|_{A^1_p(G)}, \ P_n \in \wp_n, \ p > 0 \right\} \to \inf \tag{1}$$

If $p > 0$ there exists a polynomial $P^*_{n,p} \in \wp_n$ furnishing to the problem (1), and if $p > 1$, this polynomial $P^*_{n,p}(z)$ are determined uniquely. This unique solution $P^*_{n,p}(z)$ it was called $p-$Bieberbach polynomial for the pair $(G, 0)$.

In this work we study the possibility $\left\|\varphi - P^*_{n,p}\right\|_{C(\overline{G})} \to 0$, $n \to \infty$, and estimation of the rate in regions having on the boundary both interior and exterior zero angles.

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On algebraically integrable domains

In connection with problems in celestial mechanics, Newton proved that (infinitely smooth) ovals in the plane are never algebraically integrable, meaning that the area cut off from an oval by a straight line never depends algebraically on the line. Arnold suggested to find all infinitely smooth algebraically integrable domains in the spaces $\mathbb{R}^n$ of arbitrary dimension and conjectured that the only such domains are odd-dimensional ellipsoids. The talk will be devoted to recent progress in proving this conjecture.

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The Bombieri problem for univalent functions

Since 1916, The Bieberbach Conjecture, was the basic open problem of geometric function theory till its proof by de Branges [1] in 1984. It states that for the class S of one-to-one analytic functions f defined on the unit disk D and normalized by

$$f(z) = z + \sum_{n=1}^{\infty} a_n z^n,$$

then

$$|a_n| \leq n.$$

Until 1966 only the cases $n \leq 6$ were proved.

The variational technique that was initiated in the 1940's was able to solve the local Bieberbach problem, and most of all, in 1967 the existence of positive constants α_n and β_n such that

$$\varliminf_{a_2\to 2} \frac{n-\Re\{a_n\}}{2-\Re\{a_2\}} \geq \alpha_n,\ n-\text{even}\ ;\ \ \varliminf_{a_3\to 3} \frac{n-\Re\{a_n\}}{3-\Re\{a_3\}} \geq \beta_n,\ n-\text{odd}$$

proved by Bombieri. In 1963 Bombieri [2] questioned if there exist positive constants d_n such that

$$\big|n-|a_n|\big| \leq d_n\big|2-|a_2|\big|.$$

The proof of the Bieberbach conjecture tuned the question to suit the changes. a refinement of des Branges method due to Dong opened a door to solve this problem. We shall present the problem and its solution.

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Dichotomy of global capacity density

Let φ be an outer measure in $\mathbb{R}^n$, $n \geq 2$, such that $0 < \varphi(B(x,r)) < \infty$ for every open ball $B(x,r)$ with center at x and radius r. For $E \subset \mathbb{R}^n$ and $r > 0$ define

$$\mathcal{D}(\varphi, E, r) = \inf_{x\in\mathbb{R}^n} \frac{\varphi(E\cap B(x,r))}{\varphi(B(x,r))}.$$

By definition $0 \leq \mathcal{D}(\varphi, E, r) \leq 1$. We are interested in the limit of $\mathcal{D}(\varphi, E, r)$ as $r \to \infty$.

A typical example of φ is the n-dimensional Lebesgue outer measure m. For each $c \in (0,1)$, it is easy to construct a closed set E_c such that $\lim_{r\to\infty} \mathcal{D}(m, E_c, r) = c$. If φ is a capacity, then the situation is completely different. For many capacities the limit must be either 0 or 1. By $C_\ell(E)$ we denote the logarithmic capacity of $E \subset \mathbb{R}^2$. In connection with BMOA, Stegenga proved

$$\lim_{r\to\infty} \mathcal{D}(C_\ell, E, r) = \begin{cases} 0 & \text{if } \mathcal{D}(C_\ell, E, r) = 0 \text{ for all } r > 0, \\ 1 & \text{if } \mathcal{D}(C_\ell, E, r) > 0 \text{ for some } r > 0. \end{cases}$$

In this talk we show that this *dichotomy* is enjoyed by many capacities such as Riesz capacities, weighted L^p-capacities in the Euclidean space and variational capacities in a metric measure space. On the other hand, there is a capacity that fails the dichotomy. The case when φ is Newtonian capacity is of particular interest. We observe that the dichotomy relates to the estimate of the principal frequency and intrinsic ultracontractivity.

Some parts of the talk are joint works with Tsubasa Itoh, Anders Björn, Jana Björn, and Nageswari Shanmugalingam.

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Extension of bilipschitz maps: A short survey

Let (X, d) and (Y, d') be metric spaces, and let $L \geq 1$. A mapping $f\colon X \to Y$ is L-bilipschitz if

$$d(x, y)/L \leq d'(f(x), f(y)) \leq Ld(x, y) \text{ for all } x, y \in X.$$

We give a short survey of results related to the extension problem for L-bilipschitz mappings $f\colon A \to \mathbf{R}^n$, where $A \subset \mathbf{R}^k$. The most important question is the behavior of the bilipschitz constant L' of the extension $F\colon \mathbf{R}^k \to \mathbf{R}^n$. The survey is divided into the following parts.

0. Basic examples showing why the extension may be difficult or impossible in some cases.
1. Optimal extension results with $L' = L$ for all $L \geq 1$. This case is very unusual, but can be obtained in at least two different situations.
2. General results where $L' > L$ depends quantitatively on L and possibly on the dimensions k and n.

3. The stability question: A special case of the previous part, where $L = 1+\varepsilon$ and $L' = 1+\varphi(\varepsilon)$ with $\varphi(\varepsilon) \to 0$ as $\varepsilon \to 0+$. The optimal results deal with the case $\varphi(\varepsilon) = C\varepsilon$ for some constant C.
4. Extension results in the case where the dimensions of the ambient spaces are increased. In this case extension is always possible, but the stability questions are more difficult.

We also list some open problems related to these extension properties.

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Rational and Meromorphic Approximation to certain Functions with Polar Singular Set

We consider the problem of approximating an analytic function with finitely many algebraic branchpoints, poles and essential singularities by a rational or meromorphic function of degree n on a compact subset of the domain of analyticity. We discuss how the interpolation theory developed in [2] and the solution of certain geometric extremal problems in logarithmic potential theory [1] allow to describe the n-th root error and weak pole asymtotics of best rational approximants in L^2-sense on a circle, as well as meromorphic approximants on more general curves. We further show how a refinement of the solution to the Gonchar conjecture on the degree of approximation [3], combined with AAK theory and some potential theory yields similar results in uniform approximation.

This is joint work with M. Yattselev and H. Stahl.

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Convex maps in higher dimensions

The Carathéodory prime ends theory is the most important tool for studying continuous extension at the boundary of univalent functions from the unit disc in $\mathbb{C}$. A simple consequence of the Carathéodory theory is that every univalent map from the unit disc $\mathbb{D}$ whose image $\Omega \subset \mathbb{C}$ is convex has at most two infinite singularities on the boundary $\partial\mathbb{D}$ and extends as a homeomorphism from $\overline{\mathbb{D}}$ outside the infinite singularities to $\overline{\Omega}$. Moreover, in case the infinite singularities are two, then Ω is a strip. In [2], J. Muir and T. Suffridge conjectured that a similar result holds for univalent convex maps of the unit ball $\mathbb{B}^n$ in higher dimension. Carathéodory theory extends to higher dimension for *quasi-conformal maps*, and thus the result holds for convex univalent quasi-conformal maps. However, univalent maps in higher dimensions are not in general quasi-conformal and the usual prime ends theory does not apply. In the recent paper [1], the speaker together with H. Gaussier proved the Muir-Suffridge conjecture for any convex univalent map of $\mathbb{B}^n$.

The proof uses Gromov's theory (in particular the shadowing lemma) for proving continuous extension to the boundary, and the theory of semigroups and commuting maps of $\mathbb{B}^n$ for proving the existence of at most two infinite singularities on the boundary and the boundary extension as a homeomorphism.

The aim of this talk is to give a sketch of the proof of the Muir-Suffridge conjecture.

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$0,01\%$ improvement of the Liouville property for discrete harmonic functions on $\mathbb{Z}^2$

Let u be a harmonic function on the plane. The Liouville theorem claims that if $|u|$ is bounded on the whole plane, then u is identically constant. It appears that if u is a harmonic function on the lattice $\mathbb{Z}^2$, and $|u| < 1$ on $99,99\%$ of $\mathbb{Z}^2$, then u is a constant function. Based on a joint work (in progress) with A. Logunov, Eu. Malinnikova and M. Sodin.

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An extension of Fatou's interpolation theorem and applications

Fatou's interpolation theorem states that for any closed set E of measure zero on $|z| = 1$ there exists an element in the disc algebra which vanishes precisely on E. This fundamental result has many applications in analysis. In this talk we show that Fatou's theorem implies both the Rudin-Carleson interpolation theorem and a theorem of Lohwater and Piranian on the radial limits of bounded analytic functions (see [4] and [3]). Also, some recent interpolation results will be presented. For example, using a lemma of Kolesnikov, we show that for any G_δ set F of measure zero on $|z| = 1$ there exists a function $g \in H^\infty$ non-vanishing in $|z| < 1$ such that: (i) g has non-zero radial limits everywhere on $\{|z| = 1\} \setminus F$; and (ii) g has vanishing radial limits at each point of F (see [1]). This result extends Fatou's theorem from the closed sets to G_δ sets and implies an affirmative answer to a question proposed by Rubel (see Problem 5.29 in [1], p. 168). We have shown recently that the conclusion (ii) of the formulated theorem can be slightly strengthened as follows: g has vanishing *unrestricted* limits at each point of F. Some open problems will be formulated as well.

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Connected lemniscates and distortion theorems for polynomials and rational functions

In the present talk we discuss the impact of the connectivity of some lemniscates of a rational function f on the distortion of the mapping effected by f. We consider the distortion theorems for polynomials and rational functions [1]-[2], an inequality for the moduli of derivative of a complex polynomial at its zeros [3] and an inequality for the logarithmic energy of zeros and poles of a rational function [4]. All estimates obtained are sharp. The proofs of the theorems go by an application of a certain modification of the symmetrization method [5], for which the result of the symmetrization lies on the Riemann surface of the inverse function to a Chebyshev polynomial of the first kind.

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Analyticity of semigroups on the right half-plane

This talk is devoted to semigroups of composition operators and semigroups of holomorphic mappings on the right half-plane. We establish conditions under which these semigroups can be extended in their parameter to a sector given a priori. We show that the size of this sector can be controlled by the image properties of the infinitesimal generator, or, equivalently, by the geometry of the so-called associated planar domain. We also give a complete characterization of all composition operators acting on the Hardy space H^p on the right half-plane.

The talk is based on a joint work with Fiana Jacobzon.

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Applications of the hyperbolic geometry to the Neumann problem

In this talk we give applications of the hyperbolic geometry to the spectral estimates of the Neumann-Laplace operator in a large class of simply connected planar domains. This class of domains includes domains with Hölder singularities and quasidiscs (images of the unit disc under quasiconformal homeomorphisms of $\mathbb{R}^2$). Quasidiscs can have fractal boundaries (which Hausdorff dimension can be any number into the half-interval $[1, 2)$).

Our machinery is based on the geometric theory of conformal composition operators on Sobolev spaces. These composition operators are induced by conformal and quasiconformal homeomorphisms.

(joint with Valerii Pchelintsev and Alexander Ukhlov)

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Orthogonal Polynomials on generalized Julia sets.

We extend results by Barnsley, Geronimo and Harrington about orthogonal polynomials on Julia sets to the case of generalized Julia sets. The equilibrium measure is considered. In addition, we discuss optimal smoothness of Green's functions and Widom criterion for a special family of real generalized Julia sets. The result is joint with G.Alpan.

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Diffeomorphic approximation of Sobolev homeomorphisms

Let Ω be a domain in the plane and let $f \in W^{1,1}(\Omega, \mathbb{R}^2)$ be a homeomorphism. We show that there is a sequence of smooth diffeomorphisms f_k converging to f in $W^{1,1}(\Omega, \mathbb{R}^2)$ and uniformly. This answers a Ball-Evans approximation problem in the limiting case $p = 1$. It is a joint work with A. Pratelli. We also discuss other related planar results and counterexamples to Ball-Evans approximation problem in dimension $n \geq 4$.

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Complex dilatation and the Cartan-Kähler theory

We show how the existence part of the measurable Riemann mapping theorem can also be proved by using the Cartan- Kähler theory together with standard approximation results for quasiconformal mappings in the plane. This result states that if D is a domain in the complex plane and μ is a complex–valued L^∞-function in D with $||\mu||_\infty < 1$, then there exists a quasiconformal homeomorphism f defined in D such that at almost every point $z \in D$, the Beltrami equation

$$\frac{\partial f}{\partial \overline{z}}(z) = \mu(z)\frac{\partial f}{\partial z}(z)$$

is satisfied. Several proofs are known for this result. It is well known that one possible way to prove it, is to first solve the Beltrami equation locally when μ is real analytic, and then use approximation theorems (to move from real analytic μ to measurable μ) together with the uniformization theorem for Riemann surfaces (to move from a local result to a global result). The Cartan-Kähler theory is a general method for solving systems of partial differential equations with real analytic data locally, and for finding integrability conditions in those cases where such restrictions are required.

We show how one can solve the Beltrami equation locally when μ is real analytic using the Cartan-Kähler theory. Together with the above remarks, this then provides yet another way of proving the measurable Riemann mapping theorem.

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On the John-Nirenberg 2nd result and beyond

The goal of my talk is to address some inequalities which Fritz John and Louis Nirenberg proved to be valid for certain functions defined in a cube. I will discuss the validity of similar inequalities for functions defined in an arbitrary bounded domain. My talk is based on joint work with Niko Marola and Antti V. Vähäkangas.

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Gaussian measure of a tubular neighborhood of a complex-analytic set

Let $f : \mathbb{C}^n \to \mathbb{C}^k$ be a holomorphic function with $f(0) = 0$. Set $Z = f^{-1}(0)$. We prove that for any $r > 0$,

$$\gamma_n(Z + r) \geq \gamma_n(\mathbb{C}^{n-k} + r),$$

where $Z+r$ is the Euclidean r-neighborhood of Z, where γ_n is the standard Gaussian measure in $\mathbb{C}^n$, and where $\mathbb{C}^{n-k}$ is viewed as a subspace of $\mathbb{C}^n$. Our proof is probabilistic and it relies on a technique known as Eldan's stochastic localization.

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On the growth of Lebesgue constants for convex polyhedra

Let W be a bounded set in $\mathbb{R}^d$. The following integral is called the Lebesgue constant for W

$$\mathcal{L}(W) = \int_{[0,2\pi)^d} \Big| \sum_{k \in W \cap \mathbb{Z}^d} e^{i(k,x)} \Big| \mathrm{d}x.$$

In the talk, we will present new estimates of the Lebesgue constants for convex polyhedra. In particular, we prove that if W is a convex polyhedron in $\mathbb{R}^d$ such that

$$[0, m_1] \times \cdots \times [0, m_d] \subset W \subset [0, n_1] \times \cdots \times [0, n_d],$$

then for sufficiently large $(n_1, \dots, n_d)$ we have

$$c(d) \prod_{j=1}^{d} \log(m_j + 1) \le \mathcal{L}(W) \le C(d) s \prod_{j=1}^{d} \log(n_j + 1),$$

where s is size of some triangulation of W.

Our results generalize and give sharper versions of the results of E.S. Belinsky [1], A.A. Yudin and V.A. Yudin [2], and M. Ash [3].

This is a joint work with T. Lomako (Institute of Applied Mathematics and Mechanics of NAS of Ukraine, Slov'yans'k).

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Holomorphic contractibility of Teichmüller spaces

It is known that in $\mathbf{C}^n,\ n > 1$, there exist (topologically) contractible bounded domains of holomorphy that are not holomorphically contractible (their examples have been given by Hirchkowitz, Zaidenberg and V. Lin). This fact became underlying for the problem of holomorphic contractibility of Teichmüller spaces $\mathbf{T}(0, n)$ of the spheres with $n > 4$ punctures, which arose in the 1970s in connection with solving the algebraic equations in Banach algebras and goes back to Gorin. The problem closely relates to the classical Oka-Grauert h-principle.

In the talk, we provide a positive solution of this problem.

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Analytic dependence of a periodic analog of a fundamental solution upon the periodicity parameters

We prove an analyticity result for the periodic analog of the fundamental solution of an elliptic partial differential operator upon the parameters which determine the periodicity cell. Then we show concrete applications to the Helmholtz and the Laplace operators. In particular, we show that the periodic analog of the fundamental solution of the Helmholtz and of the Laplace operator are jointly analytic in the the spatial variable and in the parameters which determine the size of the periodicity cell. The analysis of the present paper is motivated by the application of the potential theoretic method to boundary value problems corresponding to anisotropic periodic problems in which the ‘degree of anisotropy’ is a parameter of the problem.

Based on joint work with Paolo Musolino (Aberystwyth, Wales UK) musolinopaolo@gmail.com

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Transversality for critical relations of families of rational maps: an elementary proof

We give short elementary proofs of the following two results. Consider the set of rational maps of a given degree d with given multiplicities at the critical points. First, we show that this space of maps is an embedded submanifold of the space of all degree d rational maps and can be parametrized partially by the critical values. Secondly, we prove that if the map f in this manifold is not a flexible Lattes map

then one can organise the set of critical relations of f so that they unfold transversally. The latter proof uses the theory of quasi-conformal deformations.

In the talk I'll tell about the history of the problems and outline the proofs.

Joint work with Weixiao Shen and Sebastian van Strien.

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Hausdorff operators in H^p spaces, $0 < p < 1$

For the theory of Hardy spaces H^p, $0 < p < 1$, the Hausdorff operators turn out to be a very effective testing area, in dimension one and especially in several dimensions. After publication of [4], Hausdorff operators have attracted much attention. A general idea can be had of the subject from the surveys [3] and [1]. In contrast to the study of the Hausdorff operators in L^p, $1 \leq p \leq \infty$, and in the Hardy space H^1, the study of these operators in the Hardy spaces H^p with $p < 1$ holds a specific place and there are very few results on this topic. For the case of one dimension, after [2] and [6], more or less final results were given in [5]. The results differ from those for L^p, $1 \leq p \leq \infty$, and H^1, since they involve smoothness conditions on the averaging function, which seem unusual but unavoidable. To explain them will be the main purpose of the talk. This leads to better understanding even more specific difficulties in our multidimensional joint work with Akihiko Miyachi.

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On a new type of conformality in four–dimensional spaces

One of the basic properties of holomorphic functions in one complex variable is that they are conformal. The theoretical importance of this property as well as the big amount of application is well known. In this work are presented the necessary tools in order to give the analogue notion of conformality of bicomplex holomorphic functions, For this new notion, it is presented a positive hyperbolic–valued norm defined on the algebra of bicomplex numbers. As a second step it is presented the trigonometric representation of bicomplex numbers in hyperbolic terms, this means that not only the norm of the bicomplex number is positive hyperbolic–valued, but also the angle of the bicomplex number is hyperbolic–valued. The geometry that arise from these facts is analyzed, in particular the notion of hyperbolic lines and curves, and the hyperbolic angle between these objects. It is also presented a Theorem that asserts that bicomplex holomorphic functions are conformal on those point where the derivative is not zero.

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Extremal problems for modules

We consider the p-module of families of curves and surfaces in the Euclidean space $\mathbb{R}^n$ and in polarizable Carnot groups. Two important ingredients of the p-module: extremal metrics and extremal families of curves or surfaces are studied. First of all, we generalize Rodin's theorem to the Euclidean space. Given an extremal family of curves or surfaces we calculate the module of its image under homeomorphisms of certain regularity. In particular, we give some application to special functions. On the other hand, we calculate the module and find the extremal metrics and extremal families of curves and surfaces for the spherical ring domain in polarizable Carnot groups, and finally, extend Rodin's theorem to the spherical ring domain in the polarizable Carnot groups.

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Recomposing rational functions

Let A be a rational function. For any decomposition of A into a composition of rational functions $A = U \circ V$ the rational function $\widetilde{A} = V \circ U$ is called an elementary transformation of A, and rational functions A and B are called equivalent if there exists a chain of elementary transformations between A and B. This equivalence relation naturally appears in the complex dynamics as a part of the problem of describing of semiconjugate rational functions. We show that for a rational function A its equivalence class $[A]$ contains infinitely many conjugacy classes if and only if A is a flexible Lattès map. For flexible Lattès maps $L = L_j$ induced by the multiplication by 2 on elliptic curves with given j-invariant we provide a very precise description of $[L]$. Namely, we show that any rational function equivalent to L_j necessarily has the form $L_{j'}$ for some $j' \in \mathbb{C}$, and that the set of $j' \in \mathbb{C}$ such that $L_{j'} \sim L_j$ coincides with the orbit of j under the correspondence associated with the classical modular equation $\Phi_2(x, y) = 0$.

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Quasiregular extensions of cubical Alexander maps

Rickman developed in his Picard construction (1985) a method to extend (geometrically controlled) piecewise linear planar Alexander maps to the Euclidean 3-space as quasiregular mappings. This extension method, called deformation theory, is based on a trade-off between the complex defining the Alexander map and freely movable simple covers.

In this talk I will discuss a variant of Rickman's deformation theory in higher dimensions and its applications to the construction of quasiregular mappings. This is joint work with Jang-Mei Wu.

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Rough singular integrals and the maximal function: new borderline weighted estimates

Muckenhoupt-Wheeden [MW] in the seventies and Sawyer [S] in the eightees, established some one-dimensional highly nontrivial extensions of the weak type $(1, 1)$ property of the maximal function involving weights. These results were conjectured to hold for the Hilbert transform and for the maximal function in higher extensions. In the first part of this lecture we will survey about these conjectures that were proved and extended in different directions in [CMP], [OP] and [OPR]. Then we will discuss about the main open conjecture that has been recently settled in [LOP].

In the second part of this lecture we will discuss a recent work [LPRR] where we solved some conjectures for rough singular integrals and weights. The link of these two parts of the lecture is the classical good-lambda estimate between Calderón-Zygmund operators and the maximal functions due to R. Coifman and C. Fefferman leading to some strong or weak L^p estimates with A_∞ weights. We will show that a corresponding result holds for rough singular integrals and the Bochner-Riesz operator at critical level even though there is no such as good-λ in the literature. This result is key in the solution of the above mentioned conjectures combined with some extrapolation theorems for the class A_∞ [CGMP] together with a sparse formula found by Conde-Culiuc-Di Plinio-Ou. Further quite sharp qualitative and quantitative results will be presented.

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Maximal and Riesz potential operators with rough kernel in non-standard function spaces

In this talk we will discuss the boundedness of the maximal operator with rough kernel in some non-standard function spaces, e.g. variable Lebesgue spaces, variable Morrey spaces, Musielak-Orlicz spaces, among others. We will also discuss the boundedness of the Riesz potential operator with rough kernel in variable Morrey spaces. This is based on joint work with S. Samko, cf. [1, 2].

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A Godbillon-Vey type invariant for 3-dimensional manifold with a plane field

Given a smooth manifold M^3 equipped with a plane field $\mathcal{D}$ and a vector field T transverse to $\mathcal{D}$, we use 1-form ω such that $\mathcal{D} = \ker\,\omega$ and $\omega(T) = 1$ to build a 3-form analogous to defined in [1] the Godbillon-Vey class of a foliation. For any metric on M, we express this form in terms of extrinsic geometry of $\mathcal{D}$ and the curvature and torsion of its normal curves, and then derive and study Euler-Lagrange equations of associated action: for variations of a pair $(\mathcal{D}, T)$ with fixed adapted metric, and for general variations of a Riemannian metric with fixed plain field $\mathcal{D}$, [2]. Some of results are extended for Randers metrics.

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On the boundary value problems in the plane. An alternative approach.

The survey is devoted to recent advances in non-classical solutions of the main boundary value problems such as the well-known Dirichlet, Hilbert, Neumann, Poincare and Riemann problems in the plane. Such solutions are essentially different from the variational solutions of the classical mathematical physics and based on the nonstandard point of view of the geometrical function theory with a clear visual sense. The traditional approach of the latter is the meaning of the boundary values of functions at almost every boundary point in the sense of the so-called angular limits or limits along other prescribed classes of curves terminated at the boundary. This become necessary if we start to consider boundary data that are only measurable, and it is turned out to be useful under the study of problems in the field of mathematical physics, too. Thus, we essentially widen the notion of solutions and, furthermore, obtain spaces of solutions of the infinite dimension for all the mentioned boundary value problems. The latter concerns also the Laplace equation as well as to its counterparts in the potential theory for inhomogeneous and anisotropic media.

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Potential theory in the class of subharmonic functions

Pluripotential theory in the complex space $\mathbb{C}^n$ based on the well-known class of plurisubharmonic functions and on the Monge-Ampere operator $(dd^c u)^n$ is one of the actual areas of modern mathematics. Here $d = \partial + \overline{\partial}$, $d^c = \frac{\partial - \overline{\partial}}{4i}$ are standard notations. It is a direct continuation and further development of the classical potential theory, based on the Laplace operator $\Delta \sim \frac{\partial^2}{\partial z \partial \overline{z}}$. In our talk we give a potential theory in the class of $m-$ subharmonic $(m - sh)$ function.

Definition. *A function $u \in C^2(D$ is called $m - sh$, if*

$$(dd^c u)^k \wedge \beta^{n-k} \geq 0, \;\; k = 1, 2, ..., m, \;\; 1 \leq m \leq n.$$

A function $u \in L^1_{loc}$ is said to be $m-$ subharmonic in a domain $D \in \mathbb{C}^n$ if it is upper semicontinuous and, for any twice continuously differentiable $m - sh$ functions $v_1, ..., v_{m-1}$ the current(distribution) $dd^c u \wedge dd^c v_1 \wedge ... \wedge dd^c v_{m-1}$ defined as

$$\left[dd^c u \wedge dd^c v_1 \wedge ... \wedge dd^c v_{m-1} \wedge \beta^{n-m}\right](\omega) =$$
$$= \int u dd^c v_1 \wedge ... dd^c v_{m-1} \wedge \beta^{n-m} \wedge dd^c \omega, \;\; \omega \in F^{0,0}$$

is positive.

We note, that class $m - sh$ is intermediate, that is

$$sh \supset m - sh \supset psh, \;\; 1 - sh = sh, \;\; n - sh = psh.$$

$m - sh$ functions play a big role in the multidimensional complex analysis, in the studying of holomorphic functions. For deep applications we need a potential theory in the class of $m - sh$ functions. To build potential theory in the class of $m - sh$ functions we need, first of all, the definition and continuity of the operator $(dd^c u)^k \wedge \beta^{n-m}$ in the class $m - sh(D) \cap L^{\infty}_{loc}, 1 \leq k \leq m$. After we need the definition of maximal function (analog of harmonic function) and to prove a fundamental theorems of the potential theory. We will solve these problems and similar potential properties of $m - sh$ functions according to the following scheme:

1. First, we define $(dd^c u)^k \wedge \beta^{n-m}$ in the class $m - sh(D) \cap C(D)$; If $u \in m - sh(D) \cap C(D)$ and $u_j \downarrow u$ is a standard approximation, then this convergence is uniformly, $\| u_j - u \| \to 0$. This uniformly convergence allows us to prove the $(dd^c u_j)^k \wedge \beta^{n-m} \mapsto (dd^c u)^k \wedge \beta^{n-m}$.

2. Using solely the class $m - sh(D) \cap C(D)$, we define the $m-$ capacity

$$C(K) = C_m K, D =$$

$$= \inf \left\{ \int\limits_D (dd^c u)^m \wedge \beta^{n-m} : \right.$$

$$\left. : \; u \in m - sh(D) \cap C(D), \; u \mid_K \leq -1, \; \varliminf_{z \to \partial D} u\,(z) \geqslant 0 \right\}.$$

The value $C(K)$ has all the basic properties of capacities. Moreover, $C(K) = 0 \Leftrightarrow K$ is $m-$ polar.

3. We prove the potential properties of $m - sh$ functions (quasicontinuity, comparison principles, convergence of the approximation with respect to $m-$capacity, etc. In particular,

Theorem 1. (analogue of Luzin's theorem) *Arbitrary m- subharmonic function is continuous almost everywhere with respect to the capacity, that is if $u \in m - sh(D)$, then for arbitrary $\varepsilon > 0$ there exists an open set $U \subset D$ such that its capacity $C(U) < \varepsilon$ and the function u is continuous in $D \setminus U$.*

Theorem 2. *A locally $m-$ polar set is globally $m-$ polar in $\mathbb{C}^n$.*

Theorem 3. *Let $\{u_j\}$ be an increasing sequence of $m-sh$ functions such that the function $u(z) = \lim_{j \to \infty} u_j(z)$ is locally bounded from above. Then the set $\{u(z) < u^*(z)\}$, where u^* is the regularization of u, is*

$m-$ polar in D. Moreover, the sequence of currents $(dd^c u_j)^k \wedge \beta^{n-m} \mapsto (dd^c u^*)^k \wedge \beta^{n-m}$.

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On a class of holomorphic mappings in $\mathbb{C}^2$ related to bicomplex numbers

In classic multidimensional complex analysis, a holomorphic mapping in $\mathbb{C}^2$ is just a pair of holomorphic functions of two complex variables with no relations between the functions themselves. It turns out that it is possible to introduce a Cauchy–Riemann–type relation in such a way that the arising subclass of holomorphic mappings possesses a rich theory quite similar to that of functions in one variable. It will be shown that a right way of treating it is via the so–called bicomplex analysis, that is, a study of derivable bicomplex functions. The main peculiarities of this approach will be presented.

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Themes on the Bohr's phenomenon for power series

Bohr's phenomenon, first introduced by Harald Bohr in 1914, deals with the largest radius ρ_b, $0 < \rho_b < 1$, such that the inequality $\sum_{k=0}^{\infty} |a_k|\rho_b^k \leq 1$ holds whenever the inequality $|\sum_{k=0}^{\infty} a_k z^k| \leq 1$ holds for all $|z| < 1$. Bohr himself proved that $\rho_b \geq \frac{1}{6}$. The largest possible value $\rho_b = \frac{1}{3}$, which nowadays is known as *Bohr's radius*, was found independently by Friedrich Wilhelm Wiener, Marcel Riesz, and Issai Schur.

During its century long history, many authors contributed to this area of research, which now includes topics in several complex variables as well as topics in the theory of analytic functions in more general domains (different from the unit disk). Important contributions were done by Lev Aizenberg, Enrico Bombieri, Harold Boas, Dima Khavinson, Yusuf Abu Muhanna, Rosihan Ali, and others.

In this talk, I will discuss recent results published in the paper "*A note on Bohr's phenomenon for power series*" (2017) by Rosihan Ali, Roger Barnard and myself. In particular, I will discuss Bohr's phenomenon for the classes of even and odd analytic functions and for alternating series. Also, we will discuss Bohr's phenomenon for the class of analytic functions from the unit disk into the wedge domain $W_\alpha = \{w : |\arg w| < \pi\alpha/2\}$, $1 \leq \alpha \leq 2$. In particular, we find Bohr's radius for this class. Interestingly enough, our proof of the theorem on the Bohr's radius for this class depends heavily on the inequalities for Taylor coefficients of certain binomials obtained by Dov Aharonov and Shmuel Friedland.

Finally, I will mention a recent *arXiv* paper by Ilgiz Kayumov and Saminathan Ponnusamy, where the authors resolved a problem on the Bohr's radius for odd functions, which was posed in the paper by R. Ali, R. Barnard and A. Solynin mentioned above.

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An application of the Loewner theory to trivial Beltrami coefficients

A measurable function μ on the unit disk $\mathbb{D}$ of the complex plane with $\|\mu\|_\infty < 1$ is sometimes called a Beltrami coefficient. We say that μ is trivial if it is the complex dilatation $f_{\bar{z}}/f_z$ of a quasiconformal automorphism f of $\mathbb{D}$ satisfying the trivial boundary condition $f(z) = z,\ |z| = 1$. Since it is not easy to solve the Beltrami equation explicitly, to detect triviality of a given Beltrami coefficient is a hard problem, in general. In this talk, we offer a sufficient condition for a Beltrami coefficient to be trivial. Our proof is based on Betker's result on Löwner chains [1]. The present talk is based on the preprint [2].

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Some questions in Fourier analysis

The following statement generalizes the Euler-Maclaurin theorem to which it amounts when $x = 0$.

Theorem 1. *Let $n \in \mathbb{Z}$ and let for an integer $r \geq 0$ a function f and its derivative $f^{(r)}$ be of bounded variation on $[n, +\infty)$. Let also*

$\lim f^{(\nu)}(x) = 0$ *as* $x \to +\infty$ *for all* $\nu \in [0, r]$. *Then for* $0 < |x| \le \pi$

$$\sum_{k=n}^{\infty} f(k)e^{ikx} = \int_n^{\infty} f(u)e^{iux}du + 2^{-1}f(n)e^{inx}$$
$$+ e^{inx}\sum_{p=0}^{r-1} \frac{(-i)^{p+1}}{p!} h^{(p)}(x) f^{(p)}(n) + \theta\pi^{-r} V_n^{\infty}(f^{(r)}),$$

where $h(x) = x^{-1} - 2^{-1}\cot(x/2)$, *and* $|\theta| \le 3$.

The next theorem slightly strengthens the known McGehee, Pigno and Smith inequality of 1981.

Theorem 2. *There exists a constant* $c > 0$ *such that for any* $\{c_k\}_{k=1}^{\infty}$ *and* $\{n_k\}_{k=1}^{\infty}$, *where* $n_{k+1} > n_k$ *and* $n_k \in \mathbb{N}$, *we have*

$$\int_{-\pi}^{\pi} \left| \sum_{k=1}^{\infty} c_k e^{in_k x} \right| dx \ge c \sum_{s=1}^{\infty} \left(\sum_{2^{s-1} \le \nu < 2^s} \frac{|c_\nu|^2}{\nu} \right)^{1/2}$$

(the left-hand side is the $L(\mathbb{T})$ *norm of a function with the given Fourier series).*

This theorem also strengthens the classical Hardy inequality ($m_k = k$).

Theorem 3. *If for all* $x \in \mathbb{R}^m$ *we have*

$$f(x) = \int_{|x_j| \le |y_j| < \infty, 1 \le j \le m} g(y)dy$$

and

$$\int_{\mathbb{R}^m} esssup_{|x_j| \ge |y_j|} |g(x)| dy < \infty,$$

then f *is the Fourier transform of some function from* $L(\mathbb{R}^m)$.

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Quasiconformal geometry of Neumann eigenvalues in Ahlfors domains

We study estimates of integrals of derivatives of conformal mappings $\varphi : \mathbb{D} \to \Omega$ of the unit disc $\mathbb{D} \subset \mathbb{C}$ onto bounded domains Ω that satisfy the Ahlfors condition.These estimates of conformal derivatives permit us to obtain sharp Sobolev-Poincaré inequalities and by the Rayleigh quotient we have spectral estimates of the Neumann-Laplace operator in non-Lipschitz domains (quasidiscs) in terms of the quasiconformal geometry of the domains. As an example the lower estimates of the first non-trivial eigenvalues of the Neumann-Laplace operator in some fractal type domains (snowflakes) were obtained.

(Joint with Vladimir Gol'dshtein and Valerii Pchelintsev).

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Metric and quasiconformal mappings

This talk gives an overview of my recent research interests, connected with the theory of quasiconformal (qc) and quasiregular (qr) mappings in the Euclidean space R^n , $n \geq 2$. When the important parameter K , the maximal dilatation of a mapping, tends to unity, we get these classical maps, conformal maps and analytic functions as the limiting case $K = 1$. The talk will discuss the distortion theory of these mappings, i.e. how qc and qr maps transform distances between points. Some novel metrics are used for this purpose [1, 2, 3]. The talk is based on joint work with several coauthors. Also some open problems are mentioned.

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The Beltrami equation and related classes of mappings

In the talk I'll present results published jointly with Uri Srebro, Olli Martio and Vladimir Ryazanov.

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How Can Singularity Theory help in Reconstruction of Sparse Signals?

We provide an overview of some recent results on the “geometry of error amplification? in reconstructing of “sparse signals? from noisy Fourier or moments measurements. Main example is “spike-trains?, i.e. finite linear combinations of shifted delta-functions. This problem leads to a so-called “Prony system? of non-linear algebraic equations. In situations where the nodes collide (or near-collide), the Prony system encounters highly degenerated singularities. Still, some known and new tools in lines of Singularity Theory allow for a pretty accurate description of the effect of collision singularities on the reconstruction accuracy. In particular, error amplification turns out to be governed by the “Prony foliations? S_q, whose leaves are “equi-moment surfaces? in the signal parameter space. We provide some results on the geometry of error amplification, as relates to the the Prony singularities, Prony leaves, and their explicit parametrization.