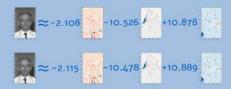
Face Recognition By Independent Component Analysis

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The Goal:

The project's goal is to propose a way to implement facial recognition using a form of Blind Source Separation called:

Independent component analysis (ICA).



As shown, the representation of both images with respect to the ICA basis is similar – A match was found.

Overview

We have our sample vactor x, ICA will find a matrix A and a source vector s such that x=As and s are statistically independent.

Step One: Preprocessing

The first step in ICA is the preprocessing step. This step is divided to 2 stages:

Centering and Whitening.

Preprocessing is done to speed up the convergence of the ICA iterative process and increase numerical precision.

Centering is the process of removing the mean from our samples: $\tilde{x}=x-E(x)=As-E(As)=A(s-E(s))=A\tilde{s}$

This process yields a new ICA problem which is easier, since $E(\tilde{x}) = E(\tilde{s}) = 0$

In effect, this is equivalent to moving our samples to the origin:



Another result of centering is that we can now assume that $Cov(\tilde{s})=I$, this is because $Cov(\tilde{s})=E((\tilde{s}-E(\tilde{s})))(\tilde{s}-E(\tilde{s}))^T)=E((\tilde{s}-0)(\tilde{s}-0)^T)=E(\tilde{s}\tilde{s}^T)=I$

Whitening is the process of restoring the shape of the samples to the shape of the original sources.



Let $Cov(x) = VDV^T$. If we define $Q := D^{+k}V^T$, we observe $Cov(Qx) = E(Qxx^TQ^T) = QCov(x)Q^T = D^{+k}V^T$ (VDV^T) $VD^{-k} = I$. The result is that Qx = QAs is a simpler than our original problem since Cov(Qx) = I and QA is orthogonal, thus Qx is merely rotated S.

Step Two: Source Separation

After the preprocessing phase, our samples x are uncorrelated, like our source vector s, but unlike s, they are not independent.

Our goal is to find an orthogonal matrix W such that y=Wx are independent. This matrix W will rotate x back to the original x.

The mutual information of y is $MI(y) = \int_{\Omega} P(y_{i_1}...y_{i_n}) log(\frac{P(y_{i_1}...y_{i_n})}{P(y_{i_1})...P(y_{i_n})}) d\vec{y}$, this function is not negative and is 0 if and only if y are independent.

Thus we want to find W such that MI(y)=MI(Wx) is minimal. From Central Limit Theorem this can be shown to be equivalent to maximizing the kurtosis: $k(w,x)=k(y_i)=E(y_i^+)-3$. Using Newton's method we can find w_i that maximizes $k(w_i,x)$ under the constraint $|w_i|=1$. In a similar fashion we can find W if we add the orthogonality constraints, and we fill our W matrix row by row in this manner, allowing us to find S=Wx.

Step Three: Facial Recognition

Let $x_{ij}x_{ij}$, x_{ij} be our images in row vector form. We insert them to matrix X such that x_i is the first row and so on.

ICA will find a source matrix S such that $S_{\mu}S_{\mu\nu}$, $S_{\mu\nu}$ are a basis that spans the face-space of our matrix X, and a coefficient matrix A such that X=AS.

Thus, the representation of X_i with respect to basis S is A_i .

Let b_i, b_i be the representation of x_i, x_i respectively in basis S.

We say that there is a match between x_i and x_i (meaning the person in image i is the same as the person in image j)

if the cosine norm $d^2(b_1,b_2) = |b_1| + |b_2| + |b_3| + |b$

Since they are normalized vectors, this is equivalent to saying the cosine of the angle between b, and b, is larger than 1-8.

